DEVELOPMENT OF DYNAMIC MODELS TO PREDICT LEAKAGES IN LONG DISTANCE NATURAL GAS TRANSMISSION PIPELINE

BY

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CERTIFICATION

This is to certify that this research work "***Development of Dynamic Models to Predict Leakages in Long Distance Natural Gas Transmission Pipeline***" was carried out by **Muoghalu Paul Ifeanyi** with registration number **20174080928** in partial fulfillment of the requirement for the award of Masters of Engineering (M.Eng) degree in chemical engineering, Federal University of Technology Owerri.

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DEDICATION

This work is dedicated to God Almighty for giving me the inspiration and grace to carry out this project successfully and also to my family for their support both spiritually and financially before and during this research work.

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I acknowledge that I am responsible for any flaw in this work and am subject to be corrected.

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NOMENCLATURES

A_R	Ratio of area of pipe to the leakage area
A	Cross sectional area of the pipeline
CFD	Computational fluid dynamics
cP	Centipoise
F	Fanning frictional factor
LDS	Leak detection system
MMSCFD	Million standard cubic feet per day
Q	Volumetric flowrate, bbl/d , ft^3/s, m^3/s
□□□	Inflow flowrate
□□□□	outflow flowrate
□□□□□	leakage flowrate
R	Ratio of inflow volumetric flowrate to the leakage flowrate
RTTM	Real time transient model
V	Fluid velocity, ft/s, m/s
□□□	Inlet/departure pressure of natural gas: psi, pa
□□□□	Exit or arrival pressure of natural gas: psi, pa
ΔP	Pressure drop,
ΔP_L	Total pressure drop due to friction across the pipe length;
ΔP_X	Total pressure drop due to friction on length X of the pipeline
ΔP_1	Pressure drop upstream of the leakage point X,
ΔP_2	Pressure drop downstream of the leakage point X,
ΔP_3	Additional pressure drop due to leakage at point X.
μ	Fluid dynamic viscosity, Pa.s
ρ	Fluid density, lb/ft^3, Kg/m^3
3LPE	Three layer polyethylene coating

ABSTARCT

A mathematical model based simulation of pipeline flow is developed and compared with result from simulation of field data to determine if the model is capable of detecting a leak in a pipeline. This research work present the development of mathematical based model for the prediction of leakages in pipelines conveying natural gas as well as collection of transient data for the testing and validation of this model. The model developed thus is given by $\square\square_{\square} = 1 + (1 - \square/\square)(\square)(\frac{1}{\square^2} - 1)(147.4993 - 4.2624\ \square/\square)$ and is developed by extending Fanning Equation due to pressure drop to be able to calculate the pressure drop along the length of natural gas transmission pipeline due to leakage. The characteristic changes in the flow mechanics and thermodynamics properties along the length of a pipeline were used in detecting the presence of leakages. Measurement of the pressure, flow and temperature data at both the upstream and downstream ends of the pipeline are used in developing the equations to govern the system in detecting the leak. The method was tested by obtaining field data from an existing pipeline infrastructure and subsequently simulating the pipeline in order to obtain the flow rate, pressure and temperature along the length of the pipeline; The 24 inch, 50KM export pipeline of AMENAM-KPONO field from AMP2 to Bonny in Niger Delta area of Nigeria was simulated using ASPEN HYSYS and the pressure profile along the length of the pipeline was obtained and compared with the pressure profile obtained by the solution of the mathematical model developed. The results of the developed model show that the approach is capable of theoretically determining the presence of leak in a pipeline.

KEY WORDS: Pipeline, Natural gas, Leak detection, Mathematical model, Pressure drop, Leakage, Flow, Temperature, Length, Flow rate.

CHAPTER ONE

INTRODUCTION

1.1 Background Information

Natural gas, a very important sources of energy is been produced from oil or gas reservoirs. Even in intermediate processing of these hydrocarbons until they are present in useable form, there is requirement for at least one or two unit operations. These operations will require connections with one another through the help of pipelines. Pipelines are therefore media required for the movement of natural gas and crude oil from reservoir, wellbore and other stations to be delivered to destination point such as separator, storage tanks, refinery, gas power plants and other points where they are required. Over time in operation, these pipelines due to ageing, corrosion and wear, design faults, operation outside design limit or deliberate damage in act of vandalism can suffer from leak defects. While pipelines are designed and constructed to maintain a high level of integrity, leakage occurrence is inevitable in a pipeline network during its lifetime. Leakages from pipeline systems have the potential to result in significant environmental damages and economic losses. As a result of the vast mileage of pipelines in the transportation of natural gases, it is important that dependable leak detection systems are used to promptly identify when and possibly where a leak has occurred so that appropriate response actions are carried out quickly. The promptness of these actions can help reduce the effects of accidents or incidents to the public, environment, and facilities.

Many new national and international pipelines are being planned and constructed per year. With the wide application of pipelines, a variety of failures of pipeline operation from water pipe rupture, gas pipe explosion to oil leakage have been reported every year. According to data available on Shell Nigeria website, for the first four months of 2019, an estimated 3200 bbl of crude have been lost by Shell due to pipeline leakages (http://www.shell.com.ng). For pipeline utility companies, pipeline leakage does not only mean a loss of product, but also high fines if the environment is impacted upon in the event of the leakage. Therefore, accurate leak detection methods that enable a quick response to pipeline failure are necessary to reduce the loss of valuable materials and to minimize the environmental damage. To address these issues, numerous research studies have developed sensing technologies, dynamic models and hydraulic simulations for detecting leakages in pipeline networks.

Leak detection techniques can range from simple visual line walking or flying over and direct mechanical drilling to sophisticated mathematical model-based techniques. Each of these methods has its merits and demerits for detecting and locating pipeline leaks effectively. Because leakage in oil and gas pipelines causes much higher financial losses and environmental damage compared with the leakage in water pipelines, most leak detection methods have been developed for oil and gas pipelines. In the review by Ghafurian in 1999, current leak detection methods were classified into three groups: simple leak detection systems, pig based monitoring systems and computer based monitoring system.

1.2 Problem Statement

The general purpose of a natural gas transmission pipeline is to deliver an appropriate quantity of natural gas, at adequate pressure, to all demand locations in the system. This cannot be achieved if there is an undetected leak on the pipeline. Over the years, there has been series of devastating effects of pipeline leakage ranging from minor lost time injuries, ill health, loss of containment, damage to property, casualties to major top events such as fire/explosions, loss of lives and property, multiple fatalities and deaths in the oil and gas industry. Generally, pipeline leakage are as a result of ageing, corrosion and wear, design faults, operations outside design limit or deliberate damage in act of vandalism. The result of this is the spillage of oil and natural gas that causes environmental pollution, economic loses, and when met with ignition sources causes fire explosions which could results to death and other multiple fatalities. This calls for a dependable leak detection system that will enable leakages to be detected on time.

1.3 Objectives

The main objective of this study is to develop a dynamic model capable of predicting the presence of leak in long distance natural gas transmission pipeline. In order to achieve this main objective, the following sub-objectives to be achieved include:

i. Collection of field data of an existing natural gas transmission pipeline for the simulation of flow parameters including temperature, pressure, density and viscosity along the length of the pipeline.
ii. Development of a transient model-based leak detection and localization model to simulate leak detection in natural gas transmission pipeline using a new approach based on pressure change (dynamic model).
iii. Validation of the mathematical model by comparing the result of the pressure drop obtained by the simulation of field data with the result of the pressure drop obtained by the model.

1.4 Justification of Study

Pipeline leaks over the years have become frequent problems to the producers and transporters of these hydrocarbons and failure to detect it can result in loss of life and facilities, loss of product and valuable time, expensive environmental cleanup and possible fines and legal suits from inhabitants. In Nigeria for instance, poor Leak Detection Systems (LDS) in the Niger Delta area have led to massive destruction to agricultural lands, source of drinking water and general destruction of the environment. Crude oil and natural gas loss due to pipeline leakages also have a negative impact on the economy of Nigeria since natural gas and crude oil is the mainstay of the economy in terms of revenue accrued to both the government and the oil companies. It is therefore of importance that an efficient and dependable leak detection (and localization) system be put in place in order to minimize the negative effects as a result of pipeline leakages for the producers, transporters and the oil producing communities.

1.5 Scope of Study

The scope of work of these research activities consists of:

Mathematical model development; in this research, a natural gas pipeline model for frictional pressure drop estimation would be developed. The natural gas pipeline model is basically pipeline network model, which simulates the natural gas flow in pipeline network and predict the pressure distribution along the pipe without and with leaking conditions, at a certain flow condition.

Collecting data from field; data from an existing natural gas transmission pipeline would be obtained and used for simulating a pipeline flow condition in order to obtain pressure drop along the length of the pipeline

The model validation will be conducted by comparing the result of the pressure drop of the field data and the pressure drop obtained by the developed model.

CHAPTER TWO

LITERATURE REVIEW

2.1 Basis of Pipeline Leakage and Detection

A leak is an unwanted opening through which fluid accidentally escapes from its container other than the normal outlet. A leakage generally is all fluid loss, either internally or to surrounding environment, which is not used for proposed operation. In this context, natural gas fails to get to the point of it usage. When pipeline leak occurs it presents economic, health and environmental degradation problems. These include: loss of valuable product, cost of cleaning up, loss of service and repair expenses, loss of man-hours, increase in hazard to life and environment. As a result of the potential dangers associated with leakages in pipelines, review of causes and methods of detection, location and control has received a continued attention over the last two decades. These activities will continue until a more reliable and robust technique is found to efficiently solve the problems of leaks in pipelines. It is expected that the preferred robust technique should be able to fulfill the three-fold objectives of detecting, locating and controlling leaks in pipelines with low cost implication.

2.2 Causes of Pipeline Leakages

There are several direct or indirect causes of leakages in long distance natural gas transmission pipelines, being the most relevant are pressure, infrastructure’s age, improper design and construction, soil characteristics, act of pipeline vandalism and frequent traffic loading. They are outlined below:

High Pressure Surge

Pressure can cause leakage as well as increase leakage rates. Firstly, pressure is the physical force applied perpendicular to the surface of an object per unit area over which that force is distributed. The continuous start-up and shut-down of pumps and boosters generate high-pressure surges. The continuous or frequent overloading of the pipes weakens pipe-fittings. A system subjected to higher pressures is much more likely to suffer accidents and ruptures. Secondly, the higher the pressure is, the greater the leakage rates are. The effect of average pressure on distributed leakage is much higher than that predicted by the classic orifice discharge law (Ramos et al., 2010).

Age of the Pipeline

Increasing pipe infrastructure age results in higher rate of leakages and burst frequency. Generally, the lifetime of the pipeline depends strongly on the pipe-material used, the method of construction, the average pressure and the operation mode. The ageing process cannot be controlled (unless by the rehabilitation of the infrastructure), and is one of the major factors causing increase leakage.

Improper Design and Construction

Oftentimes, pipe materials and pressure classes are specified according to the internal operating or extreme pressures, and not to the radial compression of the pipe due to external loading. This can lead to the pipe failure (crack). The improper construction of the pipeline support system may result in differential settlements of the pipe, and also the loss of water tightness which result in the increase in the chance of leakage.

Soil Characteristics

For underground pipelines, the characteristics nature of the surrounding soil is an important factor that leads to the occurrence of leaks and ruptures. Firstly, rain infiltration into the soil or the percolation rate of the soil tends to affect the strength of the pipeline. This may lead to differential settlement, not always compensated by the pipeline flexibility, which will result in pipe failure. Some soils may increase the external corrosion of metal pipes.

Frequent Traffic Loading

Regular and heavy traffic loading over buried pipes, mainly in urban areas, are another cause of high leakage rates. This is one of the reasons why many pipeline logistic companies have been increasingly using cast iron in their systems (e.g. EPAL, Lisbon).

Pipeline Vandalism

Pipeline vandalism is the deliberate act of damaging petroleum pipelines with the singular aim of stealing the content of the pipeline. In the Nigeria oil and gas industry, the effects of pipeline vandalism include huge economic losses from pipeline and plant shutdown, environmental pollution, fire outbreaks usually resulting in loss of lives and properties. Pipeline vandalism also results in scarcity and shortage of petroleum products which can lead to decrease in in supply of petroleum products to industries that needs them such as power plants resulting in reduced power generation. (Udofia & Joel, 2012)

2.3 Leak Detection Techniques

There are currently different available commercial leak detection techniques available for use ranging from simple physical inspection to acoustic methods. These largely depend on a leak's local properties, which entail interruption to pipe material integrity, material release and the emission of a characteristic noise or the manifestation of some other signal. Perhaps the oldest, albeit largely unsystematic, leak detection approach is visual observation. Ponding at the ground surface and anomalous vegetation growth can be indications of a pipeline leak (Colombo et al., 2009) .

The common framework to evaluate the performance of LDS include; sensitivity, reliability, accuracy, and robustness.

Sensitivity is defined as a composite measure of the size of leak that a leak detection system is capable to detect as well as the time it takes until a leak is detected and an alarm is issued. By sensitivity, a good leak detection system should be able to detect the presence of leak irrespective of the leak size.

Reliability means that false alarms are avoided but that actual leaks are reliably detected. A system is considered to be reliable if it consistently detects actual leaks without generating incorrect declarations.

Accuracy refers mainly to the localization of leaks. For example, leak location information is indicated in units of length and accurate information in percent of the entire length of the pipeline, or instrumented segment. A good leak detection system should be able to identify the exact location of leak in a pipeline.

Robustness refers mainly to the measure of the leak detection system ability to continue to operate and provide useful information, even under less than ideal operating conditions such as sensor failure. Robust LDS typically are able to tolerate sensor failures using some kind of redundancy evaluation.

Generally, there are two broad classes of leak detection systems: Hardware based methods and software based methods (Murvay & Silea, 2012b). These two groups are also called external and Internal based leak detection systems.

The different classes of leak detection are shown in the figure 2.1

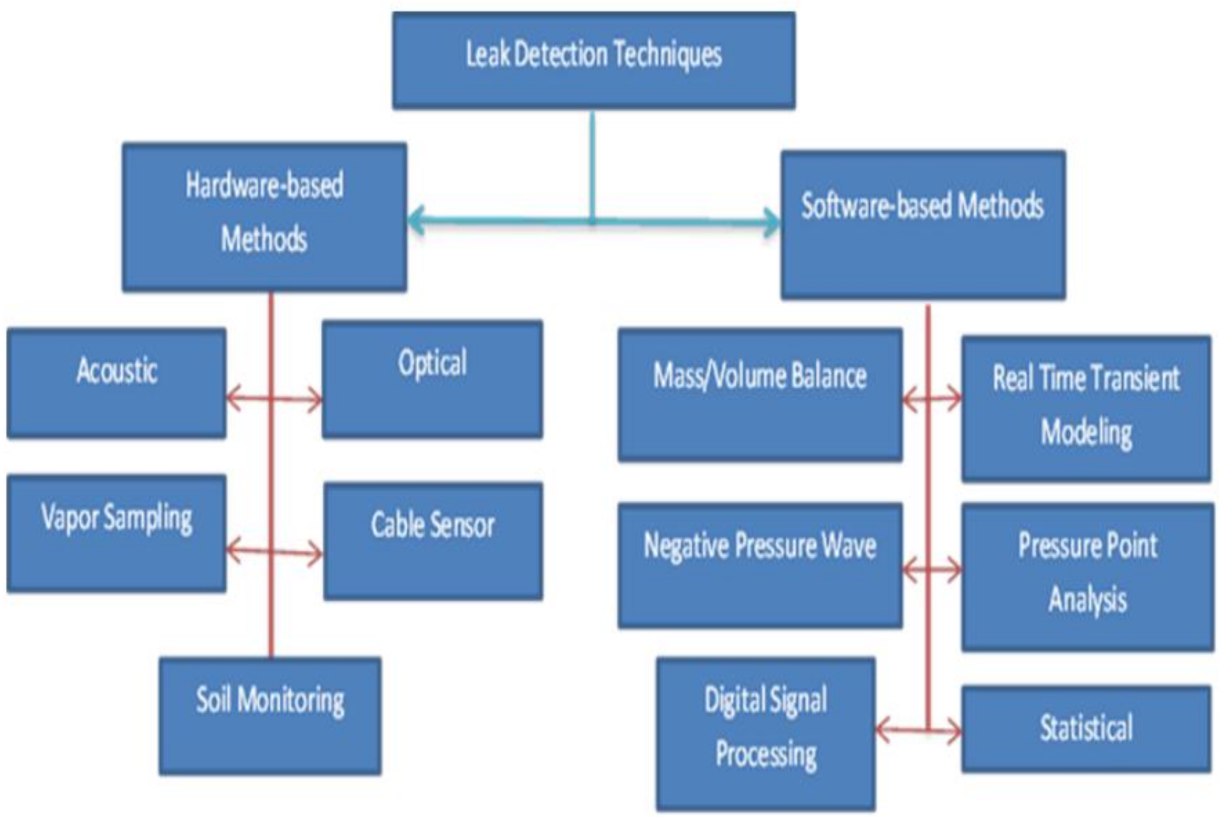

Figure 2.1 Classification of Leak Detection Techniques

2.3.1. Hardware Based Leak Detection Techniques

Hardware based leak detection methods otherwise known as external based method of leak detection and localization detect the present of leaks from outside the pipeline by visual observation or by using appropriate equipment. These kinds of techniques have a very good sensitivity to leaks and are very precise in finding the leak location. However, they are expensive and installation of their equipment is a very complex task. As a result, their usage are restricted to areas with high potential of risk such as near rivers or nature protection areas or in conditions which pipe is transferring a hazardous material (Murvay & Silea, 2012a). Hardware or Direct observation methods include the following:

2.3.1.1 Visual Observation and Surveillance

One of the simplest and most straightforward ways to detect leakages in pipeline is to patrol the pipeline if open to the atmosphere on a regular basis, making a visual analysis to ascertain the integrity of the pipeline. This involves flying over, driving or walking along the pipeline or using a specially trained tracking dog to search for abnormal fluid, odour, vegetation or humidity along the entire length of the pipeline. The effectiveness of this technique depends on the size of the leak and the inspection frequency. The leak size can only be roughly estimated as the correct actual leak size cannot be identified by mere visual analysis.

2.3.1.2. Acoustic Leak Detectors.

This leak detection system is based on the principle that when a liquid or gas escapes a leaking pipe, it produces an acoustic sound around the place of leakage which is usually different from the normal environment. Therefore, acoustic sensors such as acoustic stethoscope and ground microphones are installed outside the pipe to track and detect internal noise level and create a baseline with specific features. The self-similarity of this signal is continuously analyzed by acoustic sensors. If this signal features differs from the baseline, a leak alarm will be activated (Muggleton & Brennan, 2004) . The received signal is usually stronger near the actual point of leak on the pipeline and therefore would enable the location of the leak point. The sensors can be placed on the road surface or directly on the particular point such as fire hydrants as shown by Figure 2.2.

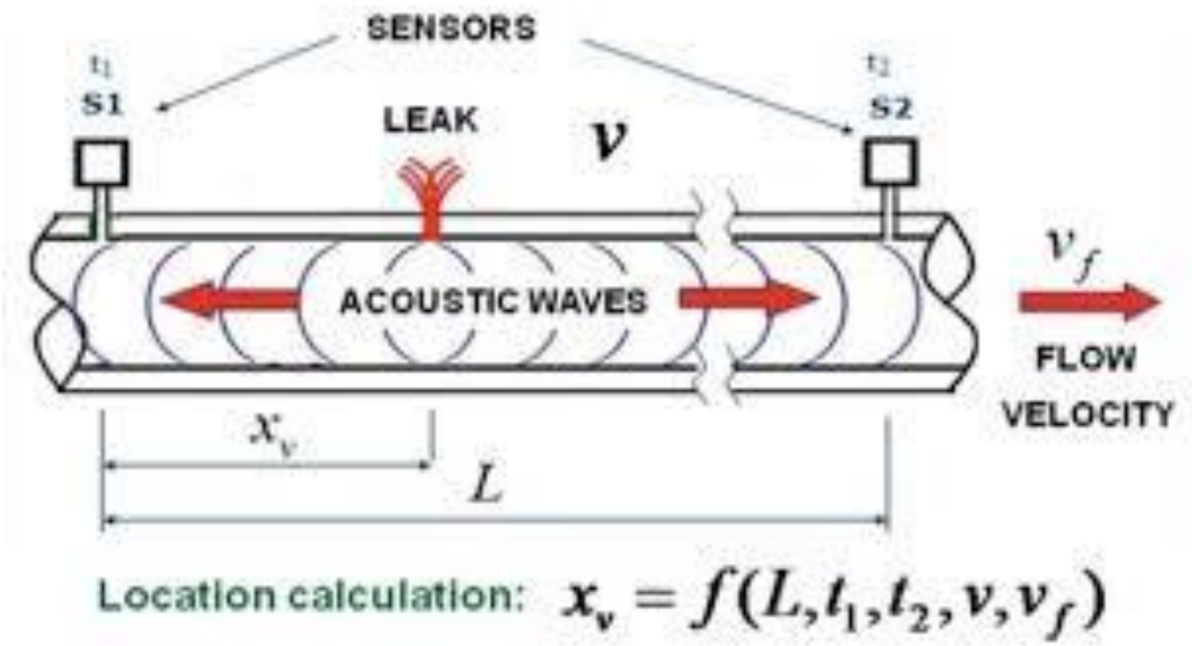

Figure 2.2 Leak Detection Using Acoustic Sensors

This method is highly accurate and reliable. However, the disadvantages of acoustic leak detection techniques include; various unwanted interference noise such as those from vehicle, wind, water and aircraft. There is also an important factor of limited application of this technique for leak detection because it is capital intensive; installing so many acoustic sensors which are needed for long pipelines inspection based on this technique is significantly expensive. There are also challenges involved in installing these acoustic sensors on subsea pipelines.

2.3.1.3 Fiber Optic Sensing Technologies

The fiber optic sensing leak detection systems involves the installation of a fiber optic cable along the length of the pipeline to act as a continuous, distributed sensor. It is based on the principle that when a leak occurs in pipeline the substance inside the pipeline gets in touch with fiber cable. So, the temperature of the cable changes due to this contact with the fluid in the pipeline. By measuring the temperature changes in fiber cable, leak could be detected.

One of the main benefits of using fiber optic leak detection method is its insensitivity to electromagnetic interference i.e a disturbance generated by an external source that affects an electrical circuit by electromagnetic induction, electrostatic coupling, or conduction. However, some disadvantages such as high costs and the stability over time limits the wide range application of this method for pipeline monitoring. Moreover, this method could not be applied for existing buried pipelines since it may need some excavation to reach the place where the optical cable should be installed for sensing purposes (Murvay & Silea, 2012b).

2.3.1.4 Vapor or Liquid Sensing Tubes.

In the vapor or liquid sensing tube based leak detection method, a tube which is in cable form is installed along the length of the pipeline. This tube is highly permeable to the fluid to be detected in the particular application. If a leak occurs, the fluid being transported through the pipeline comes into contact with the tube in the form of vapour, gas or dissolved in water. Some of the leaking fluid diffuses into the tube. After a period of time, the inside of the tube produces an accurate image of the substances surrounding the tube. Then, to assess the concentration distribution in the sensor tube, a column of air is forced into the tube past a detection unit at a constant speed. The detector unit at the end of the sensor tube is equipped with gas sensors. Every increase in gas concentration results in a pronounced "leak peak" an indication of the size of the leak (G. Geiger et al., 2003)

2.3.1.5 Chemical Based Systems

These leak detection techniques entail the use of chemical sensors of various types such as discrete chemical sensors, distributed chemical sensors, distributed fibre optic chemical sensors and chemical sensor-tube-based leak detection systems. In all of these methods, the pipeline is first inoculated with a small amount of tracer chemicals. These tracer chemicals usually used are chemicals which have properties that make for easy detection in the case of escape from the pipeline. Properties such as strong odour, non-reactive with needed fluid and reaction with air makes for a good tracer chemical. The tracer chemical will usually seep out of the pipeline in the event of leak. This is detected by dragging an instrument along the surface above the pipeline. The advantages of these methods include very low false alarms, and high sensitivity. However, the method is very expensive for use as a leakage detection method since tracer chemicals have to be continuously added to the natural gas along with cost of installing sensors along the pipeline as well as cleaning off the tracer chemicals from the fluid being transported.

2.3.2 Software Based Leak Detection Systems.

Software based leak detection systems otherwise known as internal method of leak detection make use of internal fluid measurement instruments to monitor the internal pipeline parameters associated with fluid flow in pipelines. These systems are used to continuously monitor the various properties of natural gas inside the pipeline such as pressure, flow rate, temperature, density, volume and other parameters which quantitatively characterize the released products. By combining the information conveyed from internal pipeline states, the discrepancy between two different sections of the pipeline can be used to determine the occurrence of leakage based on various methods, such as mass-volume balance, negative pressure waves, pressure point analysis, digital signal processing and dynamic modeling. Generally, the effectiveness and efficiency of the internal based methods depend on the correctness of the collected data, uncertainties associated with the system's characteristics and the operating conditions.

2.3.2.1 Mass-Volume Balance.

Mass-Volume Balance method is based on the principle of conservation of mass (Sheltami et al., 2016). The principle states that a fluid that enters a pipeline section remains inside the pipeline until it exits from the pipeline section. i.e, the quantity of mass that goes into the pipeline is same as the quantity of mass that comes out of the pipeline. In a normal cylindrical pipeline network, the inflow and outflow fluid can be metered. In the absence of leakage, the assumption is that the inflow and outflow measured at the two ends of the pipeline section must be balanced, so a discrepancy between the measured mass-volume flows at the two ends of the pipeline indicates the presence of a leakage. The inconsistency of the values in measurement can be determined using the principle of mass conservation given as follows (Gerhard Geiger, 2006a):

$$M_i(t) - M_0(t) = \frac{dM_l}{dt} \tag{2.1}$$

where $M_i(t)$ and $M_0(t)$ represent the mass flow rate at the inlet (i) and outlet (o), respectively. The mass stored across the pipeline length is denoted by M_L while L represents the length of the pipeline section. In a cylindrical pipeline system, the mass stored M_L for a pipeline of length L changes over time as a result of changes in fluid density (ρ) and cross-sectional area (A) satisfies Equation (2.2)

$$\frac{dM_l}{dt} = \frac{d}{dt}\int_0^l \rho(x)A(x)dx = \int_0^l < \rho(x)A(x)dx \tag{2.2}$$

where $\rho(x)A(x)dx$ represent the differential mass stored across the length of the pipeline (M_L) and ρ changes in accordance to the relation; $\rho(x)A(x)dx$ is measured with coordinate position *x*, $0 \leq x \leq L$. If ρ and *A* is assumed to be constant, *dM/dtL* = 0. Then Equation (2.2) becomes:

$$M_i(t) - M_0(t) = 0 \tag{2.3}$$

Similarly, according to (Dario, 2019), assuming $\rho_i(t) = \rho_i = \rho_l$ and $\rho_0(t) = \rho_0 = \rho_l$ are equal and constant for inlet and outlet mass flow, by introducing volumetric flow $\dot{V}$ with $\dot{M} = \rho V$ then;

$$\dot{V}_i(t) - \dot{V}_0(t) = 0 \tag{2.4}$$

The imbalance (R) between inlet and outlet volume can be estimated and compared as given in (2.5) and (2.6) respectively:

$$\dot{R}(t) = \dot{\dot{V}}_i(t) - \dot{V}_0(t) \tag{2.5}$$

$$R = \sum\nolimits_{\geq Rth\ if\ there\ is\ a\ leak}^{<Rth\ in\ the\ absence\ of\ leak} \tag{2.6}$$

where *Rth* is a threshold to evaluate the imbalance of the volume between inlet and outlet volume.

This approach is already commercialized and has been used in the oil pipeline industry (Golmohamadi, 2015). This method is very sensitive to pipeline instrumentation accuracy. The major demerit encountered with the mass balance leak detection method is the assumption of steady state. As a result of this assumption, the detection period has to be increased in order to prevent false alarms. As a result of this, the response time to the leak will be delayed, which is undesirable as a huge volume of the fluid must have been lost. For instance, a 1% leak needs about 60 minutes to be detected (Murvay & Silea, 2012b). Another compelling prejudice of mass balance leak detection method is that the actual location of the leak is unknown. Consequently, in the real employment of this method, other methods are required in conjunction with it after the leak has been detected to identify the location of the leak (Ghazali, 2012)

2.3.2.2. Real Time Transient Modeling (RTTM)

Using the increasing computing power of modern digital computers, it is possible to calculate in real time the profiles for flow v, pressure p and density ρ (or temperature T) along a pipeline (Gerhard Geiger, 2006b). This is called Real Time Transient Modeling (RTTM) system of leak detection and localization. This leak detection technique is based on pipe flow models which are designed using basic physical laws which the pipeline must follow (Fiedler, 2014). These physical laws which would be discussed further in subsequent sections include:

I. The principle of mass conservation, which enforces the conservation of mass within the segment of the pipeline.
II. The principle of momentum conservation, which describe the force balance on the fluid within a segment of the pipeline.
III. The principle of energy conservation, which enforces the conservation of energy within a segment of the pipeline.

These three physical principles or laws accurately describe the static and transient behavior of the flow in the pipeline. Using these equations, flow, pressure, temperature and density can be calculated and integrated in real time for each point along the pipeline (Fiedler, 2014)

The difference between the measured value and the estimated value of the flow is used to determine the presence of leaks. For constructing this model, flow, pressure and temperature measurements at both ends of the pipeline are necessary. Additionally, to design a reliable and dependable leak detection system with minimal false alarm the noise level should be continuously inspected to modify the model (Murvay & Silea, 2012b).

In as much as this leak detection system is expensive and complex, it is the most sensitive and most reliable. RTTM involves the computer simulation of pipeline conditions using advanced fluid mechanics and hydraulic modeling(Borener et al., 2000) . Calculations involving the momentum equation, the energy conservation equation and other numerous flow equations are typically used by the RTTM technique. RTTM software can predict the size and location of leaks by comparing the measured data for a segment of pipeline with the predicted modeled conditions. The analysis according to ADEC is done in a three-step process. First, the pressure-flow profile of the pipeline is calculated based on measurements at the pipeline segment inlet. Secondly, the pressure-flow profile is calculated based on measurements at the outlet. Third, the two profiles are overlapped and the location of the leak is identified as the point where these two profiles intersect. If the measured characteristics differ from the computer prediction, the RTTM system sends an alarm to the pipeline controller. The more instruments that are accurately transferring data into the model, the higher the accuracy and efficiency of the model. Worthy of note is the fact that these models rely on properly operating and calibrated instruments for optimum performance. Calibration errors can lead to false alarms or missed leaks, and the loss of a critical instrument could require system shutdown (Korbicz et al., 2004). The benefits RTTM provides over other leak detection methods are its ability to model all of the dynamic fluid characteristics (flow, pressure, temperature) and take into account the extensive configuration of physical pipeline characteristics (length, diameter, thickness, etc.), as well as product characteristics (density, viscosity, etc.) (API, 2002)

In a work by Billman, he used this approach for leak detection and localization. Their leak detection method is based on mathematical dynamic models, nonlinear adaptive state observers and a correlation detection technique. The method was tested on a 68 km gasoline pipeline. The results revealed that detection of leakage with size of 0.2% of inlet flow was feasible in 90 second. And also leak location could be estimated with accuracy of 0.9%. Verde (2021) used a linearized pipe flow model on an N-node model for leak detection. The only measurements were pressure and flow rate at both ends of the pipeline segments. Since the fluid model in the pipe is given by a set of partial differential equations, a finite dimension nonlinear model was acquired by having pressure measurements as input. The output of the model is the estimated flow rate at extremes. Verde & Visairo, (2001) in another paper extended this method to finding two simultaneous leakages in pipeline.

2.3.2.3. Negative Pressure Wave.

Negative pressure wave leak detection technique is based on the principle that when there is a leak in a pipeline, there is a pressure change and a decrease in flow speed which generates negative pressure wave at the position of the leak, and propagates the wave with a certain speed towards both upstream and downstream of the pipeline. Two pressure sensors are installed at the beginning station and the end station of the pipeline respectively. The negative pressure wave received by the two sensors can identify pipeline leak and furthermore locate the leak by calculating the time difference between the arrival times of the negative wave at each end (Hou & Zhang, 2013).

Similarly, Li et al., (2018) proposed detecting negative pressure waves with an intelligent machine learning technique using a moving windows least square support vector machine. The parameter of interest in the study centered on wave arrival time from the leak point to the end sides of the pipeline (i.e. t_1(s) and t_2(s)) using negative pressure wave signals and sensor positioning principles as shown in the Figure below.

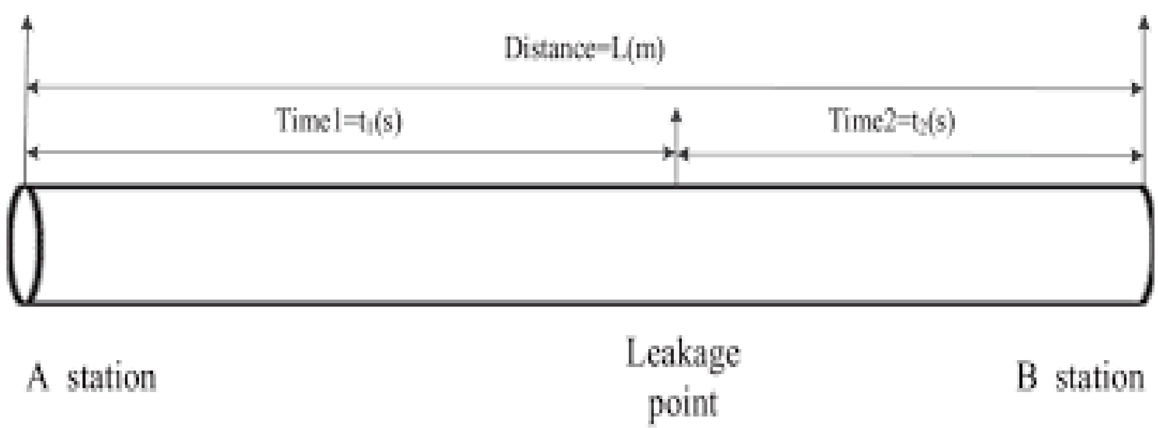

Figure 2.3 Negative pressure wave monitoring system

The location of an unknown leakage along the pipeline between stations (sensors) A and B shown in Fig 2.4 is determined using mathematical model (2.7) and (2.8) (Li, Q. et al. 2018).

$$t_1 - t_0 = \int_0^x \frac{1}{ax - v} dx \quad (2.7)$$

$$t_2 - t_0 = \int_x^l \frac{1}{ax+v}\,dx \qquad (2.8)$$

where X(m) is the distance from leak position to the sensor A, L(m) represents the distance from sensor A to B, aX (m/s) represents the propagation velocity of the negative pressure wave in the pipeline, t_0 is the time leak occur and V (m/s) is the liquid velocity. Assuming that the time difference in which the wave travelled from the first station to the end of the sensor is represented as $\Delta t = t_1 - t_2$, the above equations were reformulated and given as:

$$\Delta t = \frac{x}{a-v} - \frac{l-x}{a+v} \qquad (2.9)$$

where a is the velocity of negative pressure wave and X is the distance from a leak point to the pressure sensor A. When the fluid temperature, density and elasticity of the negative pressure wave propagation change, the fluid velocity will also change accordingly, due to this, the negative pressure wave velocity was formulated and given as:

$$a = \sqrt{\frac{\frac{k}{\rho}}{1+\left(\frac{k}{E}\right)\left(\frac{D}{e}\right)C}} \qquad (2.10)$$

where ρ (kg/m^3) is the liquid density, k (Pa) is the liquid bulk modulus of elasticity, E (Pa) is the modulus of elasticity, C is the correction factor related to the pipeline constraints, and e (m) is the pipeline thickness.

Da Silva et al.,(2005) proposed a leak detection methodology based on clustering and classification. They used a fuzzy system for classifying the running mode. Four pressure transducers were connected to a computer and leak simulated at different locations along the pipeline. The position was calculated by estimating the arrival time of the negative wave at the transducers and the knowledge of the wave speed. The drawback of the method was its incapability of finding leak location. But, this method still has not been exploited in long pipeline (Murvay & Silea, 2012a).

2.3.2.4. Pressure Point Analysis.

Pressure point analysis (PPA) method detects the occurrence of leaks by comparing the current pressure signal at a single point along the pipeline with a running statistical trend from previous pressure measurement taken over a period of time along the pipeline by pressure monitoring and flow monitoring devices (Ghazali, 2012). The principle of this method is based on the fact that there is always a pressure drop as a result of leak occurrence. Using an appropriate statistical analysis of most recent pressure measurements, a sudden change in statistic properties of pressure measurement such as their mean value is detected. If the mean of newer data is considerably smaller than the mean of older data, then a leak alarm is generated. This method is considered as one of the fastest ways of detecting the presence of leakage in a pipeline based on the fact that existence of leak always results in an immediate pressure drop at the leakage point (Arifin et al., 2018) (Murvay & Silea, 2012a). It may require high sensitive resolution but not necessarily very precise instrumentation. So, the overall installation costs are not very high. Furthermore, this method is able to identify the occurrence of leaks, but not necessarily the presence of them. Since this method use of pressure drop as a leak signature, it can yield false alarms as the pressure drop is not unique to the leak event therefore, integrating this method with other techniques such as mass-volume balance improve its effectiveness.

2.3.2.5. Statistical Analysis

A statistical leak detection system uses advance statistical technique to analyze the flow rate, pressure and temperature measurements of a pipeline. This method is appropriate for complex pipe system as it can be monitored continuously for continual changes in the line and flow/pressure instruments. In addition, this technique could be used for leak localization. Using statistical analysis is also very easy and applicable into different pipeline systems (Murvay & Silea, 2012a). The main objective of this system is to minimize the rate of false alarm. It is also suitable for real time application and has been successfully tested in oil pipeline systems (Ghazali, 2012). The main disadvantage of statistical leak detection is that noise interferes in the statistical analyses, and some leaks were hidden in the noise which prevented them from being detected.

2.3.2.6. Digital Signal Processing.

Another method for leak detection is using digital signal processing techniques. The procedure of this method is that the response of the pipeline to a known input is measured over a period of time. Afterwards, this response is compared with the later measurements. Based on comparison of their signal's features like frequency response or wavelet transform coefficients a leak alarm could be generated. Similar to statistical methods this technique does not need a pipeline model. The problem associated with using this method for leak detection is only leak occurrence could be detected not leak presence unless the size of present leak increases considerably (Murvay & Silea, 2012a).

Pipeline leak detection using digital signal processing involves five steps as illustrated in Figure (2.3). The steps are as follows:

I. Data acquisition.
II. After data acquisition, the acquired data is pre-processed to filter the background noise for efficient feature extraction.
III. In the feature extraction step, various statistical, spectral and signal transform techniques are employed to extract relevant features to monitor the state of hydrocarbon fluid transport in the pipeline.
IV. The pattern of the extracted feature is compared with the known pre-set signal or previous features for decision making.
V. Leakage detection is achieved through the comparison of the pattern with the threshold.

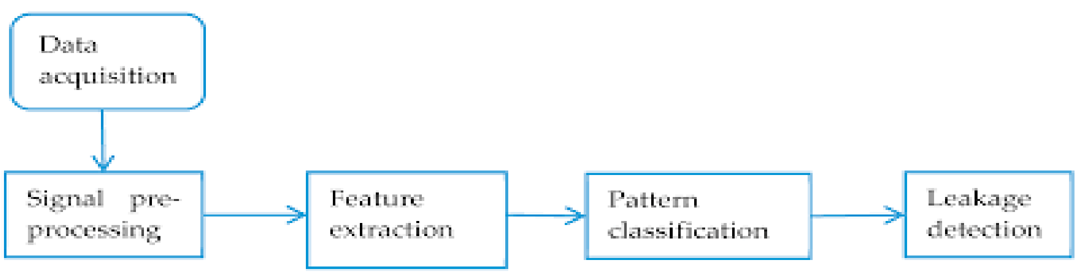

Figure 2.4 Steps in Digital Signal Processing

2.4 Mathematical Model Overview

A mathematical model is an abstract model that uses mathematical language and concept to describe the behavior of a system. Mathematical models are used primarily in natural sciences and engineering disciplines and may help to explain a system and to study the effects of different components and to make predictions about behavior.

Mathematical models can take many forms, including but not limited to dynamical systems, statistical systems or differential equations

In physical sciences, a traditional mathematical model contains most of the following elements:

I. Governing equation
II. Supplementary sub-models which include the defining equations and constitutive equation.
III. Assumptions and constraints which include initial and boundary conditions and classical constraints and kinematic equations.

Why do mathematical modeling?

Mathematical modeling has always been an important activity in science and engineering. The formulation of qualitative questions about an observed phenomenon as mathematical problems was the motivation for and an integral part of the development of mathematics from the very beginning. Although problem solving has been practiced for a very long time, the use of mathematics as a very effective tool in problem solving has gained prominence in the last 50 years, mainly due to rapid developments in computing. Computational power is particularly important in modeling chemical engineering systems, as the physical and chemical laws governing these processes are complex. Besides heat, mass, and momentum transfer, these processes may also include chemical reactions, reaction heat, adsorption, desorption, phase transition, multiphase flow, etc. This makes modeling challenging but also necessary to understand complex interactions.

All models are abstractions of real systems and processes. Nevertheless, they serve as tools for engineers and scientists to develop an understanding of important systems and processes using mathematical equations. In a chemical engineering context, mathematical modeling is a prerequisite for:

A. Design and scale-up of chemical engineering plant;
B. process control;
C. optimization;
D. mechanistic understanding;
E. evaluation/planning of experiments;
F. trouble shooting and diagnostics;
G. determining quantities that cannot be measured directly;
H. simulation instead of costly experiments in the development lab;
I. Feasibility studies to determine potential before building prototype equipment or devices.

2.4.1 Basic Pipeline Modeling Equation

In the modeling of the pipeline, the state of the pipeline at each time interval will be computed. The pipeline will be defined as a set of pressure, temperature, densities and flow that helps in describing the fluids being transported at all points and segments within the pipeline system. These quantities are found as a solution to the model equations that will be derived which describe the behavior of the pipeline. The basic fundamentals equations from which other thermodynamic equations on fluid mechanics are derived are the Continuity equation, the Momentum equation, the Energy equation.

In general, for the prediction of flow rate, pressure, temperature variation and other properties like density and viscosity of gas along pipelines, three equations of mass, momentum and energy balance are developed and solved simultaneously as shown generally below for a cylindrical pipeline at elevation angle θ from the horizontal line.

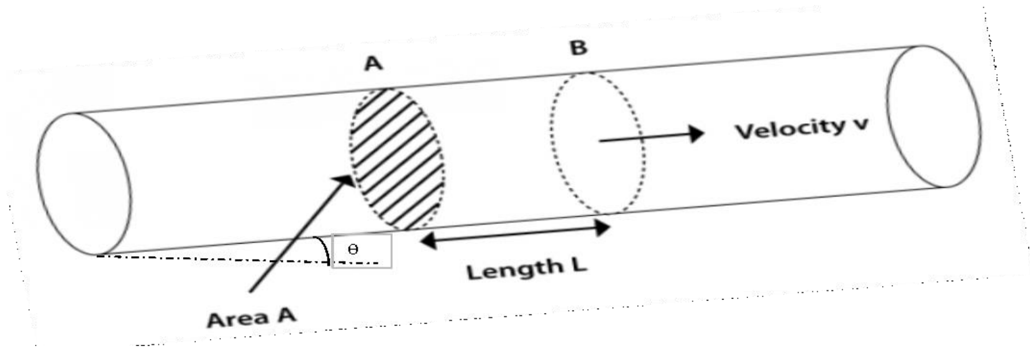

Figure 2.5 Cylindrical pipeline at elevation θ from the horizontal line.

2.4.1.1 Continuity Equation/Mass Balance Equation - It enforces the conservation of mass which states that the difference in mass flow in and out of the section of the pipeline is equal to the rate of change of mass within the section (Bird et al., 2006)

i. e Rate of Rate of Flow In = Accumulation + Rate of Flow Out

Mathematically, $\partial \frac{\rho A}{\delta t} + \partial \frac{\rho A V}{\delta x} = 0$ (2.11)

2.4.1.2 Momentum Equation – It describes the force balance on the fluid within a segment of the pipeline. Its major requirement is that any imbalance or unbalanced forces result in acceleration of the fluid element. From Navier Stokes equations, Cauchy Momentum Equations, the differential equation for the conservation of momentum in one-dimensional flow is (*Perrys-Chemical-Engineering-Handbook1*, n.d.)

$$\rho\left(\frac{\partial v}{\partial t} + V\frac{\partial v}{\partial x}\right) = \frac{-\partial P sin\theta}{\partial x} + \mu\left(\frac{\partial^2 v}{\partial x^2}\right) + \rho g \qquad (2.12)$$

Replace viscosity with Reynolds number $Re = \frac{\rho V L}{\mu} = \frac{Q D_H}{\nu A}$ (2.13)

$$\rho\frac{\partial v}{\partial t} + \rho v\frac{\partial v}{\partial x} + \frac{\partial p}{\partial x} + \rho g\frac{\partial H}{\partial x} + \rho\frac{f V^2}{2D} = 0 \qquad (2.14)$$

If Reynolds number Re < 2300, Flow is Laminar.

If Reynolds number Re > 4000, Flow is turbulent.

2.4.1.3 Energy Equation – The energy equation enforce the conservation of energy which states the rate of change of energy within a pipeline segment can be related equally to the difference in the energy flow into and out of the segment. Mathematically,

$$\rho \frac{\partial T}{\partial t} + \rho V \frac{\partial T}{\partial x} + \left(\frac{T}{c}, \frac{\partial p}{\partial T}, \frac{\partial v}{\partial x}\right) - \frac{\rho f v^2}{2c_D} + \frac{4U}{CD}\left(T - T_g\right) = 0 \tag{2.15}$$

In the above equations, the density of the real gas can be estimated as follows: $\rho = \frac{P}{ZRT}$

2.5 Review of Past Work on Mathematical Model-Based Leak Detection Method

Over the last forty years a lot of effort has been put into the special field of leak detection. Many different approaches have been taken and lately the most popular one has been software based where acquisition and advanced processing of measurements data along the pipeline helps identify and classify a leak. These methods often incorporate mass balances, momentum balances, pressure signatures of transitions from no-leak to leak and dynamic modeling. A number of pipeline leak detection models have been implemented on several pipeline systems. However, many methods are hedged in with their shortcomings, which are, long response time, and incidence of false alarm reporting, etc. (Abhulimen & Susu, 2004) some of the existing mathematical models are discussed below;

In a study, (Yousef, 2018) developed a computational fluid dynamic (CFD) model of a subsea leaking pipeline to predict the pressure and temperature profile round the pipe leak surroundings. (Yousef, 2018) develop a comprehensive model that would simulate the fluid flow through a leaked pipeline into the flowing water in one model. The model studies pipeline section with leak on the top. It considers the fluid inside the pipeline and fluid surrounding the pipeline and makes a combined simulation of the system. The developed model was used to perform studies to understand the impact of leaks on the surrounding water. The developed models are given by equations below:

$$\frac{\partial}{\partial x}(\rho V A c_v T dx) + \frac{\partial}{\partial x}\left(\frac{\rho V A P}{\rho} dx\right) + \frac{\partial}{\partial x}\left(\frac{\rho A V^2}{2} dx\right) + \frac{\partial}{\partial x}(\rho V A g z dx) - \frac{\partial}{\partial x}\left(\phi A \frac{\partial T}{\partial x} dx\right) +$$

$$K_L\left(T_{gas} - T_{amb}\right)dx + \frac{\partial}{\partial t}(\rho V A c_v T dx) + \frac{\partial}{\partial t}\left(\frac{\rho A V^2}{2} dx\right) + \frac{\partial}{\partial t}(\rho V A g z dx) = 0 \tag{2.16}$$

$$\frac{\partial(\rho\varepsilon)}{\partial t} + \partial\frac{(\rho\varepsilon V_i)}{\partial x_i} = \frac{\partial}{\partial x_j}\left(\frac{\mu eff}{\sigma_\varepsilon}\frac{\partial\varepsilon}{\partial x_j}\right) + C_{\varepsilon 1}\frac{\varepsilon}{k}(G_K + C_{\varepsilon 2}G_b) - C_{\varepsilon 2}\rho\frac{\varepsilon^2}{K} \qquad (2.17)$$

where ϕ is the thermal conductivity coefficient of fluid, *W/m K*, and K_L is the heat transfer coefficient, *W/m K.*

Equation (2.16) is the fluid hydrodynamic model showing in transient flow while equation (2.17) gives the dissipation rate of kinetic turbulent energy CFD model.

The hydrodynamic model of equation (2.16) can help to find the most critical conditions along the entire pipeline system and provides the boundary conditions for the computational fluid dynamic model of equation (2.17).

The developed model was simulated and checked for the same conditions with existing experimental findings by Ben-Mansour et al., (2012) to confirm the validity of this work. Results, shown in Figure (2.5) below, demonstrate a good match between the results of this study and the results reported in Ben-Mansour et al., (2012) and fall in the 5% of the error bars.

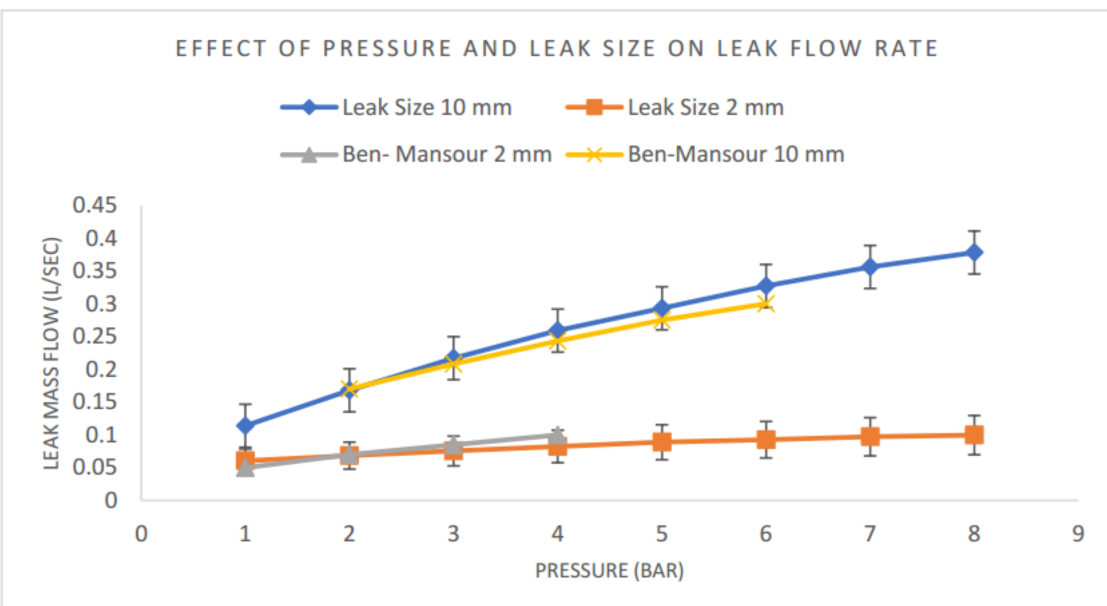

Figure 2.6 CFD Model Validation Using Ben-Mansour’s Work.

In another study, Chuka et al., (2016) described the fundamental and application of transient model-based leak detection and localization technique for crude oil pipelines. The dynamic parameters involved in this model such as pressure, flow and temperature were acquired by SCADA (Supervisory Control and Data Acquisition) system. Measurement of pressure, temperature and flow data at both the inlet and outlet of the pipeline were used in formulating the equations obtained from the inconsistency in the continuity and law of conservation of momentum equations. The characteristic changes in the flow mechanics and thermodynamics along a given length of pipeline were used in detecting, localizing and determining the flow rate of the leak.

The Leak flow rate M_l was obtained from the discrepancies between the Mass flow rates *in* and the mass flow rates *out*.

$$\dot{M}_l = x - y \tag{2.18}$$

where x is the discrepancy between the estimated mass flow rate *in (*calculated from the model) and the measured mass flow rate *in* (from the field data) and y is the discrepancy between the estimated mass flow rate *out* (calculated from the model) and the measured mass flow rate *out* (from the field data).

The leak located by the model is 11.88m behind the actual leak position as discovered during the pipeline leak remedial works and therefore, more work have to be done to improve the accuracy of the model in order to predict the exact leak location.

Abhulimen & Susu, (2007) developed a model for detecting leaks in complex pipeline network systems from the theory of Lyapunov stability theory which is used to describe the stability of a dynamic system by studying the eigenvalues of the system. The concept was applied to pipeline flow system to evolve leak detection criteria, such that if the eigenvalues of the Jacobean matrix of real pipeline flow system is less than -1, there is a leak in the system. Conversely, if the eigenvalues of the Jacobean matrix is greater than 1, there is a surge in the system. The stable point of equilibrium, where there is no surge or leak, is when the eigenvalues of the Jacobean matrix is equal to 1 in absolute terms.

The proposed model by Abhulimen and Susu is given by equation (2.19) below:

$$(1 + K_L)\frac{dv}{dt} + \frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\partial \tau}{\partial r} - g sin\theta = 0 \qquad (2.19)$$

where K_L accounts for a leak disturbance in the unsteady state equation of motion and continuity.

The model was applied to a pipeline network system of a Nigerian Oil and Gas Company that transports crude oil within and outside Nigeria. The plots of figure (2.6) and figure (2.7) below show that pressure measurements give a faster indication for leak detection than volume measurements, although volume measurements appear to respond faster for larger leak systems.

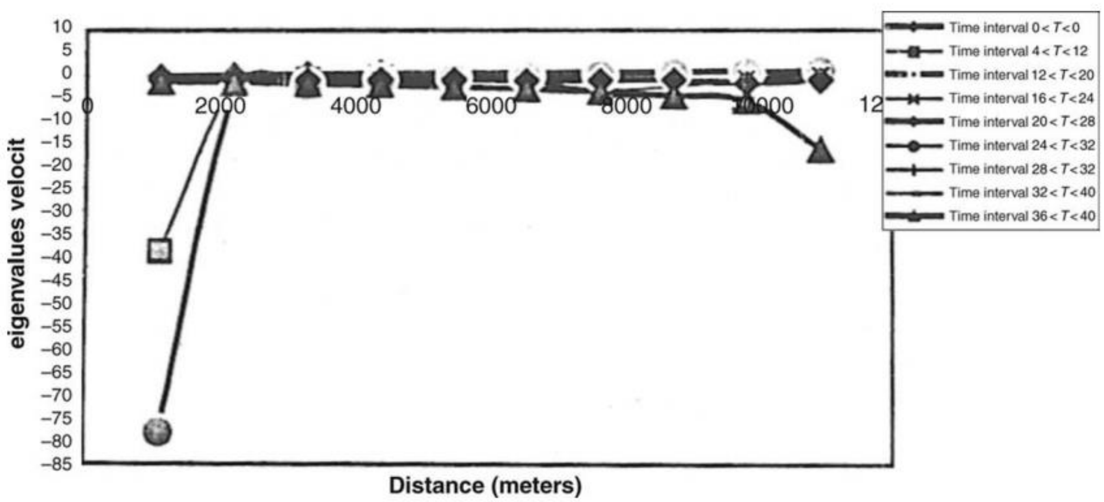

Figure 2.7 A Plot of eigenvalues of velocity in Real Time

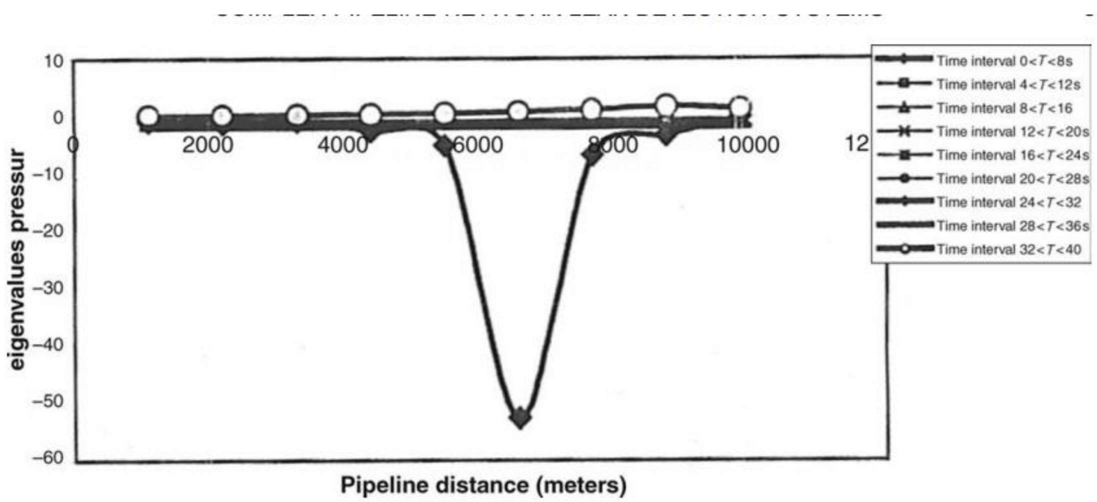

Figure 2.8 A Plot of eigenvalues with Distance in Real Time

Also, in a work by Mansy and Baghaladi (2012), they proposed a model for leak detection along a pipeline based on a one-dimensional flow analysis of an incompressible fluid flowing in laminar or turbulent regimes. The basis of the proposed model is given by equation (2.20)

$$X_c = \frac{(h_1-h_2)-k_2 L Q_2^2+\left({Q_1}/{Q_2}-1\right)(h_1-\lambda)}{k_1 Q_1^2 {Q_1}/{Q_2}+k_2 Q_2^2} \qquad (2.20)$$

Q_1 = the inlet discharge to the pipeline Q_2 = the discharge leaving the pipeline

h_1 = the static head at the inlet section h_2 = the static head at the outlet section

X = the leak location measured from the inlet section

And X_c is the calculated value for X.

The solution procedure was based on an iterative technique that led to the accurate prediction of the leak position. An experimental program was carried out to verify this model and Equation (2.20) was used to correlate the actual leak position from the experimental data. The experiment confirms the physical realism of the mathematical model since the plot of the predicted relative hole location (calculated using this model) against the relative actual hole location from experiments shows the model to be valid for various types of hole geometries since it's accuracy in calculating the leak location is estimated to be less than a few percentage for different hole geometry as shown in Figure (2.8)

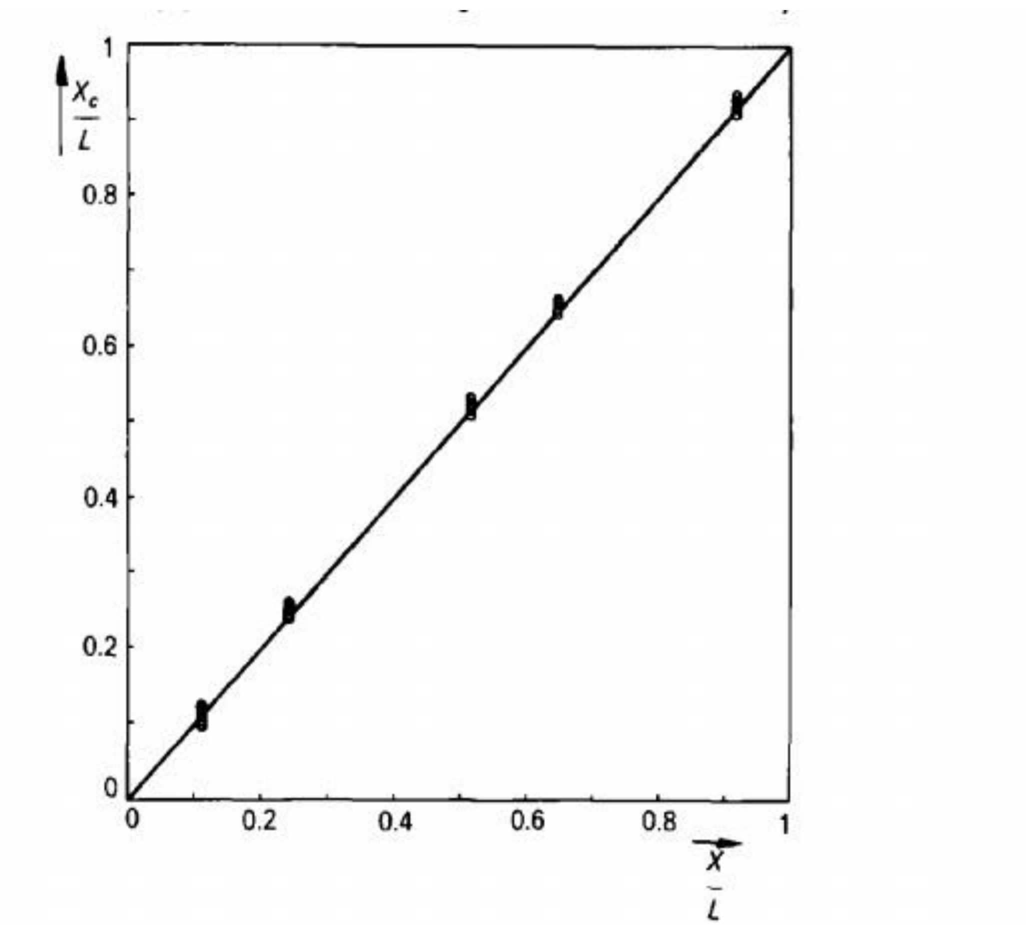

Figure 2.9 A Plot of the relative predicted leak location X_c/L versus the relative actual leak *location X/L*

Turner and Mudford in 1988 developed a method of detecting leak location in gas pipelines by using flow, pressure and temperature measurements in long distance natural gas pipelines. The method is based on an accurate simulation of the transient hydraulic behavior of the pipeline and makes use of the measurement data available. A computer program SIROLEAK simulation program in combination with Supervisory Control and Data Acquisition (SCADA) system was used in the simulation process. If the program detects a persistent mass loss in a pipeline section, then a leak alarm is triggered. The basic model developed by Turner and Mudford for the estimation of leak location is given by equation (2.21)

$$e_z^2(N_l) = \frac{e_s^2(N_l)(Z_d - Z_u)^2}{S_j^2}\left[1 + \left(\frac{s_j - 1}{s_j}\right)^2\right] \leq \frac{2e_s^2(N_l)(Z_d - Z_u)^2}{S_j^2} \tag{2.21}$$

The model was use to estimate leak location in pipeline of different leak sizes and compared with the actual values from the field, it shows that for the same pressure error trace with standard pressure error of 20 kPa. the pattern of fluctuation in the location estimates between the estimated values and actual values is almost identical, in that with increasing leak size comes more accuracy as shown in Figure (2.9) below:

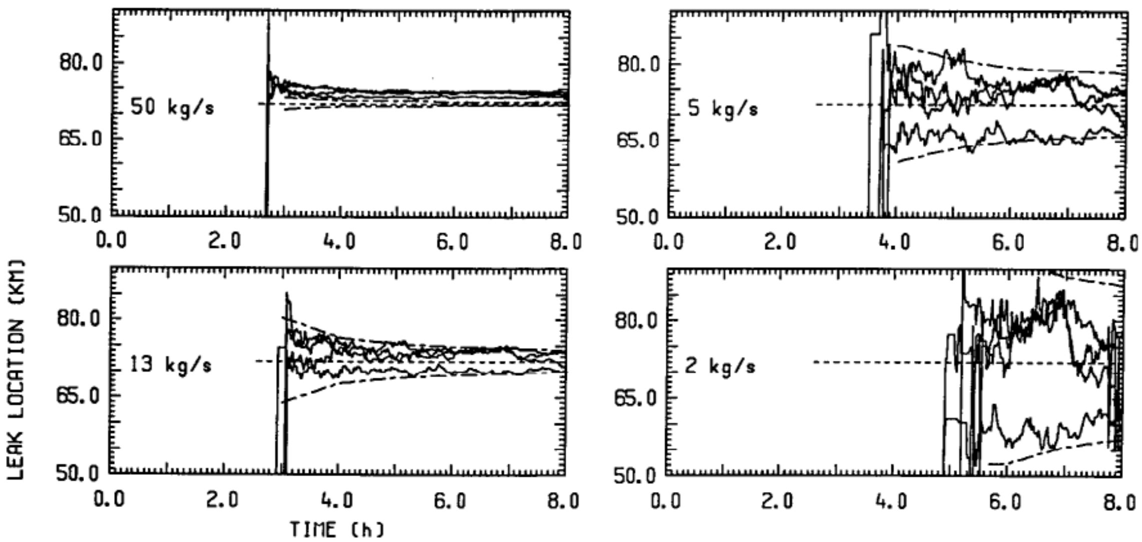

Figure 2.10 Estimated leak location for a range of pressure error trace histories. - SIROLEAK estimate; ----- true value; --- expected variation.

The evaluation of this model shows that equation (2.21) would not be suitable to accurately estimate the location of leak of small sizes since the variation between the estimated and the true value keep increasing as the leak size reduced.

Oyedeko & Balogun, (2015) used transient flow analysis to account for the imbalance in the continuity equation of equation (2.11) and the law of conservation of momentum of equation (2.14) in the development of model for leak detection in oil and gas pipeline. The pressure data for the upstream and downstream are fed as inputs into the model and the model will estimate a flow rate (without leak condition). Thus the estimated flow rate will be different from the measured flow rate at the pipeline ends when there is leak. The mass flow rate variations between the estimated and measured at both the upstream and downstream end of the pipelines is a function of the leak rate which will help in the diagnosis, location and quantification of a leak. At normal flow i.e. when there is no leak, the variations are zero

The model proposed by Oyedeko and Balogun involves the introduction of a leak term into equations (2.11) and (2.14) as follows:

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho A \upsilon)}{\partial x} + M_l \,.\, \delta(x - x_l) = 0 \tag{2.22}$$

$$\rho\frac{\partial v}{\partial t} + \left(\rho\frac{\partial v^2}{\partial x}\right) + \left(\frac{\partial P}{\partial x}\right) + \rho g\frac{\partial H}{\partial x} + \left(\frac{\rho f v^2}{2D}\right) + \rho V_l = 0 \tag{2.23}$$

where υ is the one-dimensional velocity, ρ is the density of the fluid, P is the static pressure, H is the elevation, f is the frictional factor, V_l is the leak velocity, x is the spatial space and t is the time.

The two equations (2.22 and 2.23) above is the set of one dimensional hyperbolic partial differential pipe flow equations that govern the fluid characteristics within the pipeline.

The simulation and modeling was developed using MATLAB. The evaluation of the model shows it is not good in a case of complex pipeline network and in the case of heavy data supply as shown in the MATLAB graphical plot.

Sukarno et al., (2007) proposed a model to simulate a pipeline and detect leakages in an oil transmission pipeline using a new approach based on artificial neural network (ANN). The ANN are computing systems inspired by the biological neural networks that constitute the human brain. Such systems learn to perform a given task by considering many similar examples as the task. In the case of leak detection purposes, the ANN model are trained to detect leakages in pipeline using actual existing leak data from the field which includes, flow rate, leak rate, leak location, various configuration of pipeline, intake and discharge pressure and other fluid properties. Using the prepared data the software is trained to determine the leak location and leak rate. After training, the artificial neural network would be able to predict the position of leak and the leak rate based on the information which has been fed into the system. By training the system about the condition of normal (without leaking conditions) and abnormal (with leaking conditions) pressure distribution data and flow rate data, then the leak location could be detected, when there is discrepancies between the trained flow rate and calculated flow rate.

If *i* and *j* representing intake point and discharge point, respectively, at a certain segment of pipe, once the pressures at a node are determined the flow rates Q_{ij} can be found from equation (2.24) below:

$$Q_{ij} = (P_i - P_j)\frac{\sqrt{\frac{\pi^2 D_{ij}^5}{8 f_{ij} \rho L_{ij}}}}{\sqrt{|P_i - P_j|}} \tag{2.24}$$

where f is the friction factor, the pipe has D in diameter and L in length, $P_i - P_j$ is the pressure drop from *i* to *j* and ρ is the oil density.

In the evaluation of this model, it was discovered that models developed based on ANN system can only detect leak similar to the actual leak data with which it was trained with. Therefore, it is not suitable for varying leak condition

2.6 Summary of Literature

Some of the existing models studied above indeed appeared to be useful for leak detection and isolation purposes; however, there is need for further work to be done in order to obtain a model which can result in more accurate leak detection and isolation schemes with lesser false alarm and more sensitive and accurate location capabilities especially with reference with the Nigerian peculiar conditions.

This work presents the fundamentals and application of real time transient model-based leak detection and localization technique for natural gas transmission pipeline. In this work a more accurate and precise flow model for the purpose of detection and location of such leaks in pipelines conveying natural gas with a better accuracy, sensitivity and reliability is developed.

2.7 ASPEN HYSYS Simulator

ASPEN-HYSYS is a chemical process simulator software currently developed by AspenTech used to mathematically model chemical process from unit operations to full chemical plants and refineries. Fig: 2.11 is a typical APSEN-HYSYS Screen.

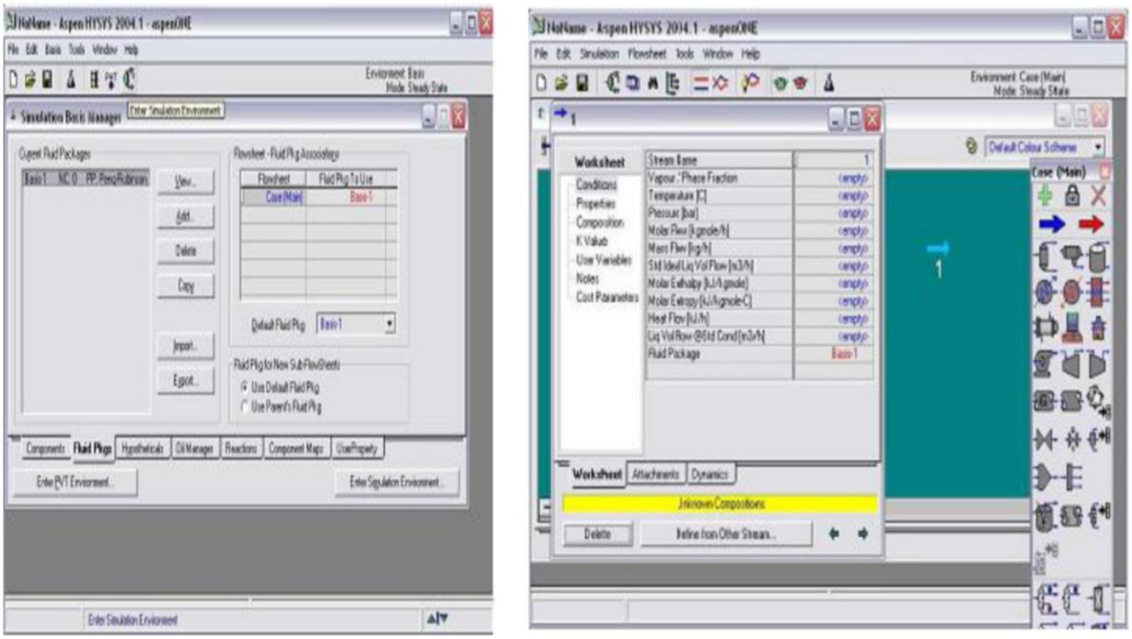

Fig: 2.11 A typical ASPEN HYSYS Screen

HYSYS is able to perform many of the core calculations of chemical engineering including those concerned with mass and energy balance, vapour-liquid equilibrium, heat and mass transfer, chemical kinetics and pressure drop. It is used extensively in industry and academia for steady-state and dynamic simulation, process design, performance modelling and optimization.

CHAPTER THREE

MATERIALS AND METHOD

This research work entails the development of predictive mathematical models for pipeline leakage detection by analyzing the flow parameters within the pipeline using a set of solution of differential equations of flow and other thermodynamic equations. The mathematical models which were developed using conservation laws (mass, momentum, energy) and thermodynamic principles, as well as Newton definition of viscosity were used to determine the property distributions in steady-state, uni-directional flow through the entire length of a natural gas transmission pipeline. The inconsistencies in the conservation laws and the thermodynamics measurement of pressure, temperature and flow data at both the inlet and outlet of the pipeline were used in formulating the predictive models for pipeline leakage detection.

3.1 MATERIALS

Natural gas is composed mainly of methane, ethane, propane or their mixtures. Molar compositions of three typical Nigeria natural gas mixtures are listed in Table 3.1 below (Obanijesu & Macaulay, 2009)

Table 3.1: Molar compositions of some Nigeria natural gas mixtures (mol %)

Chemical Compound	Kokori field study	Utoorgu gas plant	Sapele West field
Methane	68.42	90.19	68.14
Ethane	7.65	6.94	14.22
Propane	11.27	2.09	10.27
N-butane	4	0.361	3.23
I-butane	4.42	0.414	2.38
N-pentane	0.94	0.005	0.75
I-pentane	1.55	0.007	1.07
Hexane	0.18	0.00	0.00
Nitrogen	0.16	0.00	0.00

Carbon-dioxide	1.02	0.00	0.00

3.1.1 Collection of Data

The field data needed for simulation and validation of the developed model were obtained from an existing natural gas transmission pipeline; the 24 inch, 50km export pipeline of AMENAM-KPONO field from AMP2 to Bonny in Niger Delta area of Nigeria. The field related to AMENAM-KPONO project is located in offshore blocks OML 99 and OML 70, about 30km off the eastern part of Niger Delta area of Nigeria (water depth 40m). KPONO is the extension of AMENAM field located 55 km south of Bonny NLNG plant. It is one of the largest conventional offshore developments in West Africa. It is operated by Elf Petroleum Nigeria limited, in partnership with ExxonMobil and state-owned Nigerian National Petroleum Corporation (NNPC).

Figure 3.1: Amenam-Kponom field location map.

The parameters needed for the simulation of the model are the molar composition of Nigerian natural gas and the field operating data for AMP2 to Bonny 24'' natural gas pipeline. The parameters include the fluid density, inlet and outlet flow rates (of normal condition of no-leak), pressures and temperatures, length, diameter and roughness as shown in Table 3.2

Table 3.2: Field data for AMP2 to BONNY 24" gas pipeline

Items	Parameters	Value/Unit
Pipe	Nominal external diameter	24 Inch
	Nominal wall thickness	14.4 mm
Pipe Material	Steel density	7850 kg/m³
	Poisson's ratio	0.3
	Thermal expansion coefficient	0.0000117
	Overall heat transfer coefficient	37.77258 W/m²K
	Young Modulus	2.07E+11 N/m²
Coatings	Coating Layer	3LPE
	Coating thickness	3.2 mm
	Coating density	965 kg/m³
Pipeline profile	Elevation	0 m
	Horizontal distance	49500 m
Operating condition	Gas Inlet flow rate	749.8656 MMSCFPD
	Inlet temperature	45 °C
	Inlet pressure	147.5 Bar
	Inlet velocity	7.33 m/s
	Inlet density	23.97 kg/m³
	Inlet viscosity	0.015615 cP
	Outlet temperature	15 °C
	Outlet pressure	143.27 Bar
	Outlet velocity	6.7 m/s
	Outlet density	26.21 kg/m³
	Outlet viscosity	0.014487cP
Gas properties	Gas gravity	0.182 kg/m³
	Gas specific heat capacity	2170 j/(KgK)

3.2 METHOD

3.2.1 Theoretical Principle of Mathematical Model Development

Derivation and development of the model can be explained by considering a segment of a natural gas transmission pipeline, as shown in Figure 3.2

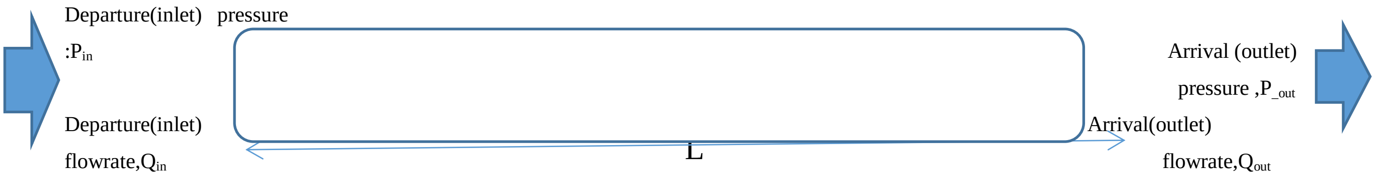

Figure 3.2: Natural gas pipeline segment

The pressure drop, ΔP_L is the discrepancies between the inlet pressure P_{in} and the outlet pressure P_{out}. It is generally known that $P_{in} > P_{out}$, as a result of pressure drop due to frictional resistance as the fluid flows along the pipe length, L. Thus

$$\Delta P_L = \quad P_{in} - P_{out} \tag{3.1}$$

Pressure drop due to frictional resistance ΔP_L has been correlated from Fanning Equation in terms of fluid properties to be generally given as:

$$\Delta P_L \quad = \frac{1}{2} f \rho v^2 \frac{L}{d} \tag{3.2}$$

where the friction factor f is a function of Reynolds number $Re = \frac{dV\rho}{\mu}$

P_L = pressure drop due to friction across the length of the pipeline

ρ = fluid density

V = fluid velocity

L = Length of the pipeline

d = density

At laminar flow regime (Re ≤ 2300), $f = \frac{64}{Re}$

while for turbulent flow (Re > 4100), $f = \frac{64}{Re}$

Suppose we break the pipe into two segments, segment 1 which we will call the upstream and segment 2 which we will call the downstream with lengths X and L-X respectively as shown in figure 3.3

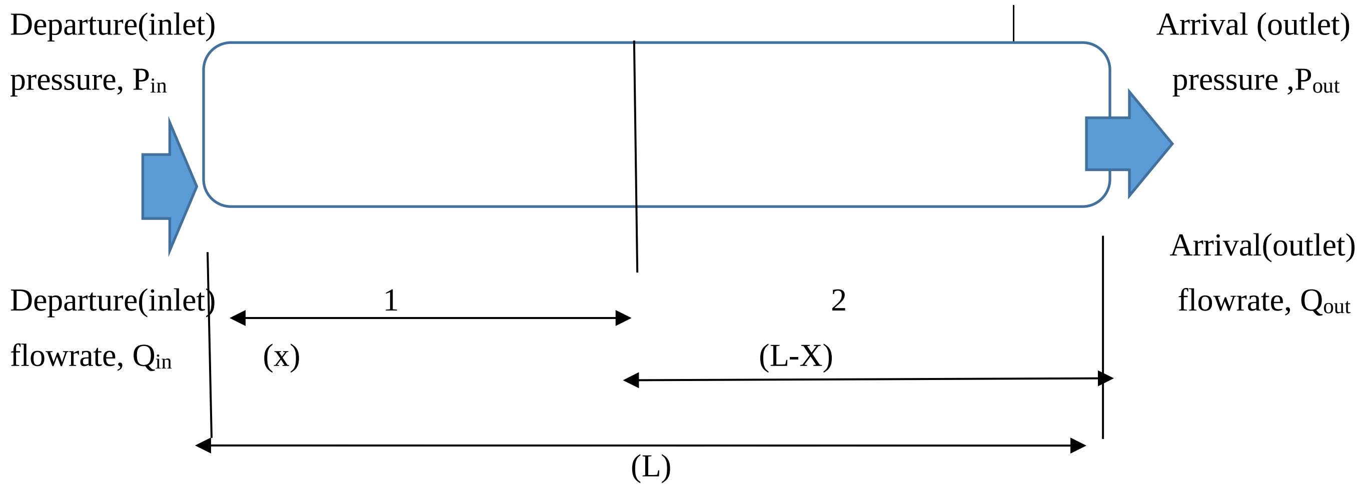

Figure 3.3: Broken natural gas pipeline segment

If there is no leakage through the valve in Figure 3.2 above then, the pressure drop in segment 1 ΔP_1 and the pressure drop in segment 2 ΔP_2 becomes,

$$\Delta P_1 = \frac{1}{2} f \rho v^2 \frac{X}{d} \tag{3.3}$$

$$\Delta P_2 = \frac{1}{2} f \rho v^2 \frac{(L-X)}{d} \tag{3.4}$$

where x = length of the pipeline of segment 1

And L-X = length of the pipeline of segment 2

Thus the total pressure drop for the whole length becomes:

$$\Delta P_1 + \Delta P_2 = \frac{1}{2} f \rho v^2 \frac{X}{d} + \frac{1}{2} f \rho v^2 \frac{(L-X)}{d} = \frac{1}{2} f \rho v^2 \frac{L}{d} = \Delta P_L \tag{3.5}$$

Therefore, frictional pressure drop for a pipeline system is additive.

Assuming that leakage occurred through the valve at the pipeline segment of length X, then, under normal conditions, the exit flow rate or downstream flow rate of the leakage point, X (between length L-X) will now be lower than the inlet or upstream flow rate before the leakage (length X). Since f and v are related to the flowrate, the frictional pressure drop in this case, ΔP_X becomes:

$$\Delta P_X = \frac{1}{2} f_1 \rho v_1^2 \frac{X}{d} + \frac{1}{2} f_2 \rho v_2^2 \frac{(L-X)}{d} = \frac{1}{2} f_1 \rho v_1^2 \frac{L}{d} \left\{ \frac{X}{L} + \frac{f_2 \rho v_2^2}{f_1 \rho f v_1^2} \left(1 - \frac{X}{L}\right) \right\} \tag{3.6}$$

Therefore:

$$\Delta P_X = \Delta P_L \left\{ \frac{X}{L} + \frac{f_2 \rho v_2^2}{f_1 \rho v_1^2} \left(1 - \frac{X}{L}\right) \right\} \tag{3.7}$$

In the case of laminar flow,

$$f_1 = \frac{64}{Re_1} = \frac{64}{(dv_1\,\rho/\mu)} \quad \text{and} \quad f_2 = \frac{64}{Re_2} = \frac{64}{(dv_2\,\rho/\mu)}$$

$but\ v_1 = \frac{Q_{in}}{A}$ and $v_2 = \frac{Q_{out}}{A}$ thus substituting into the equation (3.7) and simplifying it follows that:

$$\Delta P_X = \Delta P_L \left\{ \frac{X}{L} + \frac{64 \Big/ \left(\frac{dv_2\rho}{\mu}\right)\rho v_2^2}{64 \Big/ \left(\frac{dv_1\rho}{\mu}\right)\rho v_1^2} \left(1 - \frac{X}{L}\right) \right\} \tag{3.8}$$

$$\Delta P_X = \Delta P_L \left\{ \frac{X}{L} + \frac{64\rho v_2^2 \mu d v_1 \rho}{64\rho v_1^2 \mu d v_2 \rho} \left(1 - \frac{X}{L}\right) \right\} \tag{3.9}$$

since $v_1 = \frac{Q_{in}}{A}$ and $v_2 = \frac{Q_{out}}{A}$

$$\Delta P_X = \Delta P_L \left\{ \frac{X}{L} + \frac{Q_{out}/_A}{Q_{in}/_A} \left(1 - \frac{X}{L}\right) \right\} \tag{3.10}$$

$$\Delta P_X = \Delta P_L \left\{ \frac{X}{L} + \frac{Q_{out}}{Q_{in}} \left(1 - \frac{X}{L}\right) \right\} \tag{3.11}$$

where A = cross sectional area of the pipeline

v_1 = fluid velocity for upstream flow

v_2 = fluid velocity for downstream flow

Q_{in} = inflow volumetric flowrate

Q_{out} = outflow volumetric flowrate

Q_{leak} = flowrate of the leakage

μ = Fluid dynamic viscosity

By defining R as the ratio of the quantity leaked, Q_{leak} to the inlet flow rate Q_{in}, thus:

$$R = \frac{Q_{leak}}{Q_{in}}$$

We know that $Q_{leak} = Q_{in} - Q_{out}$

Thus, $R = \frac{(Q_{in} - Q_{out})}{Qin}$

And $Q_{out} = Q_{in}(1 - R)$

$R = 1 - \frac{Q_{out}}{Q_{in}}$ then, $1 - R = \frac{Q_{out}}{Q_{in}}$

Substituting into equation (3.11)

$$\Delta P_X = \Delta P_L \left\{\frac{X}{L} + (1-R)\left(1-\frac{X}{L}\right)\right\} \tag{3.12}$$

From the equation (3.12), it follows that, upstream pressure drop, ΔP_1 is:

$$\Delta P_1 = \Delta P_L \left\{\frac{X}{L}\right\}$$

And downstream pressure drop, ΔP_2 is:

$$\Delta P_2 = \Delta P_L (1-R)\left(1-\frac{X}{L}\right)$$

Thus, $\Delta P_X = \Delta P_1 + \Delta P_2 = \Delta P_L \left\{\frac{X}{L}\right\} + \Delta P_L (1-R)\left(1-\frac{X}{L}\right)$ (3.13)

By dividing both side of equation (3.13) by ΔP_L, we have

$$\frac{\Delta P_X}{\Delta P_L} = \frac{X}{L} + (1-R)\left(1-\frac{X}{L}\right) \tag{3.14}$$

$$\frac{\Delta P_X}{\Delta P_L} = 1 - \left(1-\frac{X}{L}\right) R \tag{3.15}$$

Since $\frac{X}{L}$ and R are less than 1, it means that the RHS is less than 1 and by extension the LHS, $\frac{\Delta P_X}{\Delta P_L}$ thus the outlet pressure when there is leakage, P_{in} - ΔP_X, will be higher than when there is no leakage, P_{in}- ΔP_L. We know this is not the usual case, as the outlet pressure will be lower when there is leakage, i.e. $\frac{\Delta P_X}{\Delta P_L}$,>1. Thus we need to include a correction term: the pressure drop due to leakage, ΔP_3.

The framework that would be adopted is to link the pressure drop due to leakage, ΔP_3 to the outlet or discharge pressure drop due to friction, ΔP_2, using the same analogy as linking the pipe fittings (valves, elbows, tees) pressure drop in terms of equivalent length of straight pipe friction pressure drop (Robert H. P., et al. 1999).

Thus recall that $\Delta P_L = \frac{1}{2} f \rho v^2 \frac{L}{d}$ and for laminar flow, reduces to:

$$\Delta P = 128 \frac{\mu Q L}{d^4}$$

$\Delta P . \frac{A^2}{Q} = 128 \mu L \left(\frac{16}{\pi^2}\right)$ = constant, K

Therefore $$\Delta P_2 . \frac{A_2^2}{Q_2} = \Delta P_3 . \frac{A_3^2}{Q_3} \tag{3.16}$$

where μ = Fluid dynamic viscosity

Q = Flow rate

Q_2 = Flow rate of the downstream section of leak point X

Q_3 = Flow rate at leakage point

A_2 = Cross sectional area of the downstream section of pipeline

A_3 = Cross sectional area of the leakage point

A_R = Ratio of area of pipe to the leakage area

$$\frac{\Delta P_3}{\Delta P_2} = \frac{A_2^2/Q_2}{A_3^2/Q_3} = \frac{A_2^2 Q_3}{A_3^2 Q_2} \tag{3.17}$$

By defining $A_R = \frac{A_3}{A_2}$, and making the following substitutions:

$Q_3 = Q_{Leak} = Q_{in}R$ and $Q_2 = Q_{out} = Q_{in}(1-R)$ into equation (3.17), we have

$$\frac{\Delta P_3}{\Delta P_2} = \frac{A_2^2 Q_3}{A_3^2 Q_2} = \frac{1}{A_R^2}\frac{Q_3}{Q_2} \tag{3.18}$$

$$\frac{\Delta P_3}{\Delta P_2} = \frac{Q_{in}/R}{Q_{in}/(1-R)} . \frac{1}{A_R^2} \tag{3.19}$$

$$\Delta P_3 = \frac{\Delta P_2 . R}{\{(1-R) . A_R^2\}} \tag{3.20}$$

Recall that $\Delta P_1 = \Delta P_L \left\{\frac{X}{L}\right\}$ and $\Delta P_2 = \Delta P_L (1-R)\left(1-\frac{X}{L}\right)$ (3.21)

Hence, $\Delta P_3 = \frac{\Delta P_L (1-R)\left(1-\frac{X}{L}\right) . R}{(1-R) . A_R^2}$ (3.22)

$$\Delta P_3 = \frac{\Delta P_L\left(1-\frac{X}{L}\right)R}{A_R^2} \tag{3.23}$$

The total pressure drop following leakage becomes,

$$\Delta P_X = \Delta P_1 + \Delta P_2 + \Delta P_3 \tag{3.24}$$

By substituting equation (3.21) and (3.23) into equation (3.24), we have that:

$$\Delta P_X = \Delta P_L \frac{X}{L} + \Delta P_L (1-R)\left(1-\frac{X}{L}\right) + \frac{\Delta P_L \left(1-\frac{X}{L}\right)R}{A_R^2} \tag{3.25}$$

Divide both sides of equation (3.25) by ΔP_L to obtain

$$\frac{\Delta P_X}{\Delta P_L} = \frac{X}{L} + (1-R)\left(1-\frac{X}{L}\right) + \frac{(1-\frac{X}{L})R}{A_R^2} \tag{3.26}$$

$$\frac{\Delta P_X}{\Delta P_L} = \frac{X}{L} + \left[1 - \frac{X}{L} - R + \frac{RX}{L}\right] + \frac{\left(1-\frac{X}{L}\right)R}{A_R^2} \tag{3.26}$$

By Collecting like terms, equation 3.26 becomes,

$$\frac{\Delta P_X}{\Delta P_L} = 1 + \frac{X}{L} - \frac{X}{L} - R + \frac{RX}{L} + \frac{\left(1-\frac{X}{L}\right)R}{A_R^2} \tag{3.27}$$

$$\frac{\Delta P_X}{\Delta P_L} = 1 - \left[R\left(1 - \frac{X}{L}\right)\right] + \left[R\left(1 - \frac{X}{L}\right)\frac{1}{A_R^2}\right] \tag{3.28}$$

Factorizing equation (3.28) since $R\left(1 - \frac{X}{L}\right)$ is common to obtain;

$$\frac{\Delta P_X}{\Delta P_L} = 1 + (R)\left(1 - \frac{X}{L}\right)\left(\frac{1}{A_R^2} - 1\right) \tag{3.29}$$

From equation (3.29), we can fit an equation to it by regression analysis and the relation used to substitute for ΔP_L in the equation above to obtain, thus:

$$\Delta P_L = a + bX/L \tag{3.30}$$

The coefficients a and b are found by regression analysis to be;

A	=	147.4993
B	=	-4.2624

Thus equation (3.22) becomes:

$$\Delta P_L = 147.4993 - \frac{4.2624X}{L} \tag{3.31}$$

By combining equation (3.29) and equation (3.31) and rearranging them, we can then obtain thus

$$\Delta P_X = 1 + (1 - \frac{X}{L})(R)\left(\frac{1}{R^2} - 1\right)(147.4993 - 4.2624\ \frac{X}{L}) \tag{3.32}$$

where $\Delta P_X = P_{in} - P_{out}$

R is the ratio of quantity leaked to the entering flowrate, i.e R = $Q_{leak}/Q_{in} = 1 - Q_{out}/Q_{in}$

Thus by knowing the inlet and outlet flowrates (Q_{in}, Q_{out}) and pressures (P_{in}, P_{out}), the LHS can be evaluated. Thus the LHS is the known while the RHS is the unknown containing the two variables; $\frac{X}{L}$ and R (which lie between 0 and 1) to be determined by numerical method like

Newton gradient methods, from where the location and the quantity of leakage can be determined.

3.2.2 Boundary and Initial Conditions

The study uses a generic model of a natural gas pipeline representing the characteristics of export gas pipelines on the Nigerian offshore location. The 24 inch carbon steel pipe has a 14.4 mm thick steel wall with a 6 mm outer plastic anti-corrosion coating. The length under study of the offshore pipeline is 50 km. The pipeline elevation profile is kept horizontally level or assumed to be so.

The number of boundary conditions needed depends on the type of the boundary. Initially, at the entrance of the pipe, L=0.0 Km

At L = 0.0 km

P_0 =147.5 bar

T_0 =45.94 °C

u_0 =7.33 m/s

ρ_0 = 23.97 kg/m³

μ_0 = 0.015615 cP

Similarly, at the exit of the pipe, L=50 Km,:

At L = 50 Km:

P_L =143.27 bar

T_L =15 °C

u_L =6.7 m/s

ρ_L = 26.21kg/m³

μ_L = 0.014487cP

3.2.3 ASPEN HYSYS Simulation

The basic pipeline modeling equations of section (2.4.1) i.e the continuity equation, momentum equation and energy equation in combination with the field data obtained was simulated using ASPEN-HYSYS software in order to generate the pressure drop along the length of the pipeline which represent the pressure drop of a real life existing natural gas transmission pipeline for the purpose of model validation. The equation of state used in this simulation is the Soave–Redlich–Kwong equation of state (SRK EoS) given by the equation (3.33)

$$P = \frac{RT}{v-b} - \frac{a}{v(v+b)} \tag{3.33}$$

We will show using finite difference method how the models develop to software in ASPEN HYSYS.

In summary, the governing equations to be solved by the ASPEN-HYSYS are:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial z} = 0 \tag{3.34}$$

$$\frac{\partial(\rho u)}{\partial t} + u\left\{\frac{\partial(\rho u)}{\partial z}\right\} = \rho g \sin\theta \; - \frac{dP}{dz} + \frac{\partial}{dz}\left\{\mu\left(\frac{\partial u}{\partial z}\right)\right\} \tag{3.35}$$

$$\frac{\partial(\rho C_p T + \frac{u^2}{2} + gh + \frac{P}{\rho})}{\partial t} + u\left\{\frac{\partial(\rho C_p T + \frac{u^2}{2} + gh + \frac{P}{\rho})}{\partial z}\right\} = -\frac{\partial}{\partial z}\left(-k\frac{\partial T}{\partial z}\right) + S \tag{3.36}$$

However we are considering a fully developed steady horizontal pipe flow, with constant k and no source terms, hence the unsteady terms disappear, and the equations reduce to:

$$\frac{\partial(\rho u)}{\partial z} = 0 \tag{3.37}$$

$$u\left\{\frac{\partial(\rho u)}{\partial z}\right\} = \rho g \; - \frac{dP}{dz} + \frac{\partial}{dz}\left\{\mu\left(\frac{\partial u}{\partial z}\right)\right\} \tag{3.38}$$

$$u\left\{\frac{\partial(\rho C_p T + \frac{u^2}{2} + \frac{P}{\rho})}{\partial z}\right\} = k\frac{d^2 T}{\partial z^2} \tag{3.39}$$

The finite difference method involves the replacement of continuous variables, with discrete variables, that is, instead of obtaining a solution which is continuous over the whole domain of interest, the finite difference method will yield values at discrete points, chosen by the analyst.

In the finite difference method, a derivative at a discrete point, the said point is evaluated using the information about discrete variables close to that point (local information). As long as the grid size is properly chosen, the finite difference approach leads to stable solutions. This section will describe the procedure for the finite difference representation in solving differential equations.

Thus based on first order and second order approximations of Taylor series expansion , the finite difference approximations for first order and second order partial differential equations (as encountered in this case)become:

$$\left(\frac{\partial y}{\partial x}\right)_i = \frac{y_{i+1} - y_{i-1}}{2\Delta x} \tag{3.40}$$

$$\left(\frac{\partial^2 y}{\partial x^2}\right)_i = \frac{y_{i+1} - 2y_i + y_{i-1}}{(\Delta x)^2} \tag{3.41}$$

where we divide the spatial domain into N equal intervals with the grid size being Δx given as:

$$\Delta x = \frac{1}{N} \tag{3.42}$$

And the point x_i along the interval as:

$$\Delta x_i = i(x) \quad for \quad i = 0,1,2,3, \dots, N$$

Application

Using the discrete formula for the first and second derivatives, applied to each conservation equation, as follows:

For

$$\frac{\partial(\rho u)}{\partial z} = 0 \tag{3.43}$$

This is rearranged as

$$\frac{u\partial\rho}{\partial z} + \frac{\rho\partial u}{\partial z} = 0 \tag{3.44}$$

The finite difference transformation becomes:

$$u_i\left(\frac{\rho_{i+1}-\rho_{i-1}}{2\Delta z}\right)\ +\rho_i\left(\frac{u_{i+1}-u_{i-1}}{2\Delta z}\right)=0 \tag{3.45}$$

For

$$u\left\{\frac{\partial(\rho u)}{\partial z}\right\}=\rho g\ -\frac{dP}{dz}+\frac{\partial}{dz}\left\{\mu\left(\frac{\partial u}{\partial z}\right)\right\} \tag{3.46}$$

This is rearranged as:

$$u^2\frac{\partial\rho}{\partial z}+\rho u\frac{\partial u}{\partial z}=\rho g\ -u\frac{\partial P}{\partial z}+\mu\frac{\partial^2 u}{\partial z^2}+\left(\frac{\partial u}{\partial z}\right)\left(\frac{\partial\mu}{\partial z}\right) \tag{3.47}$$

The finite difference transformation becomes:

$$u_i^2\left(\frac{\rho_{i+1}-\rho_{i-1}}{2\Delta z}\right)+\ \rho_i u_i\left(\frac{u_{i+1}-u_{i-1}}{2\Delta z}\right)=\rho_i g-\ u_i\left(\frac{\rho_{i+1}-\rho_{i-1}}{2\Delta z}\right)+\mu_i\left(\frac{u_{i+1}-2u_i+u_{i-1}}{(\Delta z)^2}\right)+$$
$$\left(\frac{u_{i+1}-u_{i-1}}{2\Delta z}\right)\left(\frac{\mu_{i+1}-\mu_{i-1}}{2\Delta z}\right) \tag{3.48}$$

For

$$u\left\{\frac{\partial(\rho C_p T+\frac{u^2}{2}+\frac{P}{\rho})}{\partial z}\right\}=k\frac{d^2T}{\partial z^2} \tag{3.49}$$

This is rearranged as:

$$uC_p\rho\frac{\partial T}{\partial z}+uC_pT\frac{\partial\rho}{\partial z}+u^2\frac{\partial u}{\partial z}+\frac{u}{\rho}\frac{\partial P}{\partial z}-\frac{uP}{\rho^2}\frac{\partial\rho}{\partial z}=k\frac{d^2T}{\partial z^2} \tag{3.50}$$

The finite difference discretization becomes:

$$C_p\rho_i u_i\left(\frac{T_{i+1}-T_{i-1}}{2\Delta z}\right)+\ C_pT_iu_i\left(\frac{\rho_{i+1}-\rho_{i-1}}{2\Delta z}\right)+u_i^2\left(\frac{u_{i+1}-u_{i-1}}{2\Delta z}\right)+\ \frac{u_i}{\rho_i}\left(\frac{P_{i+1}-P_{i-1}}{2\Delta z}\right)+\ \frac{u_iP_i}{\rho_i^2}\left(\frac{\rho_{i+1}-\rho_{i-1}}{2\Delta z}\right)=$$
$$k\left(\frac{T_{i+1}-2T_i+T_{i-1}}{(\Delta z)^2}\right) \tag{3.51}$$

In summary, the finite difference discretization of the governing equations is as follows

$$u_i\left(\frac{\rho_{i+1}-\rho_{i-1}}{2\Delta z}\right)\ +\rho_i\left(\frac{u_{i+1}-u_{i-1}}{2\Delta z}\right)=0 \tag{3.52}$$

$$u_i^2\left(\frac{\rho_{i+1}-\rho_{i-1}}{2\Delta z}\right)+\ \rho_i u_i\left(\frac{u_{i+1}-u_{i-1}}{2\Delta z}\right)=\rho_i g-\ u_i\left(\frac{\rho_{i+1}-\rho_{i-1}}{2\Delta z}\right)+\mu_i\left(\frac{u_{i+1}-2u_i+u_{i-1}}{(\Delta z)^2}\right)+$$
$$\left(\frac{u_{i+1}-u_{i-1}}{2\Delta z}\right)\left(\frac{\mu_{i+1}-\mu_{i-1}}{2\Delta z}\right) \tag{3.53}$$

$$C_p\rho_i u_i\left(\frac{T_{i+1}-T_{i-1}}{2\Delta z}\right)+\ C_p T_i u_i\left(\frac{\rho_{i+1}-\rho_{i-1}}{2\Delta z}\right)+u_i^2\left(\frac{u_{i+1}-u_{i-1}}{2\Delta z}\right)+\ \frac{u_i}{\rho_i}\left(\frac{P_{i+1}-P_{i-1}}{2\Delta z}\right)+\ \frac{u_i P_i}{\rho_i^2}\left(\frac{\rho_{i+1}-\rho_{i-1}}{2\Delta z}\right)=$$
$$k\left(\frac{T_{i+1}-2T_i+T_{i-1}}{(\Delta z)^2}\right) \tag{3.54}$$

Rearranging and simplifying these equations give the finite-difference equations:

$$u_i(\rho_{i+1}-\rho_{i-1})+\rho_i(u_{i+1}-u_{i-1})=0 \tag{3.55}$$

$$2\Delta z u_i^2(\rho_{i+1}-\rho_{i-1})+2\Delta z\rho_i u_i(u_{i+1}-u_{i-1})-4(\Delta z)^2 g\rho_i+2\Delta z u_i(\rho_{i+1}-\rho_{i-1})-$$
$$4\mu_i(u_{i+1}-2u_i+u_{i-1})-(u_{i+1}-u_{i-1})(u_{i+1}-u_{i-1})=0 \tag{3.56}$$

$$\Delta z C_p\rho_i^3 u_i(T_{i+1}-T_{i-1})+\Delta z C_p\rho_i^2 u_i(\rho_{i+1}-\rho_{i-1})+\Delta z\rho_i^2 u_i^2(u_{i+1}-u_{i-1})+\Delta z u_i\rho_i(P_{i+1}-$$
$$P_{i-1})+\Delta z u_i P_i(\rho_{i+1}-\rho_{i-1})-2k\rho_i^2(T_{i+1}-2T_i+T_{i-1})=0 \tag{3.57}$$

These are coupled nonlinear algebraic equations in terms of the parameters. To solve we divide the spatial domain into N equal intervals with the grid size being Δz given as:

$$\Delta z=\ \frac{1}{N}$$

And the point z_i along the interval as:

$$\Delta z_i=i(z)\quad for\quad i=0,1,2,3,\dots,N$$

In this case of N=50, we will have 50 equations in each case, which obviously cannot be solved analytically, hence the use of software.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 NUMERICAL RESULTS

A natural gas transmission pipeline system; the 24 inch, 50kM export pipeline of AMENAM-KPONO field from AMP2 to Bonny in Niger Delta area of Nigeria was considered to investigate the leak detection method using the mathematical model developed and the procedure explained above in preceding chapter.

Based on the field data provided in Table 3.1 as boundary conditions in combination with the basic pipeline modeling equation, simulation is carried out using ASPEN HYSYS to find the pressure, temperature, velocity, density and viscosity head at the different section of the pipeline. The output of the simulation is shown in Table 4.1

Table 4.1 Simulation output for the Pipeline for length 50km

Pipe length	elevation	Pressure	Temperature	Velocity	density	Viscosity
(km)	(m)	(bar)	(°C)	(m/s)	(kg/m³)	(cP)
0	0	147.5	45.94	7.33	23.97	0.015615
0.99	0	147.41	30.83	6.94	25.32	0.015075
1.98	0	147.32	23.1	6.73	26.07	0.014799
2.97	0	147.24	19.14	6.63	26.47	0.014658
3.96	0	147.16	17.12	6.58	26.67	0.014586
4.95	0	147.07	16.09	6.56	26.77	0.014548
5.94	0	146.99	15.56	6.55	26.81	0.014529
6.93	0	146.9	15.28	6.54	26.83	0.014519
7.92	0	146.82	15.15	6.54	26.83	0.014513
8.91	0	146.74	15.07	6.55	26.82	0.01451
9.9	0	146.65	15.04	6.55	26.81	0.014509
10.89	0	146.57	15.02	6.55	26.8	0.014507

11.88	0	146.49	15.01	6.56	26.79	0.014507
12.87	0	146.4	15.01	6.56	26.77	0.014506
13.86	0	146.32	15	6.56	26.76	0.014505
14.85	0	146.24	15	6.57	26.74	0.014505
15.84	0	146.15	15	6.57	26.73	0.014504
16.83	0	146.07	15	6.57	26.71	0.014504
17.82	0	145.98	15	6.58	26.7	0.014503
18.81	0	145.9	15	6.58	26.68	0.014503
19.8	0	145.82	15	6.58	26.67	0.014502
20.79	0	145.73	15	6.59	26.65	0.014502
21.78	0	145.65	15	6.59	26.64	0.014501
22.77	0	145.56	15	6.6	26.62	0.014501
23.76	0	145.48	15	6.6	26.61	0.0145
24.75	0	145.39	15	6.6	26.59	0.0145
25.74	0	145.31	15	6.61	26.58	0.014499
26.73	0	145.23	15	6.61	26.56	0.014499
27.72	0	145.14	15	6.61	26.55	0.014498
28.71	0	145.06	15	6.62	26.53	0.014498
29.7	0	144.97	15	6.62	26.52	0.014497
30.69	0	144.89	15	6.63	26.5	0.014497
31.68	0	144.8	15	6.63	26.49	0.014496
32.67	0	144.72	15	6.63	26.47	0.014496
33.66	0	144.63	15	6.64	26.46	0.014495
34.65	0	144.55	15	6.64	26.44	0.014495
35.64	0	144.46	15	6.64	26.43	0.014494
36.63	0	144.38	15	6.65	26.41	0.014494
37.62	0	144.29	15	6.65	26.4	0.014493
38.61	0	144.21	15	6.66	26.38	0.014493
39.6	0	144.12	15	6.66	26.37	0.014492

40.59	0	144.04	15	6.66	26.35	0.014492
41.58	0	143.95	15	6.67	26.34	0.014491
42.57	0	143.87	15	6.67	26.32	0.014491
43.56	0	143.78	15	6.67	26.3	0.01449
44.55	0	143.7	15	6.68	26.29	0.01449
45.54	0	143.61	15	6.68	26.27	0.014489
46.53	0	143.53	15	6.69	26.26	0.014489
47.52	0	143.44	15	6.69	26.24	0.014488
48.51	0	143.36	15	6.69	26.23	0.014488
49.5	0	143.27	15	6.7	26.21	0.014487

Table 4.1 shows the result obtained from the ASPEN HYSYS simulation of pipeline flow parameters using the boundary conditions presented by the field data.

4.2 Analysis and Discussion

In Figures 4.1, a graph of pressure against length, pressure decreases linearly with increasing length; pressure is inversely proportional to the length.

In Figure 4.2, a graph of temperature against length, temperature decreases with increasing length up to 15°C and becomes constant with increasing length. From the plots below, it shows that both pressure and temperature decrease with increase in distance of the pipeline from the point of departure to the point of exist, however pressure drop along the length of the pipeline is less significant (147.5 bar to 143.27 bar) when compared to the temperature drop along the length of the pipeline (45.94^{o}C to 15^{o}C) which is a very significant drop.

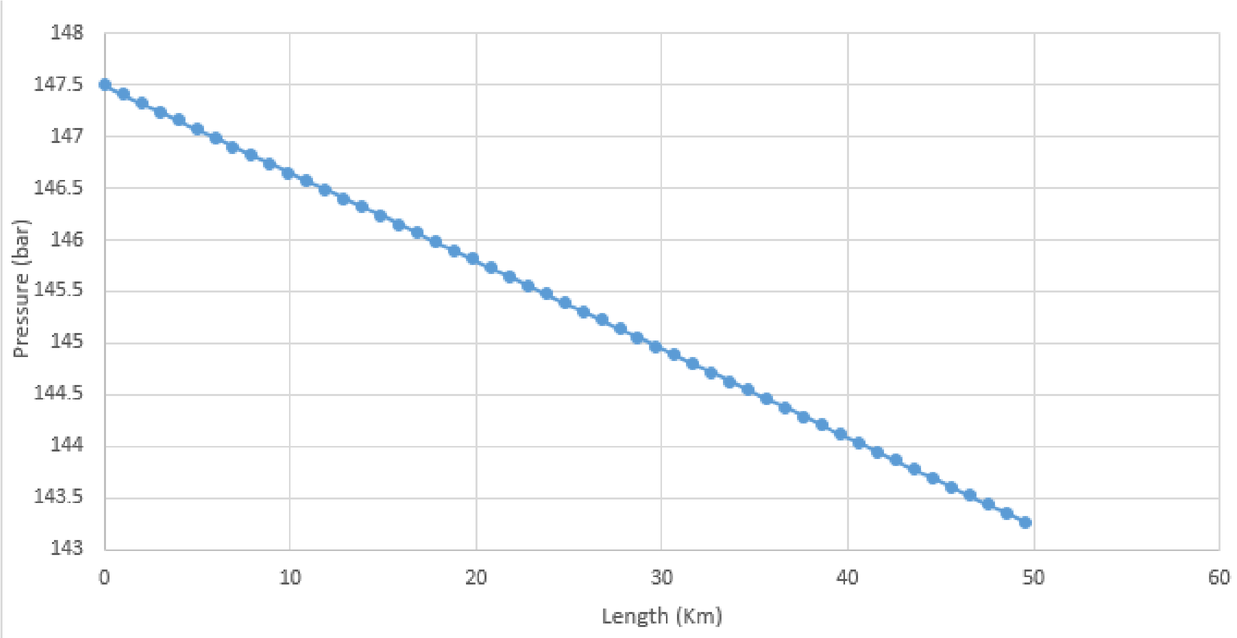

Figure 4.1: Pressure Profile along the length of the pipeline

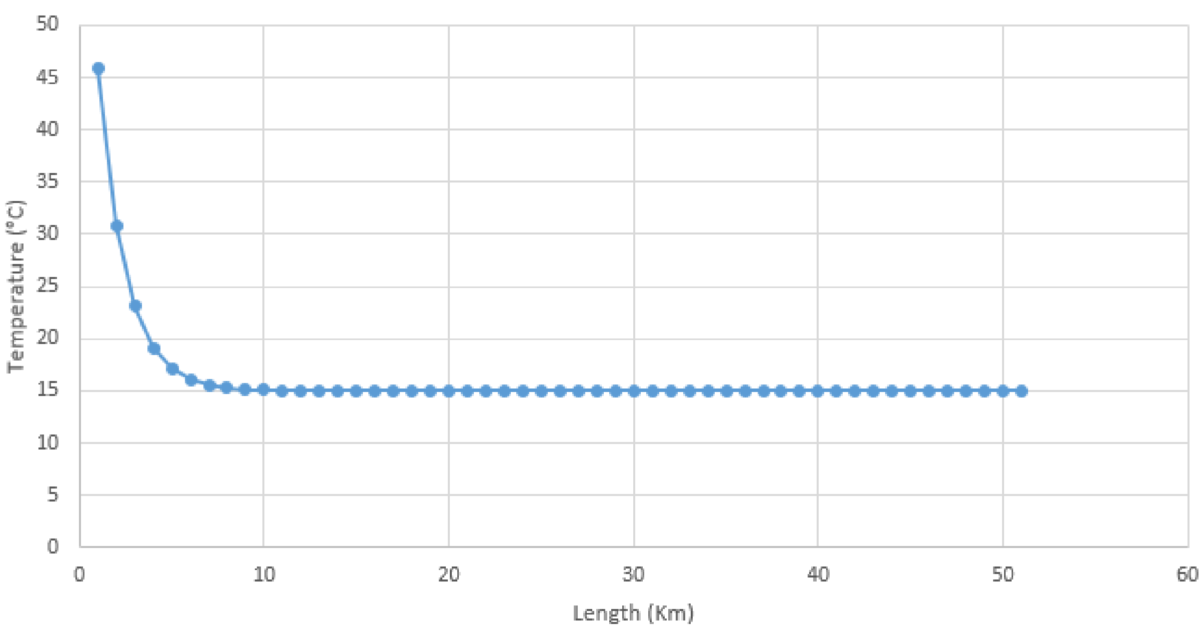

Figure 4.2: Temperature Profile along the length of the pipeline

Similarly, in Figure 4.3 a graph of velocity against length, there is a sharp fall of velocity up to 6.55m/s at 50km. In Figure 4.4, a graph of density against length, density increases sharply up to $26.8 km/m^3$at $7km$ before gradual linear fall to $26.2 kg/m^3$ at $50km$. From the data plotted, it shows that both the density and velocity fluctuate along the length of the pipeline due to the turbulent nature of the flow in the pipeline.

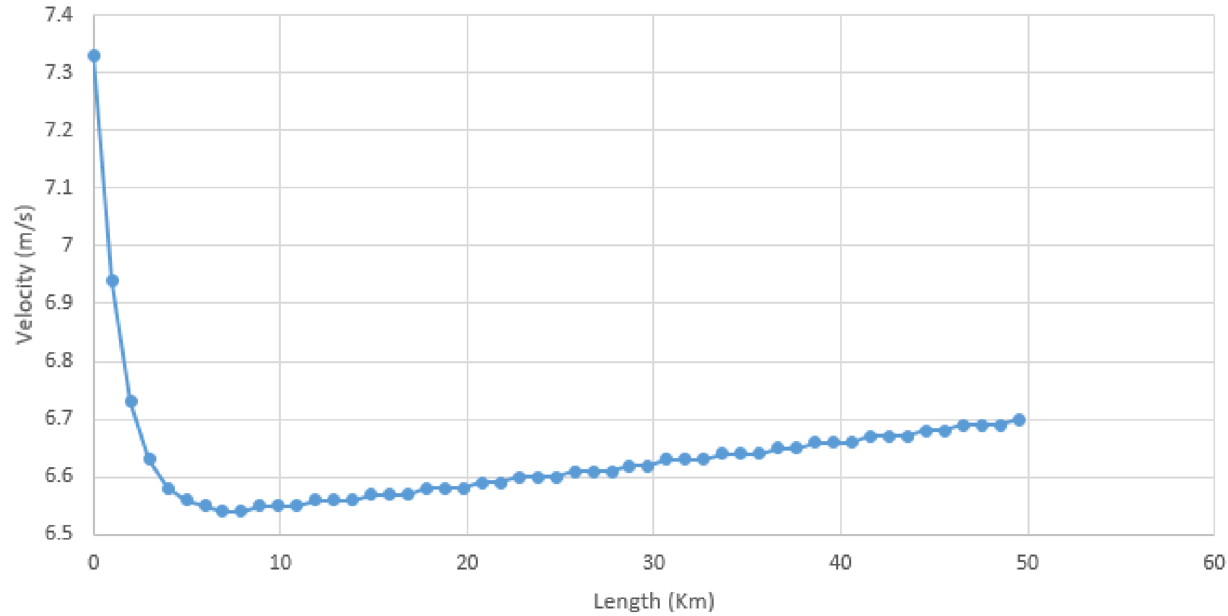

Figure 4.3: Velocity distribution along the length of the pipeline

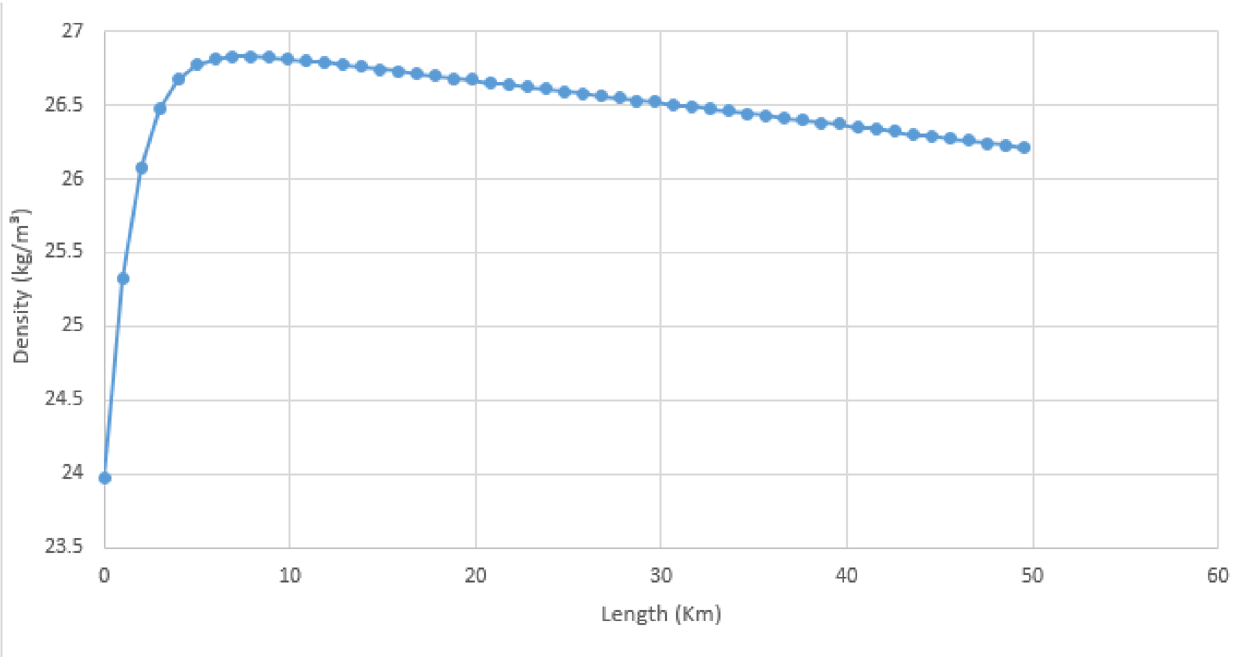

Figure 4.4: Density distribution along the length of the pipeline

Similarly, In Figure 4.5, a graph of viscosity against length. There is a sharp fall of viscosity to $0.014cp$ at $7km$ before remaining constant as length increases to $50km$.

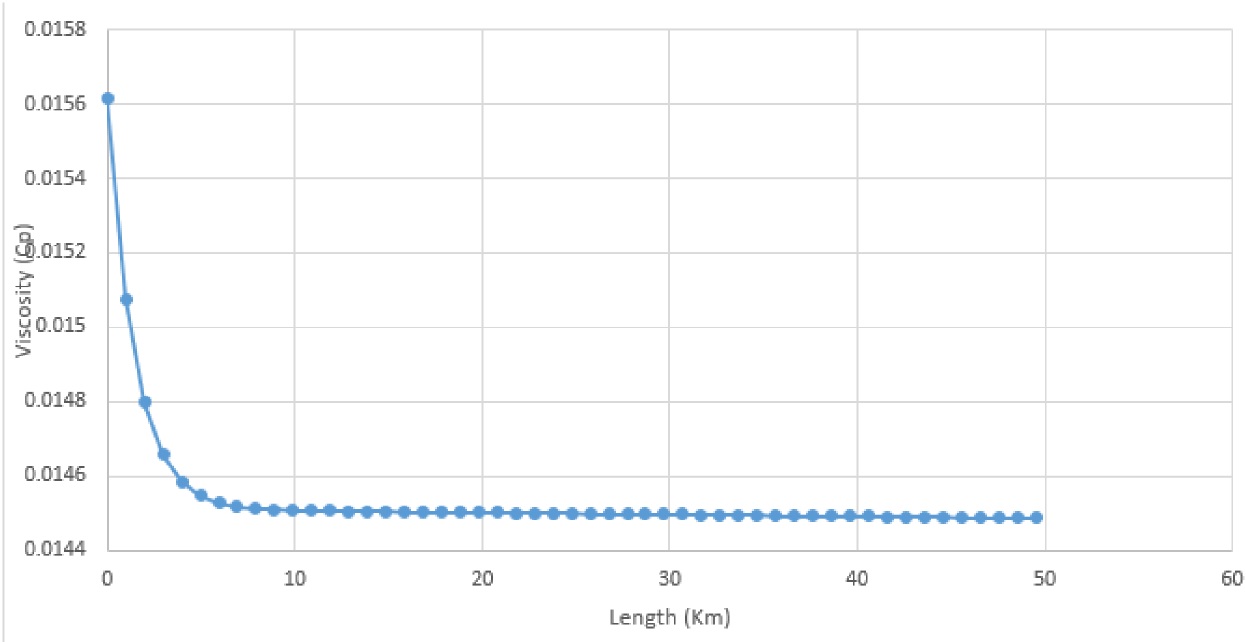

Figure 4.5: Viscosity distribution along the length of the pipeline

The effect of pressure on the various parameters of flow of the natural gas in the pipeline is analyzed as follows.

In Figure 4.6, a graph of temperature against pressure, plotted from the reverse end of the pipeline, the temperature remained constant at 15℃ increasing pressure up to 147 bar before a sharp rise to 147.5 bar at 45℃. That means as temperature is increasing, the molecules in the natural gas move faster hence, increasing the pressure.

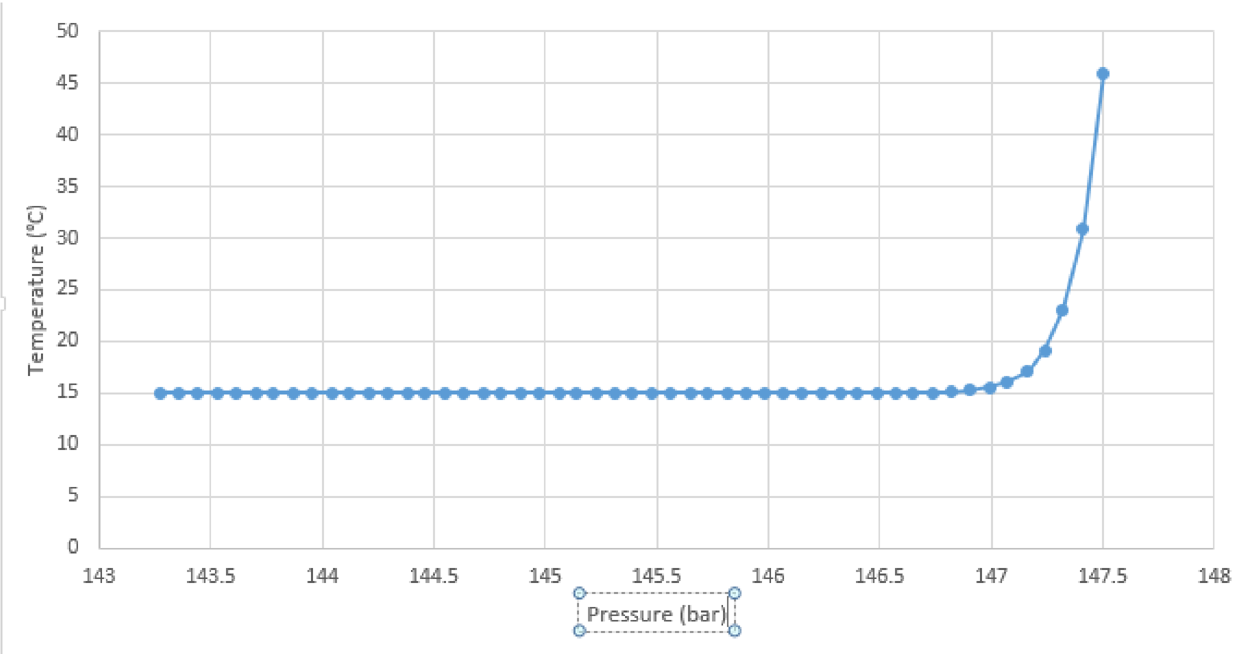

Figure 4.6: Temperature variation with pressure along the length of the pipeline

In Figure 4.7, a graph of velocity against pressure, pressure is slightly inversely proportional to velocity from the longer arm of the pipeline up to 147 bar at 6.55m/s after which velocity shoot up with increasing pressure to 147.5 bar at 7.34m/s at the shorter arm of the pipeline i.e the higher the velocity of a fluid the lower the pressure it exerts.

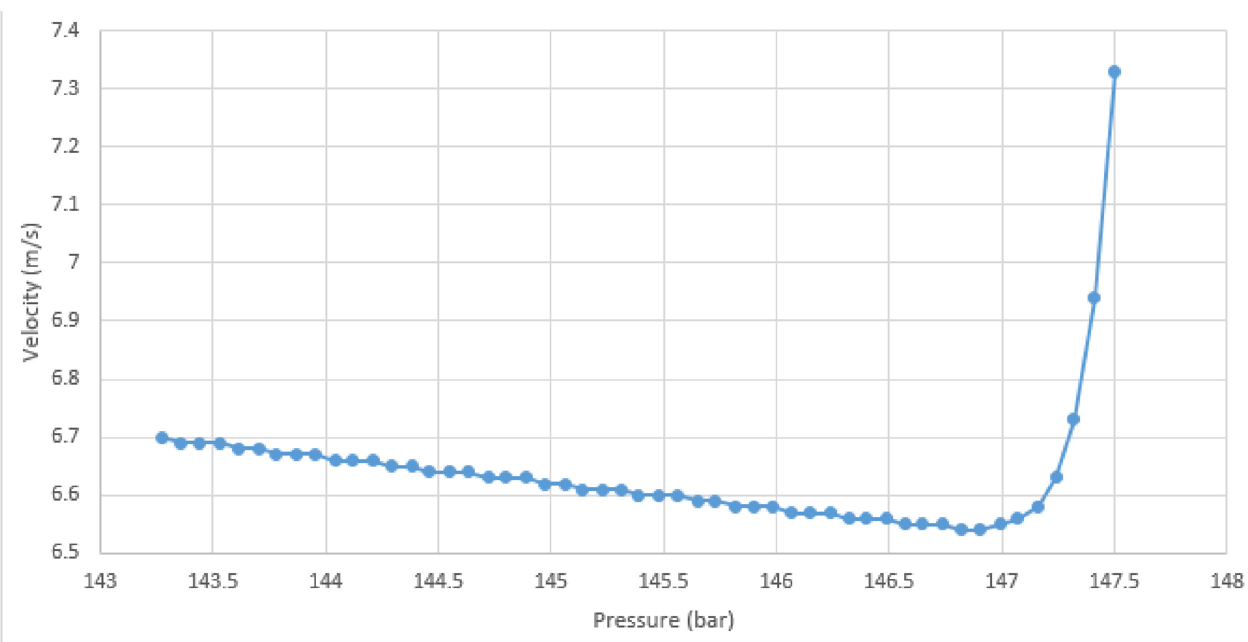

Figure 4.7: Velocity Variation with Pressure along the length of the pipeline

In Figure 4.8, a graph of density against pressure, the pressure is directly proportional to the density in the longer arm of the pipeline. The density falls sharply with increasing pressure at the shorter arm of the pipeline i.e the higher the density of a fluid the higher the pressure.

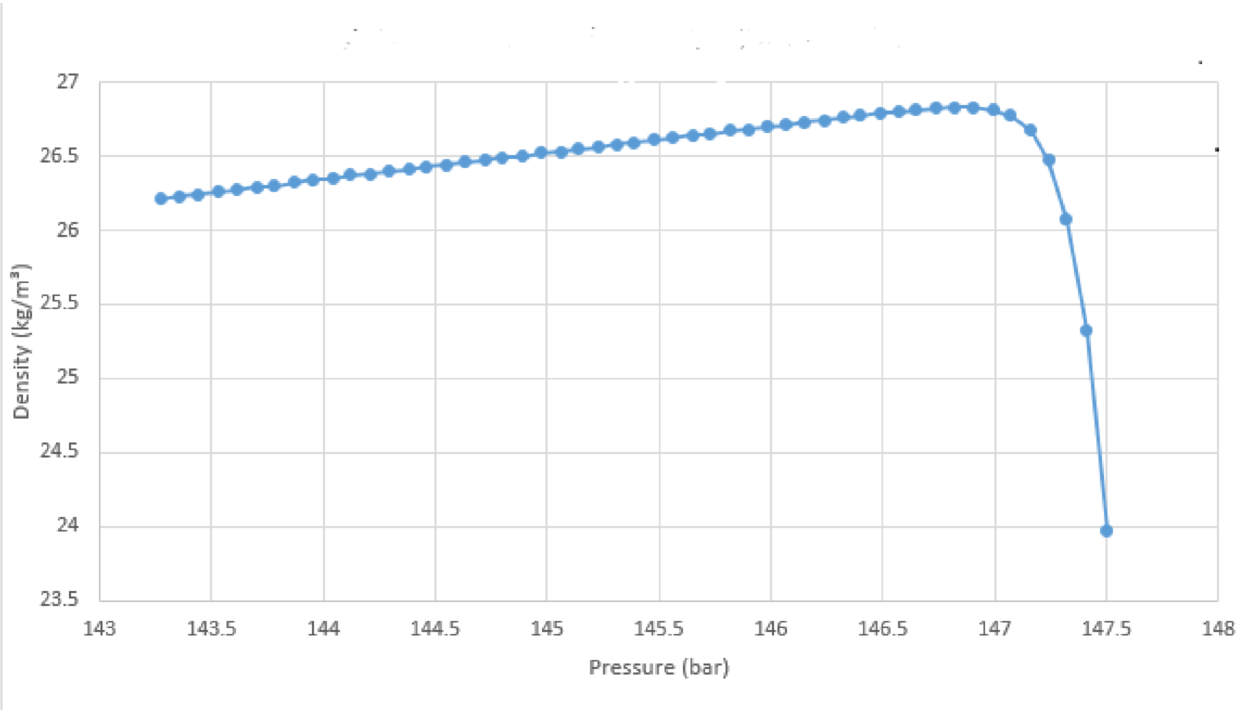

Figure 4.8: Density Variation with Pressure along the length of the pipeline

In Figure 4.9, a graph of viscosity against pressure, the viscosity increases infinitesimally with increasing pressure in the longer arm of the pipeline. It shows that viscosity increases with increasing pressure.

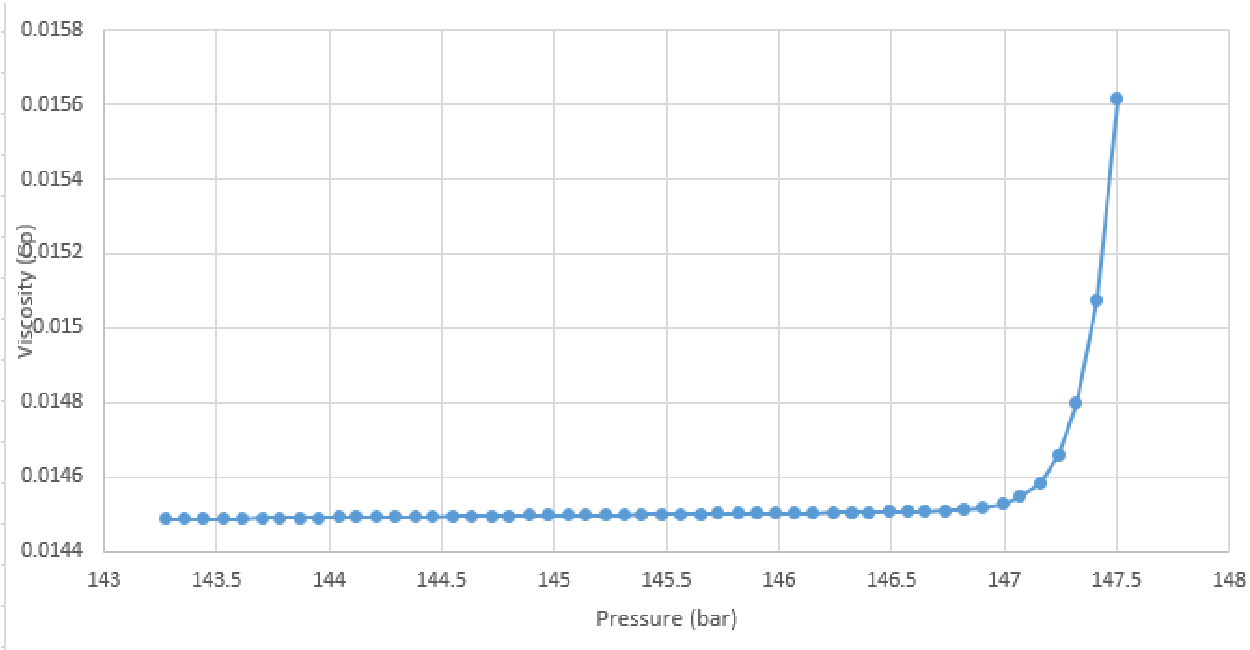

Figure 4.9: Viscosity Variation with Pressure along the length of the pipeline

4.3 Model Validation and Assessment of Error

The pressure drop ΔP_L of the present mathematical model from equation 3.16 above is calculated and result tabulated against the ΔP_L from the simulation result from the field data obtain. The result below shows that the model and the field data are significantly close.

Table 4.2 Result of the model validation.

X/L	ΔP_L (DATA.)	ΔP_L (MODEL)
0	147.5	147.4993
0.0198	147.41	147.4149
0.0396	147.32	147.3305
0.0594	147.24	147.2461
0.0792	147.16	147.1617
0.099	147.07	147.0773
0.1188	146.99	146.9929
0.1386	146.9	146.9085
0.1584	146.82	146.8241

0.1782	146.74	146.7397
0.198	146.65	146.6553
0.2178	146.57	146.5709
0.2376	146.49	146.4865
0.2574	146.4	146.4021
0.2772	146.32	146.3178
0.297	146.24	146.2334
0.3168	146.15	146.149
0.3366	146.07	146.0646
0.3564	145.98	145.9802
0.3762	145.9	145.8958
0.396	145.82	145.8114
0.4158	145.73	145.727
0.4356	145.65	145.6426
0.4554	145.56	145.5582
0.4752	145.48	145.4738
0.495	145.39	145.3894
0.5148	145.31	145.305
0.5346	145.23	145.2206
0.5544	145.14	145.1362
0.5742	145.06	145.0518
0.594	144.97	144.9674
0.6138	144.89	144.883
0.6336	144.8	144.7987
0.6534	144.72	144.7143
0.6732	144.63	144.6299
0.693	144.55	144.5455
0.7128	144.46	144.4611
0.7326	144.38	144.3767

0.7524	144.29	144.2923
0.7722	144.21	144.2079
0.792	144.12	144.1235
0.8118	144.04	144.0391
0.8316	143.95	143.9547
0.8514	143.87	143.8703
0.8712	143.78	143.7859
0.891	143.7	143.7015
0.9108	143.61	143.6171
0.9306	143.53	143.5327
0.9504	143.44	143.4483
0.9702	143.36	143.3639
0.99	143.27	143.2796

By comparing the values of the pressure drop from data simulation and the pressure drop from the model, we can deduce that the model can be used to correctly predict the presence of leakage in a natural gas transmission pipeline. With R^2 value of 99.1 % the model is a good fit to the data as shown in figure

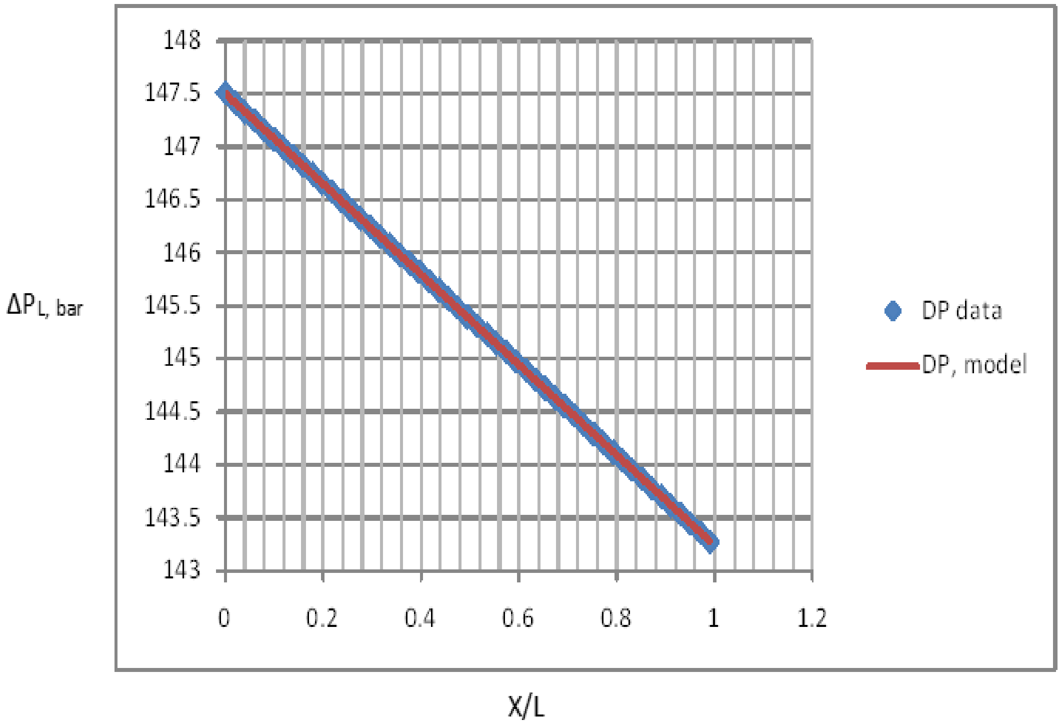

Figure 4.10: Comparison of model and simulation results for pressure drop

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

This research work focuses on the study of the structure of a mathematical model which should help to detect the presence of leakages in natural gas transmission pipelines. Natural gas leakage have been part of oil and gas industry operational problems. It normally occurs as a result of crack in the pipeline due to fatigue, aging or corrosion and the late detection of these leakages have compounded the problem leading to progression into explosion and occasionally fire with the attendant huge costs and destruction. Thus in this project, a simplified mathematical model was developed to efficiently predict or detect the presence of leakage in natural gas transmission pipelines. The model was shown to be physically realistic. The model developed thus is:

$$\Delta P_X = 1 + (1 - X/L)(R)\left(\frac{1}{R^2} - 1\right)(147.4993 - 4.2624\,X/L)$$

Where $\Delta P_X = P_{in} - P_{out}$

R is the ratio of quantity leaked to the entering flowrate, i.e $R = Q_{leak}/Q_{in} = 1 - Q_{out}/Q_{in}$

Upon evaluation, the value of pressure drop obtained from the model and that from the simulation result shows close similarity thus the mathematical model is realistic and can be used to predict the presence of leakage, the location in the pipeline where the leakage occurs (leakage point), and the extent of pipe damage (leakage area). This will ensure early and prompt remedial action that will minimize losses and damage occasioned by oil leaks.

5.2 Recommendation

The aim of this work has been to develop a mathematical model for leak detection and localization. I recommend the mathematical model developed by this study for it swiftness and ease of use. However, the accuracy of the leakage location is still an issue. Based on this, the following recommendations were made:

I. More work should be done to factor in parameter to quantify the leak size.
II. For this work, a zero inclination was assumed; further work should be carried out for pipelines with inclination.

III. For this work, a unidirectional flow was assumed; further study should be carried out for multi directional flow network of pipeline.

5.3 Contribution to Knowledge

This work has been able to bridge a knowledge gap on mathematical model based leak detection technique with the effective prediction of the pressure profile along a given length of a natural gas transmission pipeline. When there is variation above the acceptable error threshold in the pressure in and pressure out at any given segment of the natural gas transmission pipeline, it shows the presence of the leakage.

REFERENCES

Abhulimen, Kingsley E., & Susu, A. A. (2004). Liquid pipeline leak detection system: Model development and numerical simulation. *Chemical Engineering Journal*, *97*(1), 47–67.

Abhulimen, Kingsley. E., & Susu, Alfred. A. (2007). Modelling Complex Pipeline Network Leak Detection Systems. *Process Safety and Environmental Protection*, *85*(6), 579–598.

API. (2002). *API 1130: Computational Pipeline Monitoring for Liquid Pipelines*. https://global.ihs.com/doc_detail.cfm?document_name=API%201130&item_s_key=00405640

Arifin, B. M. S., Li, Z., Shah, S. L., Meyer, G. A., & Colin, A. (2018). A novel data-driven leak detection and localization algorithm using the Kantorovich distance. *Computers & Chemical Engineering*, *108*, 300–313. https://doi.org/10.1016/j.compchemeng.2017.09.022

Ben-Mansour, R., Habib, M. A., Khalifa, A., Youcef-Toumi, K., & Chatzigeorgiou, D. (2012). Computational fluid dynamic simulation of small leaks in water pipelines for direct leak pressure transduction. *Computers & Fluids*, *57*, 110–123. https://doi.org/10.1016/j.compfluid.2011.12.016

Bird, R. B., Stewart, W. E., & Lightfoot, E. N. (2006). *Transport Phenomena, Revised 2nd Edition | Wiley*. https://www.wiley.com/en-us/Transport+Phenomena%2C+Revised+2nd+Edition-p-9780470115398

Borener, S. S., Patterson, T., & E.G.& G Dynatrend. (2000). *SPILL REDUCTION TECHNOLOGY:* https://webcache.googleusercontent.com/search?q=cache:Mm9RQU-yuv8J:https://rosap.ntl.bts.gov/view/dot/9011/dot_9011_DS1.pdf%3F+&cd=13&hl=en&ct=clnk&gl=ng

Chuka, C. E., Freedom, I. H., & Anthony O, U. (2016). Transient Model-Based Leak Detection and Localization Technique for Crude Oil Pipelines: A Case of N.P.D.C, Olomoro. *Saudi Journal of Engineering and Technology*, *1*(2), 37–48. https://doi.org/10.21276/sjeat.2016.1.2.2

Colombo, A., Lee, P., & Karney, B. (2009). A selective literature review of transient-based leak detection methods. *Journal of Hydro-Environment Research*, *2*, 212–227. https://doi.org/10.1016/j.jher.2009.02.003

Da Silva, H. V., Morooka, C. K., Guilherme, I. R., da Fonseca, T. C., & Mendes, J. R. P. (2005). Leak detection in petroleum pipelines using a fuzzy system. *Journal of Petroleum Science and Engineering*, *49*(3–4), 223–238.

Dario, C. (2019). A method to obtain precise determinations of relative humidity using thin film capacitive sensors under normal or extreme humidity conditions. *Journal of Cultural Heritage*, *37*, 166–169. https://doi.org/10.1016/j.culher.2018.11.003

Fiedler, J. (2014, February 15). AN OVERVIEW OF PIPELINE LEAK DETECTION TECHNOLOGIES | American School of Gas Measurement Technology. *American School of Gas Measurement Technology | American School of Gas Measurement Technology*. https://asgmt.com/paper/an-overview-of-pipeline-leak-detection-technologies/

Geiger, G., Werner, T., & Matko, D. (2003). Leak Detection And Locating—A Survey. *Undefined*, 15–17. /paper/Leak-Detection-And-Locating-A-Survey-Geiger-Werner/186baf0a41e92b1fc500a23e46ae1610ba371aa3

Geiger, Gerhard. (2006a). State-of-the-Art in Leak Detection and Localisation. *Oil and Gas Journal; (United States)*, 1–26.

Geiger, Gerhard. (2006b). State-of-the-Art in Leak Detection and Localisation. *Pipeline Technology 2006 Conference*, 25.

Ghazali, M. F. (2012). *Leak detection using instantaneous frequency analysis* [Phd, University of Sheffield]. https://etheses.whiterose.ac.uk/2545/

Golmohamadi, M. (2015). Pipeline leak detection. *Masters Theses*. https://scholarsmine.mst.edu/masters_theses/7397

Hou, C. X., & Zhang, E. H. (2013). Pipeline Leak Detection Based on Double Sensor Negative Pressure Wave. *Applied Mechanics and Materials*, *313–314*, 1225–1228. https://doi.org/10.4028/www.scientific.net/AMM.313-314.1225

Korbicz, J., Kościelny, J., Kowalczuk, Z., & Cholewa, W. (2004). *Fault diagnosis. Models, artificial intelligence, applications*. https://doi.org/10.1007/978-3-642-18615-8

Li, Q., Du, X., Zhang, H., Li, M., & Ba, W. (2018). Liquid pipeline leakage detection based on moving windows LS-SVM algorithm. *2018 33rd Youth Academic Annual Conference of Chinese Association of Automation (YAC)*. https://doi.org/10.1109/YAC.2018.8406462

Muggleton, J., & Brennan, M. (2004). Leak noise propagation and attenuation in submerged plastic water pipes. *Journal of Sound and Vibration - J SOUND VIB*, *278*, 527–537. https://doi.org/10.1016/j.jsv.2003.10.052

Murvay, P.-S., & Silea, I. (2012a). A survey on gas leak detection and localization techniques. *Journal of Loss Prevention in the Process Industries*, *25*(6), 966–973. https://doi.org/10.1016/j.jlp.2012.05.010

Murvay, P.-S., & Silea, I. (2012b). A survey on gas leak detection and localization techniques. *Journal of Loss Prevention in the Process Industries*, *25*(6), 966–973. https://doi.org/10.1016/j.jlp.2012.05.010

Obanijesu, E., & Macaulay, S. (2009). West African Gas Pipeline (WAGP) Project: Associated Problems and Possible Remedies. In *Appropriate Technologies for Environmental Protection in the Developing World—Selected Papers from ERTEP 2007* (pp. 101–112). https://doi.org/10.1007/978-1-4020-9139-1_12

Oyedeko, K. F., & Balogun, H. A. (2015). Modeling and Simulation of a Leak Detection for Oil and Gas Pipelines via Transient Model: A Case Study of the Niger Delta. *Journal of Energy Technologies and Policy*, *5*(1), 16.

Perrys-chemical-engineering-handbook1. (n.d.). Retrieved June 5, 2021, from https://chembugs.files.wordpress.com/2015/12/perrys-chemical-engineering-handbook1.pdf

Ramos, H., Covas, D., Borga, A., & Loureiro, D. (2010). Surge damping analysis in pipe systems: Modelling and experiments. *Journal of Hydraulic Research*, *2004*, 413–425. https://doi.org/10.1080/00221686.2004.9641209

Sheltami, T., Bala, A., & Shakshuki, E. (2016). Wireless sensor networks for leak detection in pipelines: A survey. *Journal of Ambient Intelligence and Humanized Computing*, *7*. https://doi.org/10.1007/s12652-016-0362-7

Sukarno, P., Sidarto, K. A., Trisnobudi, A., Setyoadi, D. I., Rohani, N., & Darmadi, D. (2007). Leak Detection Modeling and Simulation for Oil Pipeline with Artificial Intelligence Method. *Journal of Engineering and Technological Sciences*, *39*(1), 1–19. https://doi.org/10.5614/itbj.eng.sci.2007.39.1.1

Udofia, O. O., & Joel, O. F. (2012, August 6). *Pipeline Vandalism in Nigeria: Recommended Best Practice of Checking the Menace*. Nigeria Annual International Conference and Exhibition. https://doi.org/10.2118/162980-MS

Verde, C. (2001). Multi-leak detection and isolation in fluid pipelines. *Control Engineering Practice - CONTROL ENG PRACTICE*, *9*, 673–682. https://doi.org/10.1016/S0967-0661(01)00026-0

Verde, C., & Visairo, N. (2001). Bank of nonlinear observers for the detection of multiple leaks in a pipeline. *Undefined*. /paper/Bank-of-nonlinear-observers-for-the-detection-of-in-Verde-Visairo/45568f750c464bbeacd1d71de4f50c920da1220a

Yousef, Y. A. (2018). *Offshore pipeline leak modeling using a computational fluid dynamics approach* [Masters, Memorial University of Newfoundland]. https://research.library.mun.ca/13506/

APPENDIX

Spreadsheet Program for Regression Analysis (Least Square Method)

```
Private Sub CommandButton1_Click()
''On Error GoTo 351
'nn = n
Dim summX(100, 100), summY(100, 100), B(1000), A(100, 100), F(100, 100), YM(1000) As Single, MF(1000)
Dim X(1000), Y(1000) As Single, XX(1000) As Single

Let P = 51     '10 ' mm '3 10 ' 2 '2 '12 '11 '12 '10 ' 12 ' 11 '12
pp = P
Let n = 2      '3 '2 'mm - 1 '2 '3 '2 '10 '9 '8 '7 '6 '5 '4 '3 '2 '8 ' 2 ' '4 '3 '2 '2 '2 '5 '20 '3 '2 '4 ' 2 '9 '3
Let m = n + 1
For i = 1 To P
'READ X1(I), X2(I), Y(I)
'Debug.Print X(i), Y(i)
X(i) = Sheet1.Cells(1 + i, 1) / 50 '= (X(i))
Y(i) = Sheet1.Cells(1 + i, 2) '= (Y(i))
Next i
' End
'GoTo 351
 '====================================
For i = 1 To n
For j = 1 To (m - 1)
summX(i, j) = 0
summY(i, m) = 0
Next j
Next i
For i = 1 To n
```

```
For j = 1 To (m - 1)
For k = 1 To pp
summX(i, j) = summX(i, j) + X(k) ^ (i + j - 2)
summY(i, m) = summY(i, m) + X(k) ^ (i - 1) * Y(k)
Next k
Next j
Next i
For i = 1 To n
summY(i, m) = summY(i, m) / n
Next i
'====================================
For i = 1 To n
For j = 1 To (m - 1)
Let A(i, j) = summX(i, j)
Let A(i, m) = summY(i, m)
Next j
Next i
Let C = 1
For i = C To n
For j = C To m
Let F(i, j) = A(i, j)
Let B(i) = A(i, C)
'Debug.Print A(i, j),
Sheet2.Cells(P + 3 + i, j) = A(i, j)
Next j
Sheet2.Cells(P + 3 + i, n + 1) = "a" & i - 1
Sheet2.Cells(P + 3 + i, n + 2) = "="
Sheet2.Cells(P + 3 + i, n + 3) = A(i, m)
'Debug.Print "  "
Next i
```

```
'GoTo 351
60 For k = C To (n - 1)
For LL = (k + 1) To n
If Abs(A(k, C)) >= Abs(A(LL, C)) Then GoTo 20
EMAT = A(k, C)
A(k, C) = A(LL, C)
A(LL, C) = EMAT
20 Next LL
Next k
Let EMAT = A(C, C)
For R = C To n
If EMAT = B(R) Then GoTo 10
 Next R
10 Let R = R
For k = C To m
 E = F(C, k)
 F(C, k) = F(R, k)
 F(R, k) = E
 Next k
Let AAs = C + 1
 For R = AAs To n
 MF(R) = F(R, C) / F(C, C)
 Next R
For R = AAs To n
 For Yy = C To m
Next Yy
 Next R
 If AAs = n Then GoTo 50
 C = AAs
 For i = C To n
```

```
 For j = C To m
 Let A(i, j) = F(i, j)
 Let B(i) = A(i, C)
 Next j
 Next i
 GoTo 60
'Debug.Print "=============================="
50  For i = 1 To n
For j = 1 To m
'Debug.Print F(i, j),
Next j
'Debug.Print "   "
Next i
XX(n) = F(n, m) / F(n, n)
 For i = (n - 1) To 1 Step -1
 summ = 0
 For j = (i + 1) To n
 summ = summ + F(i, j) * XX(j)
 Next j
 XX(i) = (F(i, m) - summ) / F(i, i)
 Next i
For i = 1 To n
 Let AAs = (i)
 'Debug.Print "C"; AAs; "="; XX(i)
  Sheet2.Cells(P + n + 3 + 2, 1) = "Solving:"
 Sheet2.Cells(P + n + 3 + 2 + i, n + 1) = "a" & i - 1
Sheet2.Cells(P + n + 3 + 2 + i, n + 2) = "="
 Sheet2.Cells(P + n + 3 + 2 + i, n + 3) = XX(i)
Next i
'GoTo 351
```

```
For i = 1 To n
'Let C(i) = XX(i)
Next i
'TO WRiTE THE MODEL
'Debug.Print "Y = "
For i = 1 To n
'Debug.Print "+"; XX(i); "X"; "^"; (i - 1); "'+";
Next i
'TO VERiFY THE MODEL
Sheet2.Cells(P + n + n + 3 + 3 + 3 + 2, 1) = "Verification:"
Sheet2.Cells(P + n + n + 3 + 3 + 3 + 3, 1) = "X"
Sheet2.Cells(P + n + n + 3 + 3 + 3 + 3, 2) = "Y(EXPT.)"
Sheet2.Cells(P + n + n + 3 + 3 + 3 + 3, 3) = "Y(MODEL)"
 'Debug.Print "0000000000000000000000000000"
' Debug.Print "0000000000000000000000000000"
 'Debug.Print "0000000000000000000000000000"

'Debug.Print "X", "Y(EXPT.)", "Y(MODEL)"
'Debug.Print " "
For j = 1 To pp
summ = 0
For k = 1 To n
summ = summ + XX(k) * X(j) ^ (k - 1)
Next k
Let YM(j) = summ
Debug.Print X(j), Y(j), YM(j)
Sheet2.Cells(P + n + n + 3 + 3 + 3 + 3 + j, 1) = X(j)
Sheet2.Cells(P + n + n + 3 + 3 + 3 + 3 + j, 2) = Y(j)
Sheet2.Cells(P + n + n + 3 + 3 + 3 + 3 + j, 3) = YM(j)
Next j
```

```
 Debug.Print "++++++++++++++++++++++++++++++++++++++++++++++++++"
'End
'kk = 12,  '10 '9 '8 '7 '6 '5 '4 '3 '2 '1
 't1 = n1 + (kk - 1) * (n2 - n1) / (12 - 1)
'DYdt = XX(1) + XX(2) * t1
y1 = (-XX(1) + (XX(1) + XX(2) * Y_tau) * Exp(XX(2) * (t - tau))) / XX(2)
'Y1 = Y0 - Yt / DYdt
'summ1 = 50 - 2 * Exp(-0.196 * t) '1 / (1 - t)
'summ1 = 5 / 2 * Exp(2 * t) - 3 / 2 ' 2 / 5 * Exp(-3 * t)
'summ1 = 2 / 3 * Exp(-3 * t) + 1 / 3 '3 / 2 ' 2 / 5 * Exp(-3 * t)
summ1 = 1 / 4 * t ^ 2 - 1 / 3 * t + 1 / 2 + 1 / (12 * t ^ 2)
'Debug.Print Y0, DYdt, Yt, Y1
 Debug.Print t, summ1, y1
'End
'Next kk
End
```