FORCED VIBRATION ANALYSIS OF RECTANGULAR CLAMPED PLATE SUBJECTED TO HYDROSTATIC LOADS

BY

KOLAWOLE SUNDAY ADEDUNTAN (B.ENG.)

REG NO. 20104771068

A THESIS SUBMITTED TO THE POSTGRADUATE SCHOOL, FEDERAL UNIVERSITY OF TECHNOLOGY, OWERRI

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF MASTER OF ENGINEERING (M.ENG) DEGREE IN CIVIL ENGINEERING (STRUCTURES)

JULY, 2015.

CERTIFICATION

I certify that this work **"Forced Vibration Analysis of Rectangular Clamped Plate Subjected to Hydrostatic Loads"** was carried out by Adeduntan, K.S. (B. Eng.) with Reg. No. (20104771068) in partial fulfillment of the requirements for the award of Master of Engineering (M.Eng.) degree in Civil Engineering (Structures) in the School of Engineering and Engineering Technology, Federal University of Technology, Owerri, Nigeria.

....	16/07/2015 ....
Engr. Dr. J. C. Ezeh Principal Supervisor	Date
....	16/07/2015 ....
Engr. Dr. O. M. Ibearugbulem Co - Supervisor	Date
....	28/08/15 ....
Engr. Dr. Mrs. B. U. Dike Head of Department	Date
....	31/08/15 ....
Engr. Prof. G. I. Nwandikom Dean of SEET	Date
....	....
Engr. Prof. K. B. Oyoh Dean of Postgraduate School	Date
....	16/7/15 ....
Engr. Prof. O. O. Ugwu External Examiner	Date

DEDICATION

To God Almighty, I dedicate this research work, for all his goodness in my academic endeavors.

ACKNOWLEDGEMENTS

I wish to express my profound gratitude and indebtedness to my supervisor, Engr. Dr. J.C Ezeh for initiating and supervising this research project. I am also grateful to my other supervisor Engr. Dr. O. M. Ibearugbulem for his valuable guidance, constant encouragement and kind co-operation throughout the period of work, which has been instrumental in the success of this research work. I acknowledge the Dean of School of Engineering and Engineering Technology Engr. Prof. E. E. Anyanwu and the Dean of Postgraduate School Engr. Prof. Mrs. K.B. Oyoh for their tireless work to uphold academia. My special thanks go to my lecturers Engr. Dr. D. O. Onwuka, Engr. Dr. C. Okere, Engr. Dr. B. U. Dike, Engr. Dr. B.C. Okoro, Engr. Prof. O.O. Ugwu, Engr.Dr. B. Mama, Engr. Dr. Rev. L.O. Ettu, and Engr. M.S. Mbajiorgu. I am equally grateful to Engr. U.C. Anya, Engr. Dr. J. Osuagwu, Engr. Dr. H. Nwoke, Engr. F. Njoku, Engr J. Arimanwa, Engr. Lewechi Anyaogu, Engr. K. C. Onyema, Engr. C. Awodiji, Engr. K.C. Nwachukwu, Engr N.L Nwakwasi, Engr. A. P. Amanze and Engr. A.N. Nwachukwu for sacrificing their time to give me important advice that encouraged me during this research project.

I would like to thank all the members of the Department of Civil Engineering, Federal University of Technology, Owerri (FUTO), who helped me by providing the necessary resources and in various other ways for the completion of my work. I also acknowledge the effort of my good parents Mr. and Deaconess Z. A. Adeduntan for their encouragement, love and financial assistance.

I also acknowledge the efforts of my Boss Engr. O. S. Azeez who inspired me to take up this course, my lovely wife Mrs. M.O. Adeduntan, for her understanding and encouragement throughout the course of this project works. I would like to thank Engr. Oguaghamba Onyedikachi.A. and all those who have directly or indirectly supported me in carrying out this research work successfully. Finally, I thank God almighty for everything.

Adeduntan, K.S.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF PLATES

DEFINITION OF NOTATIONS

NOTATION	MEANING
∇^4	Biharmonic operator
a and *b*:	Rectangular plate dimensions
D:	Flexural Rigidity
E:	Young's modulus
h:	Plate thickness
L:	Linear differential operator
W:	Deflection
Π:	Functional
ρ:	Material density
λ :	Fundamental Natural frequency
Fi(x, y):	Variable of the shape function
$\sqrt{\bar{n}}$:	Co-efficient of fundamental natural frequency
A:	Shape function
p:	Aspect ratio
F (t):	Applied force
u:	Displacement
R&Q:	Non dimensional parameter of x and y respectively
C:	Damping factor
8^2:	Force frequency
υ	Poisson's ratio
s & p	In – plane str

ABSTRACT

This study presents forced vibration analysis of rectangular clamped plate subjected to hydrostatic load. Galerkin's equilibrium equation of plate under forced vibration was used. Orthogonal polynomial deflection equation of a plate under hydrostatic load was also used. The deflection equation was substituted into the Galerkin equation and integrated within the closed domain. After integration, the natural frequency of the plate was determined for free vibration. The coefficient of the deflection for the various cases of forced vibration was also determined. Different percentages (0%, 20%, 40%, 60%, 80% and 100%) of the fundamental natural frequency were used as various forcing frequencies. With these frequencies, this study obtained deflection, bending moment and shear force of the plate for different values of aspect ratios (p=a/b), where a and b are the plate dimensions along x and y-axes. The values of fundamental natural frequency obtained were compared with those from Ventsel and Krauthemmer, Galin and Janich. For a square plate, the fundamental natural frequency obtained is 35.9982Hz and those of Ventsel and Krauthemmer, Galin and Janich are 36.3485Hz, 36.000Hz and 37.2500Hz respectively. For the same square plate, the mid span deflection using 0%, 20%, 40%, 60%, 80% and 100% of fundamental natural frequency are 0.00131m, 0.00164m, 0.00218m, 0.00327m and 0.00655m respectively. The edge bending moments of the square plate for same forcing frequencies are 0.0503KNm, 0.0621KNm, 0.0828KNm, 0.1257KNm and 0.2513KNm respectively. It was observed that deflection, bending moment and shear force increase as the forcing frequency increases.

Keywords: Forced Vibration, Uniformly Distributed Load, Indirect Variational Method; Galerkin Method; Fundamental Natural Frequency; Characteristic Orthogonal Polynomials.

CHAPTER ONE
INTRODUCTION

Liquid containing structures, storage reservoirs, or tanks are used to store water, liquid, petroleum, petroleum products and similar liquids. Depending upon the location of the tank, the tank can be named as overhead, on ground or underground. Overhead water tank and underground water reservoirs are the most effective storing facilities used for domestic or even industrial purpose. Types of storage tank based on placing and shape are given in Figure 1.1. Circular tanks have minimum surface area when compared to other shapes for a particular capacity of storage required. Hence, the quantity of material required for circular water tank is less than required for other shapes. But the formwork for a circular tank is very complex and expensive when compared with other shapes. Square and Rectangular tanks are generally used underground or on the ground.

The tanks can be made of reinforced concrete or even of steel. Liquid containing structures (LCS) when used as part of environmental engineering facilities are primarily used for water and sewage treatment plants and other industrial wastes. In this way, they are generally constructed of reinforced concrete in the form of rectangular or circular configurations. The overhead tanks are usually elevated from the rooftop through column. In the other hand, the underground tanks are rested on the foundation. One of the vital considerations for design of tanks at present is that the structure should have adequate resistance to cracking and has adequate strength to resist tension and moments that act on the component. Present design practices assume that the load generated by the water load is only from static. As it has being the practice in statics to assume that all external forces are applied slowly: so slowly that the loads and the resulting stresses and deformations are independent of time. Therefore, the trend in carrying out design for such a system has never been elaborate as to include this additional load generated due to fluid transmission.

Reservoir tanks are recharged by either gravity pressure or machine pump which induced excitation. During this process of recharge, the force exerted on the walls of the tank will increase or vary over the period, which the tank is recharged. Hence, the water exerts a time – dependent load on the walls of the reservoir. That is, the weight exerted by the fluid at any time t, varies. These loads are generally regarded as dynamic loads. Also, according to Mitchell, (1972); and Bangash, (1993), most loads acting on structures are dynamic in origin. In his work, concurred that all loadings, from whatever source are variable with time, due either to variation of field strength, variations in the structure or its contents or inherent nature of the loadings.

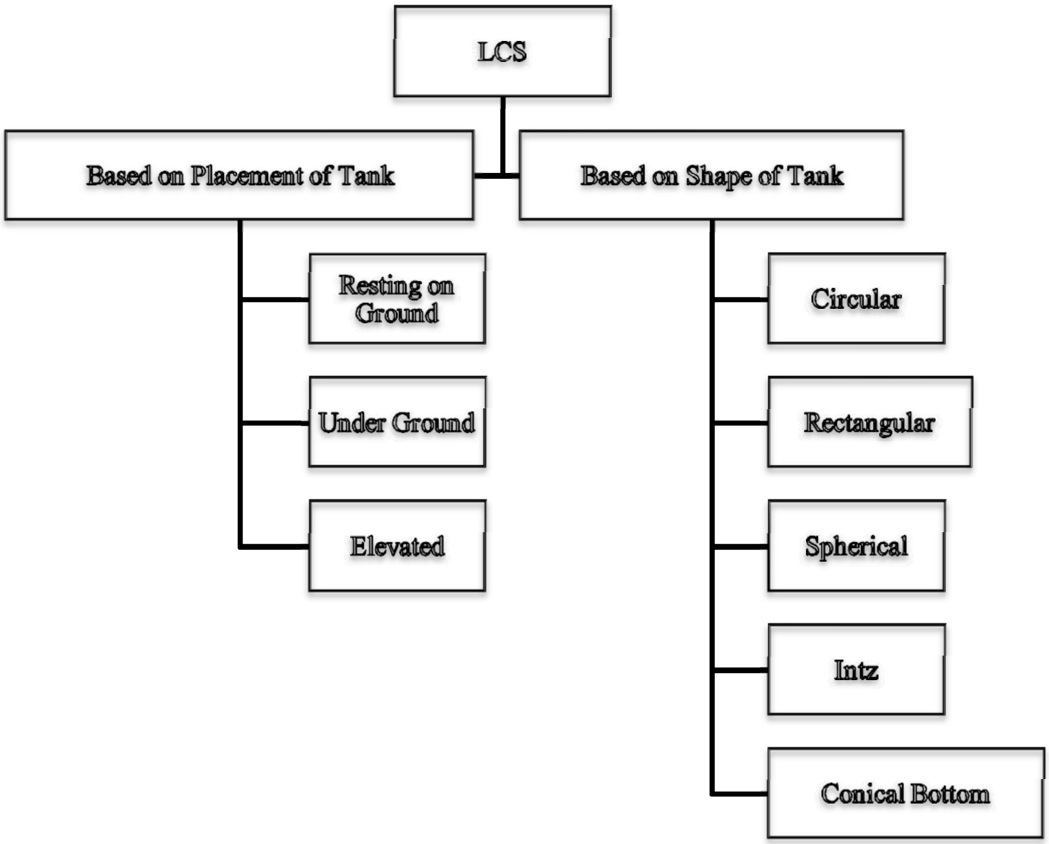


Figure 1.1: Types of Water Tank Based on Placing and Shape

Structural dynamics deals with time-dependent motions of structures, primarily, with vibration of structures, and analyses of the internal forces associated with them. Thus, this study intends to determine the dynamic effect on sloshing effects on rectangular liquid containing structures using Galerkin's method. The simplest form of water tank is circular tank for the same amount of storage the circular tank requires lesser amount of material. However, rectangular tanks are more economical than circular because the construction of circular tanks requires complicated and costly formwork. More over compartmentalization in a rectangular tank is much easier than the circular tanks. Moreover, users of rectangular tanks make the full use of the space available. This study would focus on rectangular tank, because of not only the outlined reasons but also its predominance use.

The main components of a rectangular tank are sidewalls, base slab and roof slab. These components are rigidly (fixed) jointed as to achieve high-level fluid content – tightness. The sidewall which is subjected to hydrostatic pressure distribution that varies from zero at top to maximum value at base represents a thin rectangular plate (C-C-C-C) fixed at edges and subjected to varying loads. The bottom of the tank represents a thin rectangular plate clamped on all edges (C-C-C-C) and subjected to uniformly distributed load. The roof slab represents a thin rectangular plate clamped on all edges (C-C-C-C) and subjected to self-weight due to gravity. All these components of the tank would be subjected the dynamic effect during the recharge of the tank.

This study aims at studying the vibration of these componential parts of the tank during its recharge and the resultant dynamic stresses due this dynamic effect would be established, contrary to the usual design where the stress analysis are done on the premise that the dynamic effect are negligible. Thus, the dynamic analysis of water storage tanks boils down to the vibrations analysis of rectangular plate subjected to self-weight, varying load and uniformly distributed loads.

The dynamics of plates, which are continuous elastic systems, can be modeled mathematically by partial differential equations based on Newton's laws or by integral equations based on the considerations of virtual work. In practical applications, only the lateral vibration is of interest, and the effects of extensional vibrations in the middle plane may be neglected. Therefore, the inertia forces, associated with the lateral translation of the plate, are considered.

Plate 1.2: Rectangular Liquid Retaining Tank Resting On Ground

Plate1.3: Elevated Rectangular Liquid Retaining Tank

Three approaches have been established for the solutions of vibration of elastic thin plates. They are the equilibrium (Euler) approach, the energy (approximate) approach and the numerical approach (Ibearugbulem, 2012). The Euler approach tends to find solution of the governing differential equation by direct integration and satisfying the boundary conditions of the four edges of the plate. Direct integration leads to the exact solution. An exact solution of the governing differential equation in closed form is possible only for a limited number of cases regarding a plate's geometry and its boundary conditions. As mentioned previously, the natural fundamental frequencies are of the greatest importance in practice. Therefore, one has to analyze the vibration of thin rectangular plates by the approximate methods. Numerical approach is a good alternative to the Euler approach. Some examples of this approach include truncated double Fourier series, finite difference, finite strip, Runge-Kutta and finite element methods among others. Methods of numerical approaches have the capacity of handling plates of various boundary conditions. It has been shown from past works that in most cases, the solution from numerical approach approximate closely those of the exact approach Ventsel and Krauthammer, (2001). The problem with these numerical solutions is that the accuracy of the solution is dependent on the amount of work to be done.

For instance, if one is using finite element method, the more the number of elements used in the analysis the closer the approximate solution to the exact solution. Hence, when a plate has to be divided into several elemental plates for an accurate solution to be reached, then the extensive analysis is involved, requiring enormous time to be invested. Outside the time input, extensive analysis and the great volume of data generated will be difficult to condense into design charts and tables. In view of this, bandwidth control becomes necessary in matrix formulation. A sound knowledge in mathematics and skillful experience in computer programming is inevitable in this case. At this point one will see vividly that the problem one is trying to avoid in equilibrium approach is still found in numerical approach.

Energy approach is another method that can be used. This approach is quite different from Euler and numerical approaches. The solution from it agrees approximately with the exact solution. Typical examples of energy approaches are Ritz, Raleigh-Ritz; Garlekin, minimum potential energy etc. These methods are called variational methods. Ritz and Raleigh – Ritz are grouped as direct variational methods while Galerkin’s forms the indirect variational method. They seek to minimize the total potential energy functional in other to solve the differential equation. The total potential energy function is a function of plate deflection function. The accuracy of the solution is dependent on the accuracy of the approximate deflection function (shape function). For Raleigh – Ritz, the approximate shape function is substituted in the total potential energy function, and the resulting equation is partially differentiated. The total potential energy will be said to be minimized when its partial derivative is equated to zero. This implies that the difference between the approximate and exact solutions is zero (Iyengar, 1988). But, in Galerkin’s method, the conditions orthogonality is applied. In this case, the approximate solution and the governing differential equation of plate in dynamic regime forms the two set of mutually orthogonal functions. Whereas the shape function is approximate, the two set of functions would be longer orthogonal. Hence, the functions would be required to be minimum. This requirement is equivalent to the condition that the above function should be orthogonal to some bounded set of functions: first, to the approximating function. This leads to Galerkin’s minimum total potential energy for a rectangular plate in vibration regime. However, the Galerkin method can be derived from the general variational principle of virtual work. The more the approximate shape function gets closer to the exact shape function the more the approximate solution gets closer to the exact solution. Many scholars have used trigonometric series in this approach. For instance, trigonometric series can be used to formulate approximate shape function for a plate, whose four edges are simply supported or clamped. It can also be used for a plate whose opposite edges are clamped and the other opposite edges are simply supported. However, it is extremely difficult to formulate a shape function for plates using trigonometric series when opposite edges are clamped and simply supported like propped cantilever beams. Examples of plates, whose shape function cannot be formulated using trigonometric series, include two edge simply supported and two edge clamped plate (SSCC), one edge simply supported and three edge clamped plate (SCCC), one edge clamped and three edge supported plate (CSSS), two edge simply supported, one edge clamed other edge fixed plate (SSCF), one edge clamped, two edge simply supported and the other edge fixed plate (CSSF) etc. Some other boundary conditions make it difficult to use the trigonometric series Ugural, (1999), Iyengar, (1988), Ventsel and Krauthammer, (2001).

Because of these limitations of energy method using trigonometric series to formulate the approximate shape functions, one will be tempted to use the numerical approach. In the light of the above problems, researches in thin plate vibration are going on so as to obtain solutions that are very close approximation to exact solution, and at the same time reduce the volume of computation. Consequently, this research sought to formulate shape function of all edge clamped plate (C-C-C-C) subjected to uniformly and triangular varying loads using characteristic orthogonal polynomials. The resulting approximate shape functions are substituted into the total potential energy function, which is then minimized to analyze the vibration of the components part of the rectangular storage tank during its recharge. It is in attempt to analyze the dynamic - regime of rectangular storage tank during recharge using indirect variational method that gave birth to the research topic “Forced Vibration Analysis of Rectangular Clamped Plate Subjected to Hydrostatic Loads using Galerkin’s Method”

1.1 Statement of Problem

One of the vital considerations for design of tanks at present is that the structure should have adequate resistance to cracking and adequate strength. In statics, it is assumed that all external forces are applied slowly; so slowly that the loads and the resulting stresses and deformations are independent of time. This design approach has led to wrong design specifications. Dynamic loads may be created by moving vehicles, wind gusts, seismic disturbances, unbalanced machine vibrations, flight loads, sound, fluid transfer into a reservoir etc.

Reservoir tanks are recharged by either gravity pressure or machine pump induced pressure, depending on the location of the tank. During this process of recharge, the force exerted on the walls of the tank will increase or vary over the period, which the tank is recharged. Hence, the water exerts a time – dependent load on the walls of the reservoir. With all these problems, the present research use energy approach in the form of a Galerkin's indirect variational principle to analyze the effect of forced dynamic regime on the walls of rectangular tank. The approximate shape function will be formulated by using characteristic orthogonal polynomials as against using trigonometric functions.

1.2 Objectives of Study

The main objective of this study is forced vibration analysis of rectangular clamped plate subjected to hydrostatic loads. The specific objectives are:

a) To use characteristic orthogonal polynomial theorem in Galerkin's method for vibration analysis of rectangular water tank, which is composed of thin rectangular flat plates.

b) To find how close the obtained solutions of rectangular plate clamped on all edges using characteristic orthogonal polynomials is to exact and other approximate solutions.

c) To carry out comprehensive study of rectangular plate in dynamic regime due to uniformly and varying load excited forced vibration using Galerkin's indirect energy theorem applied to characteristic orthogonal polynomials for plate clamped on all edges.

1.3 Justification of Study

Outside addressing the problems stated here in this research, the research is also making the following contributions:-

a) It is contributing in addressing problem of dearth of literature in the study of thin rectangular plate under forced vibration.

b) It is contributing in addressing problem of finding an adequate and accurate approximate shape functional for the plate under free vibration problems using characteristic orthogonal polynomial theorem.

c) It exposed the potentials of the use of characteristic orthogonal polynomial theorem as against trigonometric series in analysis of plates and shells problems.

1.4 Scope of Study

The main components of a rectangular tank are sidewalls, base slab and roof slab. These components are rigidly (fixed) jointed as to achieve high-level fluid content – tightness. The sidewall which is subjected to hydrostatic pressure distribution that varies from zero at top to maximum value at base represents a thin rectangular plate fixed at edges(C-C-C-C) and subjected to varying loads. The bottom of the tank represents a thin rectangular plate clamped on all edges (C-C-C-C) and subjected to uniformly distributed load. The roof slab represents a thin rectangular plate clamped on all edges (C-C-C-C) and subjected to self-weight due to gravity. All these components of the tank would be subjected to dynamic effect during the recharge of the tank. This study aims at studying the vibration of these component parts of the tank during its recharge and the resultant dynamic stresses due to this dynamic effect would be established, contrary to the usual design where the stress analysis are done on the premise that the dynamic effect are negligible. Thus, the dynamic analysis of water storage tanks boils down to the vibrations analysis of rectangular plate subjected to self-weight, varying load and uniformly distributed loads. Literature reviews of past works in this regard were made in this research. This enabled the research to discover some unanswered theoretical questions, and know the extent past scholars had gone in this direction. This was to avoid repetition of a study that had been made before now.

CHAPTER TWO

LITERATURE REVIEW

2.1 Historical Development in Plate Vibration Problems

Plates are structural elements that are frequently subjected to vibration and controlling the frequency at which plate vibrates is very important to structural designers. Vibration by definition is referred to as a periodic motion of a body or system of interest when it passes through the equilibrium point each cycle. Alternatively, it is also defined as a phenomenon that involves alternating interchange of kinetic and potential energies. This requires system to have a compliance component that has the capacity to store the potential energy (spring for example) and component that has capacity to store the kinetic energy (mass or inertia). The study of vibration is an important engineering aspect because of both useful and deprecating effects of vibration of machine elements and structures. In many engineering applications, vibratory motion is required as in the case of hoppers, compactors, clocks, pile drivers, and vibratory conveyers, material sorting systems, vibratory finishing process and many more. On the other hand, undesirable vibrations result in premature failure of many components such as blades in turbines and aircraft wings. Vibration also causes a high rate of wear in machine elements, as well as annoys human operators due to noise. Failures of bridges, buildings and dams are commonly attributed to vibration caused by earthquakes or with vibrations caused by wind loads. This explains the necessity to understand vibration behaviour of various systems and eliminate or otherwise reduce vibrations by change in design or by designing suitable control mechanisms.

The analysis of the free vibration of plates were well documented by Leissa (1973) and includes a variety of boundary condition, and aspect ratio using trigonometric series. Gorman (1982) solved problem on free vibration analysis of all edge simple supported plate (SSSS) and two edge clamped and the other two edge simply supported plates (CSCS) plates, using single Fourier series of solutions by the super position method, in which the natural modes were expressed in trigonometric and hypermetric series, and the number of terms in series depended on the requirements of precision. Free and forced vibrations of circular plates having rectangular orthotropic was investigated by (Laura and Sanchez, 1998), vibrations of circular plate with variable thickness on elastic foundation Topalian et al (1997), vibrations of circular plate having polar anisotropy having concentric circular support (Gutierrez and Laura, 2000), vibrations of free-free annular plate .Vera et al (1999), are some of many works on plate vibrations by Laura.

The solution methods for solving the plate vibrations have also evolved with time, conditions having an isotropy or variable thickness. Laura applied the method of differential quadrature to the circular plate vibrations (Laura and Gutierrez, 1995), the same method was applied then by various researchers including Wu et al (2002) for circular plates with variable thickness, Gupta et al (2006), for non-homogeneous circular plates having variable thickness.

Hsu (2003) modelled the vibration response of isotropic and orthotropic plates with mixed boundary conditions numerically using a solution that is based on the differential quadrature method (DQM). The DQM uses the basis of the Gauss method in deriving the derivative of a function. Chen et al (2008) modelled seismic design of concrete rectangular Liquid Containing Structure (LCS) using the generalized single degree of freedom (SDF) system. The model considered the effect of flexibility of tank wall on hydrodynamic pressures and used the consistent mass approach. The model can be simply used in seismic design of LCS.

In the novel superposition method introduced by Kshirsagar and Bhaskar, (2008), the intrinite series counterparts of conventional levy-type closed – form expressions were used to simplify the solution procedure without any compromise on accuracy. Filipich and Rosales, (2000) developed a variational method for the frequency analysis of a free rectangular thin plate within the Germain- Lagrange theory, and Seok et al, 2004) studied the free vibration of a cantilevered plates based on a variation approximation approach, in which the differential equation and the conditions on either side of the width were satisfied exactly, and the conditions the free and fixed edges were satisfied variationally. Hedrih (2006), Investigated the free and forced transversal vibrations of an elastically connected double- plate system based on the classical theory of thin pltes and mode superposition method. Ouyang and Zhong (1993), Zhong and Zhang (2006), Bao and Deng (2005) applied Hamilton dual method to the analyses of modes and frequencies of thin plates. Venksel and krauthammar (2001) used circular polynomial function, which satisfied only the geometric boundary condition as shape function and applied it to Rayleighh Ritz method which reduced to fundamental natural frequency of all edge clamped (CCCC) rectangular plates.

Ezeh et al (2014) used theoretical formulation based on Ibearugbulem's shape function and application of Ritz method. The free vibration of simply supported panel with on free edge was analysed. The Ibearugbulem's shape function derived was substituted into the potential energy functional which was minimized to obtain the fundamental natural frequency.

Free vibrations of rectangular plates that are clamped on all edge have been studied by many researchers in the past with the aim of calculating the natural frequencies and this they have done using numerical approaches Lee, (2004); Shi, (1990); Werfalli; and Karoud, (2005) and Misra, (2012) and energy variational method. (lal et al., 2009), Shu et al., (2007),Sakata et al., (1996). Njoku, (2013) used Taylor series peculiar shape function for CCC isotropic thin rectangular plates on Galerkin's functional to determine the fundamental frequency of the plate under vibration. none of the existing from past works go beyond using numerical, vibrational method, and Galerkin's method for free vibrational analysis of thin rectangular plates with the aim of calculating its natural frequency beyond calculating fundamental natural frequency under free vibration but to present force vibration analysis of rectangular clamped plate subjected to hydrostatic loads. Galerkin's equilibrium equation of plate under force vibration was used with orthogonal polynomial deflection equation a plate under hydrostatic load.

2.2 Types of Plates

Geometrically, plates are bounded either by straight or curved boundaries. The distance between the plane faces is called the thickness, h of the plate Ventsel and Krauthammer (2001).A plate resists transverse loads by means of bending, exclusively. The flexural properties of a plate depend greatly upon its thickness in comparison with other dimensions. Plates, they added may be classified into three groups according to the ratio

a/h where a is a typical dimension of a plate in a plane and h is a plate thickness.

These groups are:

The first group presented by thick plates having ratios $a/h \leq 8\ldots10$. The analysis of such bodies they said includes all the components of stresses, strains and displacements as for solid bodies using the general equations of three-dimensional elasticity.

The second group refers to plates with ratios $a/h \geq 80\ldots100$. These plates are referred to as membranes and they are devoid of flexural rigidity. Membranes, according to (Timoshenko and Woinowsky – Krieger, 1970) carry the lateral loads by axial tensile forces N (and shear forces) acting in the plate. These forces they called membrane forces;

which produce projection on a vertical axis and thus balance a lateral load applied to the plate – membrane.
The most extensive group represents an intermediate type of plate, so called thin plate with 8…10 $\leq a/h \leq$ 80 . . . 100. Depending on the value of the ratio w/h, the ratio of the maximum deflection of the plate to its thickness, the part of flexural and membrane forces here may be different. Therefore, this group, Ventsel and Krauthammer (2001) said, in turn, may also be subdivided into two different classes.

Stiff Plates: A plate can be classified as a stiff plate, they said, if $w/h \leq 0.2$. Stiff plates are flexurally rigid thin plates. They carry loads two dimensionally, mostly by internal bending and twisting moments and by transverse shear forces, they disclosed. Continuing, they opined that for such a plate, the middle plane deformation and the membrane forces are negligible.

Flexible Plates: If the plate deflections are beyond a certain level $w/h \geq 0.3$, then, the lateral deflections will be accompanied by stretching of the middle surface. Such plates are referred to as flexible plates. These plates represent a combination of stiff plates and membranes and carry external loads by the combined action of internal moments, shear forces, and membrane (axial) forces. "Such plates, because of their favourable weight – to – load ratio, are widely used by aerospace industry;" they added. When the magnitude of the maximum deflection is considerably greater than the plate thickness, the membrane action predominates. So, if $w/h > 5$, the flexural stress can be neglected compared with the membrane stress. Consequently, the load – carrying mechanism of such plates becomes the membrane type, i.e., the stress is uniformly distributed over the plate thickness Ventsel and Krauthammer (2001).In this study, only small deflections of thin plates would be considered. That is type 3a.

2.3 Vibration of plates

The vibration of plates is a special case of the more general problem of mechanical vibration. Rectangular plates have wide application in Civil and Mechanical Engineering. The dynamic characteristics of rectangular plates are important in engineering designs. The equations governing the motion of plates are simpler than those for general three dimensional objects because one of the dimensions of a plate is much smaller than the other two. This suggests that a two dimensional plate theory will give an excellent approximation to the actual three – dimensional motion of a plate like object, and indeed that is found to be true. (Reddy, 2007).
There are several theories that have been developed to describe the motion of plates. The

most commonly used are the Kirchhoff- Love theory (Love, 1888) and Mindlin – Reissner theory. Solutions to the governing equation predicted by these theories can give us insight into the behaviour of plate like objects both under free and forced conditions. This includes the propagation of waves and the study of standing waves and vibration modes in plates.

2.3.1 Isotropic Kirchhoff- love plates.

For an isotropic and homogeneous plate, the stress- strain relations are

$$\begin{bmatrix} a_{11} \\ a_{22} \\ a_{12} \end{bmatrix} = \frac{E}{1-v^2} = \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & 1-v \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{22} \\ s_{12} \end{bmatrix} \tag{2.1}$$

Where $s \& p$ are the in-plane strains. The strain – displacement relations for Kirchhoff- love plates are

$$s \& p = \frac{1}{2}(\mu_\alpha, p + \mu_p, \alpha) - x_3 w, \alpha p \tag{2.2}$$

Therefore, the resultant moments corresponding to these stresses are

$$\begin{bmatrix} M_{11} \\ M_{22} \\ M_{12} \end{bmatrix} = \frac{2h}{3(1-v^2)} = \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & 1-v \end{bmatrix} \begin{bmatrix} w_{11} \\ w_{22} \\ w_{12} \end{bmatrix} \tag{2.3}$$

If we ignore the in plane displacements $\mu_{\alpha p}$, the governing equations reduces to

$$D\nabla^2(\nabla^2 w) = -q(x,t) - 2\rho h\ddot{w} \tag{2.4}$$

The above equation can also be written in an alternative notion;

$$u\Delta\Delta w + q + qwtt = 0 \tag{2.5}$$

The relation can be derived in an alternative manner by considering the curvature of the plate, courant et al (1953).

In solid mechanics a plate is often modelled as a two dimensional elastic body whose potential energy depends on how it is bent from a planar configuration, rather than how it is stretched (which is the instead the case for a membrane such as a drum head).

In such situations, a vibrating plate can be modelled in manner analogous to a vibrating drum. However, the resulting partial differential equation for the vertical displacement W of a plate from its equilibrium position is fourth order, involving the square of the laplacian of W, rather than second order, and its qualitative behaviour is fundamentally different from that of the circular membrane drum.

2.3.2 Mindlin Theory for Isotropic Plates

The Mindlin – Reissner theory of plates is an extension of Kirchhoff Love plate theory that takes into account shear determinations through the thickness of a plate. The theory was proposed in 1951 by Raymond Mindlin. A similar but not identical, theory had been proposed earlier by Eric Reissner in 1945. Both theories are intended for thick plates in which the normal to the mid surface remain straight but not necessarily perpendicular to the mid surface.

The Mindlin – Reissance theory is used to calculate the deformations and stress in a plate whose thickness is of the order of one tenth the planar dimensions. While the Kirchoff – Love theory is applicable to thinner plates.

The form of Mindlin- Reissner plate theory that is most commonly used is actually due to Mindlin and is more properly called Mindlin plate theory Wang et al (2001).

The Reissner theory is slightly different. Both theories include in plane shear strains and both are etensions of kirchhof- Love plate theory incorporating first order shear effects.

Mindlin's theory assumes that there is a linear variation of displacement across the plate thickness but that the plate thickness does not change during deformation. An additional assumption is that the normal stress through the thickness is ignored; an assumption which is also called the plane stress condition. On the other hand, Reissner's theory assumes that the bending stress is linear while shear stress is quadratic through the thickness of the plate.

This leads to a situation where the displacement through the thickness is not necessarily and where the plate thickness may change during deformation. Therefore Reissner's theory does not invoke the plane stress condition. The Mindlin-Reissner theory is often called the first order shear deformation theory of plates. Since a first order shear deformation theory implies a linear displacement variation through the thickness, it is in compatible with Reissner's plate theory.

For uniformly thick, homogenous and isotropic plates, the stress- strain relations in the plane of the plate are

$$\begin{bmatrix} a_{11} \\ a_{22} \\ a_{12} \end{bmatrix} = \frac{E}{1-v^2} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & 1-v \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{22} \\ s_{12} \end{bmatrix} \qquad (2.6)$$

Where E is the young's modulus, V is the Poisson's ratio, and s & p are the in- plane strains. The through the thickness shear stresses and strains are related by

$$a_{31} = 2Gs_{31} \text{ and } a_{32} = 2Gs_{32} \qquad (2.7)$$

G= $\frac{E}{2(1+V)}$ is the shear modulus.

The relations between the stress resultants and the generalized deformations are

$$\begin{bmatrix} n_{11} \\ n_{22} \\ n_{12} \end{bmatrix} = \frac{2Eh}{1-v^2} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & 1-v \end{bmatrix} \begin{bmatrix} u_{1.1} \\ u_{2,2} \\ \frac{1}{2}(u_{1,2}+u_{2,2}) \end{bmatrix} \qquad (2.8)$$

$$\begin{bmatrix} M_{11} \\ M_{22} \\ M_{12} \end{bmatrix} = \frac{2Eh^3}{3(1-v^2)} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & 1-v \end{bmatrix} \begin{bmatrix} Q_{1.1} \\ Q_{2,2} \\ \frac{1}{2}(Q_{1,2}+Q_{2,2}) \end{bmatrix} \qquad (2.9)$$

And

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = kGh \begin{bmatrix} w_{1}-Q_1 \\ w_{2}-Q_2 \end{bmatrix} \qquad (2.10)$$

The bending rigidity is defined as the quality

$$D = \frac{2Eh^3}{3(1-v^2)}$$

(2.11)

Governing Equation

If we ignore the in-plane extension of the plate, the governing equations are

$$M_{\alpha\beta}\beta - Q_{\alpha} = 0 \quad (2.12)$$

$$Q_{\alpha,\alpha} + q = 0 \quad (2.13)$$

In forms of the generalized deformations, these equations can be written as

$$\nabla^2 \left(\frac{6Q_1}{6s_1} + \frac{6Q_2}{6s_2}\right) = -\frac{q}{D} \qquad (2.14)$$

$$\nabla^2 w^o - \left(\frac{6Q_1}{6s_1} - \frac{6Q_2}{6s_2}\right) = -\frac{q}{kGh} \qquad (2.15)$$

$$\nabla^2 \left(\frac{6Q_1}{6s_2} - \frac{6Q_2}{6s_1}\right) = \frac{2kGh}{D(1-v)}\left(\frac{6Q1}{6s_2} - \frac{6Q2}{6s_1}\right) \qquad (2.16)$$

The boundary conditions along the edges of a rectangular plate are

For simply supported

$$w^o = 0, \text{(or } M_{11} = 0 (or\ M_{22} = 0), Q_1 = 0 (Q_2 = 0) \qquad (2.17)$$

For Clamped

$$w^o = 0, Q_1 = 0, Q_2 = 0 \qquad (2.18)$$

2.4 Free Vibration of Plates

Consider a rectangular plate with arbitrary support. Let us assume that certain transverse surface loads distributed on the surface cause the particles, located in the middle surface, to attain the deflections and velocities directed perpendicularly to the initial (under formed) middle surface. At a certain time, which is assumed to be the initial deflection and velocity, begin to vibrate. The particles located in the middle surface move in the direction perpendicular to the plate and, as a result, the plate becomes curved. Such vibrations are called free or natural transverse vibrations. The plate will execute free or natural lateral vibrations.

Natural vibrations are functions of the material properties of the elastic plate, independent of any load. Thus for natural or free vibrations, p(x, y, t) is set equal to zero.

$$D\nabla^2\nabla^2 w(x, y, t) + qh\frac{6^2 w}{6t^2}(x, y, t) = 0 \qquad (2.19)$$

Deflection W must satisfy the boundary conditions at the practically do not differ from those in the case of static equilibrium and the following initial conditions

When t = 0. W = w_o (x, y), $\frac{6w}{6t} = v_o(x, y)$, (2.20)

Where and are the initial deflection and initial velocity of point (x, y).

Eq. (2.19) is the governing, fourth – order homogeneous partial differential equation of the undamped, free, linear vibrations of plates. A complete solution of the problem of a freely vibration plate is reduced to determining the deflection s at any point for any moment of time. However, the most important part of the problem of free flexural and the mode shapes of the vibration (deflection surfaces in two dimensions) associated with each natural frequency. For such a problem (like in buckling), Equation (2.19) is an Eigen value problem. The natural frequencies are the Eigen functions. Values of these parameter are necessary for establishing the dynamic stresses caused by a variable load. A solution of Eq. (2.19) can be obtained by applied the plate bending problems.

Let us describe a general analytical method (the Fourier method) for determining the natural frequencies of a freely vibrating plate. To solve Eq. (2.19) and obtain W(x, y, t) in general, one can assume the following solution;

$$W(x, y, t) = (A\cos wt + B\sin wt)\, W(x, y), \qquad (2.21)$$

Which is a separable solution of the shape function W(x, y) describing the modes of the vibration and some harmonic vibration which is related to vibration period T by the relationship W=2π/T

Introducing Eq. (2.21) into Eq. (2.19), we have

$$\Delta\nabla^2\nabla^2 w(x, y) - w^2 qhW = 0 \qquad (2.22)$$

Let us represent a solution of this equation in the form of a Fourier series. Require the function W to satisfy the boundary conditions and to be the solution Eq. (2.22), we obtain a system of homogeneous equations for the unknown constants. This system has solutions that differ from zero, the case when its determinant

$$\Delta\ (w) = 0 \qquad (2.23)$$

This equation will have an infinite number of solutions which constitute the frequency spectrum for a given plate. In general, the frequency will depend on two parameters, m and n (m=1, 2....., n=1, 2 ...). The lowest frequency is called the frequency of the fundamental mode or the fundamental natural frequency and all other frequencies are called the frequency of higher harmonic, or overtones. For each frequency Wmn there is a corresponding shape function $W_{Nn}(s,y)$ that, on the basis of the homogeneous equations, is as being equal to unity). For example, in the case of a rectangular, simply supported plate, the shape function maybe taken as

$$W(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} \sin \frac{m\alpha x}{a} \sin \frac{n\alpha y}{b} \tag{2.24}$$

Where a and b are the plate dimensions and is the vibration amplitude for each value of m and n

Substitution of Eq. (2.24) into Eq. (2.22) results in the homogenous algebraic equation.

$$\frac{N^4 n^4}{a^4} + 2\frac{N^2 n^2}{a^2}\frac{n^2 n^2}{b^2} + \frac{n^4 n^4}{b^4} - \frac{w^2 qh}{\Delta} = 0 \tag{2.25}$$

Solving this equation for W gives the natural frequencies

$$w_{nN} = n^2 \left(\frac{N^2}{a^2} + \frac{n^2}{b^2}\right) \sqrt{\frac{D}{qh}} \tag{2.26}$$

The fundamental natural frequency can be obtained by letting m=1, n=1

For a square plate of dimension an Eq. (2.26) becomes

$$w_{nN} = \frac{n^2}{a^2} \sqrt{\frac{D}{qh}} \left(N^2 + n^2\right), \tag{2.27}$$

And the fundamental natural frequency is

$$w_{11} = \frac{2n^2}{a^2} \sqrt{\frac{D}{qh}} \tag{2.28}$$

2.5 Forced Vibrations of Plates

The equation of motion of plates under a variable, time dependent, transvers load P(x, y, t) is given as

$$D\nabla^2\nabla^2 w(x,y,t) = e(x,y,t) - qh\frac{\partial^2 w}{\partial t^2}(x,y,t) \qquad (2.29)$$

The solution of the above non homogenous partial differential equation must satisfy the prescribed boundary and initial conditions. An exact solution can be obtained by using the following procedure. First let us solve the problem of free vibrations of a plate, and determine the natural frequencies Wnn and the corresponding mode shapes Wmn. Then, let us introduce a load p(x, y, t) in the form of series extended in Eigen function (the mode shapes), i.e.,

$$P = \sum_{N=0}^{\infty} \sum_{n=1}^{\infty} F_{mn}(t) W_{mn}(x, y). \quad (2.30)$$

We seek a solution of Eq. (2.29) in the form

$$W = \sum_{N=0}^{\infty} \sum_{n=1}^{\infty} F_{mn}(t) W_{mn}(x, y) \quad (2.31)$$

The following equation takes place for the function Fmn;

$$F_{mn}t \; w^{2}_{mn} F_{mn} = \frac{1}{\rho h} F_{mn}(x, y)$$

From which

$$F_{mn} = A_{mn} \cos w_{mn} t + B_{mn} \sin w_{mn} t + F^{(p)}_{mn}(t) \quad (2.34)$$

And

$$W = \sum_{N=0}^{\infty} \sum_{n=1}^{\infty} [A_{mn} \cos w_{mn} t + B_{mn} \sin w_{mn} t + F^{(p)}_{mn}(t)] \, w_{mn}(x, y) \quad (2.35)$$

Where $F^{(p)}_{mn}$ is a particular solution of Eq. (2.29); its form depends or Fmn, i.e, how a given load P varies with time. The constants Amn and bmn are determined from initial conditions. Namely, at t=0

$$W = w_0(x, y) \text{ and } \frac{\partial w}{\partial t} = v_0(x, y) \quad (2.36)$$

CHAPTER THREE
METHODOLOGY

3.1 Galerkin's Method

The exact solutions to the governing equations are generally not available. Hence, numerical solutions are introduced to obtain approximate solutions. In solving nonlinear PDE's, we often apply approximation. One of the approximation methods: Galerkin Method, invented by Russian mathematician Boris Grigoryevich Galerkins, is quite complicated, its physical interpretation is relatively simple. Thus, the method formulated by Galerkin can be applied successfully to diverse types of problems of applied elasticity including the plate bending problems. It is the general strategy for solving nonlinear PDE's. The method is a valuable approximation tool for the solution of Partial Differential Equations (PDE) when the analytical solutions are difficult or impossible to obtain due to complicated geometry or boundary conditions.

Such a Method uses a spatial discretization and a weighted residual formulation to transform the governing PDE (strong form) into an integral equation (weak form) that upon variational treatment yields to the solution of a system of matrix equations. The mathematical expression of the method is obtained as follows

Let a differential equation of a given 2D boundary value problem be of the form:-

$$L[w(x,y)] = P(x,y) \text{ in some 2D domain } \Omega \qquad (3.1)$$

Where,

w = w(x, y) is an unknown function of two variables (of course, the method can be applied to 3D problem also)

P = a given load term defined also in the domain, which is zero for a plate without externally applied load

$L =$ a symbol indicating either a linear or non – linear differential operator.

For instance, for plate free vibration problems in orthotropic regime,

$$L[w(x,y)] \equiv D\,\nabla^2\nabla^2(W(x,y)) - \rho h\omega^2[W(x,y)] - F \qquad (3.2)$$

The function w, must satisfy the prescribed boundary conditions on the boundary of that domain. An approximate solution of Eq. (3.1) is sought in the following form;

$$w_N(x,\ y) = \sum_{i=1}^{N} \alpha_i f_i\,(x,y) \qquad (3.3)$$

Where, α_i are unknown coefficients to be determined.

$f_i(x, y)$ are the linearly independent co-ordinate functions (they are also called trial) that satisfy all the prescribed boundary conditions but not necessarily satisfy Eq. (3.1).

From Calculus, any two functions $f_1(x)$, $f_2(x)$ are called mutually orthogonal in the interval (a, b) if they satisfy the condition:

$$\int_a^b f_1(x) f_2(x)\, dx = 0 \qquad (3.4)$$

For example, a set of functions: 1, Sin x, Cos x, Cos 2x, Sin 2x. . . Cos Kx, Sin Kx, . . . is orthogonal in the interval (0, 2π) because any two functions from the set satisfy the condition in the above interval. If one of the functions – for example, $f_1(x)$ - is identically equal to zero, then the condition 3.4 is satisfied for any function $f_2(x)$. Thus, if a function $w(x, y)$ is an exact solution of the given boundary value problem, then, the function $[L(w) - P]$ will be orthogonal to any set of functions. Since the deflection function, $w_N(x, y)$ in the form of Equation 3.3 is an approximate solution only if Equation 3.1, $[L(w) - P] \neq 0$, and it is no longer orthogonal to any set of functions. However, we can require that the magnitude of the function $[L(w) - P]$ be minimum. This requirement is equivalent to the condition that the above function should be orthogonal to some bounded set of functions: first of all, to the trial functions $f_i(x)$. It leads to the following Galerkin equation given by Ventsel and Krauthmmer (2001).

$$\iint_A [L(w_N) - P]\, f_i(x, y)\, dx\,dy = 0 \qquad (3.5)$$

Expanding equation (3.5), we have,

$$\iint_A [L(w_N) - P]\, f_i(x, y)\, dxdy - \iint_A (P.f(x, y)dxdy \qquad (3.6)$$

Where L (w_N) is equal to $D\ \nabla^4 w$, P is inertial force and $f_i(x, y)$ *is the trial function.*

The equation that delineates the flexural vibration of thin isotropic rectangular plates derived by Njoku, (2013) from the principles of the theory of elasticity is expressed as

$$\iint_A w\, [D[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w(x, y)}{\partial y^4}] = \bar{m}h\, \omega^2 w(x, y)]\, dxdy \qquad (3.7)$$

Where W is displacement in position Z- direction, $\bar{m}$ is the mass per unit area of the plate, h is the fundamental natural frequency, the flexural rigidity is expressed as

$$D=\frac{Eh^3}{12(1-v)} \tag{3.8}$$

E= modulus of elasticity, h= plate thickness, V= Poisson's ratio.

Equation (3.7) can be represented in the form expressed as

$$\nabla^4 W(x, y) - \bar{N}h^2 w(x, y)=0 \tag{3.9}$$

∇^4=B, harmonic differential operator and $\bar{N}h^2$=inertia force

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + \frac{2\,\partial^4}{\partial x^2\,\partial y^2} + \frac{\partial^4}{\partial y^4} \qquad (3.10)$$

Substituting equation (3.9) into equation (3.5), we have.

$$D \iint_A^g (\nabla^{4w)}\, f_i(x,y)\, dxdy - \bar{N}h^2 \iint_A^g (f_i(x,y)dxdy = 0 = G \qquad (3.11)$$

Substituting for Equation (3.11)

$$\iint_{A}^{g} {}_{6}{}^{4}{}_{w}[D[\frac{}{a^4\, 6R^4} + 2\frac{6^4 w}{a^2b^2\, 6R^2\, 6Q^2} + \frac{6^4 w}{b^4\, 6Q^4}] - \rho h\omega^2[W(x,y)] - F(t)]\text{-}f_i(x,y)dx = 0 \qquad (3.12)$$

Where $\bar{N} = qh$ and the function $f_i(x,y)$ (shape function) is related to deflection by the expression

$$f_i(x,y) = \frac{W(x,y)}{A} \qquad (3.\ 13)$$

And A, equate with the constant proportionality in the shape function of the plate as obtained by (Ibearugbulem, 2012)

$$W(x,y) = A.f_i(x,y) \qquad (3.14)$$

In non- dimensional form.

Where X= aR, Y= bQ, P=a/b (aspect Ration),

$h^2 = m^2$(Fundamental natural frequency)

R and Q are non – dimensional axis parallel to x and y axis respectively

$$fl_A^{g}{}_{w}\, D[\frac{\partial^4}{a^4\,\partial R^4} + 2\frac{\partial^4 w}{a^2b^2\,\partial R^2\,\partial Q^2} \quad \frac{\partial^4 w}{b^4\,\partial Q^4}]\, f_i(x,y)abdRdQ = 0$$

+

$$g \quad w \quad \partial^4 \quad \partial^4 w \quad \partial^4 w$$

$$D ab \iint_A \left[\frac{}{a^4 \partial R^4} + 2 \frac{}{a^2 b^2 \partial R^2 \partial Q^2} + \frac{}{b^4 \partial Q^4} \right] f_i(x, y)\, dR\, dQ$$

$$+$$

$$-ab \iint_A \rho h \omega^2 [W(x, y)] \cdot f_i(x, y)\, dR\, dQ - ab \iint_A F(t) \cdot f_i(x, y)\, dR\, dQ = 0$$

Dividing through by ab:

$$D \iint_0^1 \left[\frac{1}{p^4} \frac{\partial^4 W}{\partial R^4} + 2 \frac{1}{p^2} \frac{\partial^4 W}{\partial R^2 \partial Q^2} + \frac{\partial^4 W}{\partial Q^4} \right] f_i(x, y)\, dR\, dQ$$

$$- \iint_0^1 \rho h \omega^2 [W(x, y)] \cdot f_i(x, y)\, dR\, dQ - \iint_0^1 F(t) \cdot f_i(x, y)\, dR\, dQ$$

$$= 0 \tag{3.15}$$

Substituting Equation (3.14) into Equation (3.15):

$$\frac{DA}{b^4}\int_0^1\int_0^1 \left[\frac{1}{p^4}\frac{\partial^4 f_i}{\partial R^4} + \frac{2}{p^2}\frac{\partial^4 f_i}{\partial R^2 \partial Q^2} + \frac{\partial^4 f_i}{\partial Q^4}\right] f_i \, dRdQ - \rho h\omega^2 A \int_0^1\int_0^1 f_i \cdot f_i dRdQ - F(t)\int_0^1\int_0^1 f_i \, dRdQ = 0$$

$$\frac{DA}{b^4}\int_0^1\int_0^1 \left[\frac{1}{p^4}\frac{\partial^4 f_i}{\partial R^4} + \frac{2}{p^2}\frac{\partial^4 f_i}{\partial R^2 \partial Q^2} + \frac{\partial^4 f_i}{\partial Q^4}\right] f_i \, dRdQ - \rho h\omega^2 A \int_0^1\int_0^1 f_i^2 \, dRdQ - F(t)\int_0^1\int_0^1 f_i dRdQ = 0 \quad (3.16)$$

Making A the subject of formular

$$A = \frac{F(t)\iint_0^1 f_i dRdQ}{\frac{D}{b^4}\iint_0^1 \left[\frac{1}{p^4}\frac{\partial^4 f_i}{\partial R^4} + \frac{2}{p^2}\frac{\partial^4 f_i}{\partial R^2 \partial Q^2} + \frac{\partial^4 f_i}{\partial Q^4}\right] f_i \, dRdQ - \rho h\omega^2 \iint_0^1 f_i^2 \, dRdQ} \quad (3.17)$$

From equation (3.17)

Let

$$K1 = F(t)\int_0^1\int_0^1 f_i dRdQ \quad (3.18)$$

$$K2 = \frac{D}{b^4}\int_0^1\int_0^1 \left[\frac{1}{p^4}\frac{\partial^4 f_i}{\partial R^4} + \frac{2}{p^2}\frac{\partial^4 f_i}{\partial R^2 \partial Q^2} + \frac{\partial^4 f_i}{\partial Q^4}\right] f_i \, dRdQ \quad (3.19)$$

1

$$K3 = \rho h \omega^2 \iint_0 f_i^2 \, dR dQ \quad (3.20)$$

$$A = \frac{K1}{K2 - K3} \quad (3.21)$$

Where K1 represent the load, K2 represents the material stiffness and K3 represent the inertia stiffness.

Equation 3.21 is the Galerkin’s minimum total potential energy for a rectangular plate in vibration regime, which had been derived from the special mathematical condition of orthogonality of two functions.

3.2 Characteristic Orthogonal Polynomials Shape Functions

Although it is advantageous to apply the well-known method of Galerkin's method in various engineering problems, it is often difficult to obtain the meaningful deflection shape functions in the said method. So, a class of characteristic orthogonal polynomial (COPs) can be constructed using Gram–Schmidt process and then these polynomials are employed as deflection functions in the Galerkin's method. The orthogonal nature of the polynomials makes the analysis simple and straightforward. Moreover, ill conditions of the problem may also be avoided.

Galerkin's method is a very powerful technique that can be used to predict the natural frequencies and mode shapes of vibrating structures. The method requires a linear combination of assumed deflection shapes of structures in free harmonic vibration that satisfy at least the geometrical boundary conditions of the vibrating structure. Expressions for the maximum kinetic and potential energies are obtained in terms of the arbitrary constants in the deflection expression. By equating the maximum potential and kinetic energies, it is possible to obtain an expression for the natural frequency of the structure. Applying the condition of stationarity of the natural frequencies at the natural modes, the variation of natural frequencies with respect to the arbitrary constants is equated to zero to obtain an eigenvalue problem (Meirovitch, 1967).

Solution of this eigenvalue problem provides the natural frequencies and mode shapes of the system. The assumed deflection shapes were normally formulated by inspection and sometimes by trial and error until Bhat (1985a, b) proposed a systematic method of constructing such functions in the form of Characteristic Orthogonal Polynomials (COPs). The restrictions on the series are the following:

a. They satisfy the geometrical boundary conditions.

b. They are complete.

c. They do not inherently violate the natural boundary conditions.

When the above conditions are met, the numerical solutions converge to the exact solution and it depends on the number of terms taken in the admissible series. Different series types, viz., trigonometric, hyperbolic, polynomial, give different results for the same number of terms in the series and the efficiency of the solution will depend to some extent on the type of series chosen (Brown and Stone; (1997)).

The choice of the functions F(x) or G(y) may be anything like, algebraic, trigonometric, hyperbolic and so on or a combination of these and will depend mainly on the boundary conditions of the plate.

For example, if the two opposite edges of the plate at $x = 0$ and $x = a$ are simply supported, the deflected surface on $x - z$ plane may be conveniently taken as a trigonometric function in the form of sine or cosine series as:

$$F(x) = \sum_{N=1}^{œ} F_N \sin \frac{Nnx}{a} \qquad (3.22)$$

In which, $F_{i\text{-}s}$ are the coefficients (like F_1, F_2 . . . F_m) to be determined. It is clearly seen that Equation 3.22 satisfies the boundary conditions viz, F(x) = 0 at x = 0 and at x = a.

If the other two edges viz, at y = 0 and y = b are also simply supported, the function G(y) obviously can be written as:

$$G(y) = \sum_{n=1}^{œ} G_n \sin \frac{nny}{b} \qquad (3.23)$$

If the boundary conditions at y = 0 and y = b are other than simply supported or a combination of simply supported, clamped, free or so on, the function G(y) should be selected accordingly to satisfy the prescribed boundary conditions.

Thus, the displacement function for the rectangular plate is therefore assumed as a product of two functions; one of which is a pure function of x and the other is of y so that:

$$W(x,y) = F(x) \cdot G(y) \qquad (3.24)$$

$$W(x,y) = \sum_{N=1}^{œ} F_N \sin \frac{Nnx}{a} \cdot \sum_{n=1}^{œ} G_n \sin \frac{nny}{b} \qquad (3.25)$$

$$W(x,y) = \sum_{m=1}^{œ} \sum_{n=1}^{œ} {}_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \qquad (3.26)$$

Where,

$$C_{Nn} = F_N G_n \qquad (3.27)$$

This double Fourier series suits well for plates simply supported at all the four edges. This type of solution is due to (Navier, 1823).

In 1900 Levy developed a method for solving rectangular plate bending problems with simply supported two opposite edges and with arbitrary conditions of supports on the two remaining opposite edges using single Fourier series. This method is more practical because it is easier to perform numerical calculations for single series than for double series and it is also applicable to plates with various boundary conditions. The single Fourier series solution in which the plate should at least two opposite edges simply supported of the form:

$$w = w(x, y) = \sum_{N=1}^{\infty} G(y) \sin \frac{Nny}{a} \tag{3.28}$$

The assumed deflection either in the form of infinite series or in a combination of a finite number of functions is selected in such a way as to resemble closely the actual deflected shape.

COP Method for Finding Shape Functions in rectangular plates for quick analysis and design is simple but approximate. Thus, the popular method essentially considers the compatibility of the deflection at any particular point of two in-dependent plate strips spanning along the two directions at right angles to each other through that point. The deflection behaviour of the individual plate strip assumed that of a beam with the identical boundary conditions at the two ends as that of the plate along that corresponding direction.

This method further considers that the given intensity of the loading on the plate is shared by the beam strips as the uniform loading on them. The method is as follows:

Consider a rectangular plate of dimension, a along x and b along y. If the deflection pattern of the plate along x is represented by a beam strip qualitatively, the beam function along x is taken as f(x). Similarly, the corresponding beam function along y is taken as g(y).

The solution for prismatic beam of constant EI and length spanning along x is given as:

$$w_s = F(x) = \sum_{N=1}^{\infty} X_N x^N \qquad (3.29)$$

Similarly technique in the y-direction we shall obtain:

$$w_y = G(y) = \sum_{n=1}^{\infty} Y_n y^n \qquad (3.30)$$

Where,

X_m and Y_n are constant parameters in x and y directions respectively.

m and n are series to infinity limit.

Thus, the displacement function for the rectangular plate is therefore assumed as a product of two functions; one of which is a pure function of x and the other is of y so that:

$$w(x, y) = F(x).G(y) = w_x.w_y \qquad (3.31)$$

$\infty \quad \infty$

$$w_s = F(x) = \sum_{N} \sum_{n} X_N x^N Y_n y^n \qquad (3.32)$$

Expressing Equations 3.29 through 3.32 in the form of non-dimensional parameters, say R and Q for x and y directions respectively:

$$x = aR \tag{3.33}$$

$$y = bQ \tag{3.34}$$

Then, Equations 3.29 through 3.32 become

$$w_s = F(R) = \sum_{N=1}^{æ} X_N a^m R^N \tag{3.35}$$

$$w_y = G(Q) = \sum_{n=1}^{æ} Y_n b^n Q^n \tag{3.36}$$

Let:

$$A_N = X_N a^m \tag{3.37}$$

$$B_n = Y_n b^n \tag{3.38}$$

$$w_s = F(R) = \sum_{N=1}^{æ} A_N R^N \tag{3.39}$$

$$w_y = G(Q) = \sum_{n=1}^{æ} B_n Q^n \tag{3.40}$$

Also,

$$w(R, Q) = \sum_{m=1}^{æ} \sum_{n=1}^{æ} A_N B_n R^N Q^n \tag{3.41}$$

Since Equation 3.2, which is dynamic plate's equation in vibration regime is a fourth order differential and the density of the plate is constant, then, the value of m and n in Equation 3.37 must be of 4th order. If the variation of loading is linear or a second degree parabola, the value of the power n will be of 5th or 6th order respectively and so on (Onyeyili, 2012).

Expanding Equation 3.39 through 3.41 to 4th series, we shall obtain:

$$w_s = F(R) = \sum_{N=1}^{4} A_N \beta^N = (A_0 + A_1 R + A_2 R^2 + A_3 R^3 + A_4 R^4) \quad (3.42)$$

$$w_y = G(Q) = \sum_{n=1}^{4} B_n Q^n = (B_0 + B_1 Q + B_2 Q^2 + B_3 Q^3 + B_4 Q^4) \quad (3.43)$$

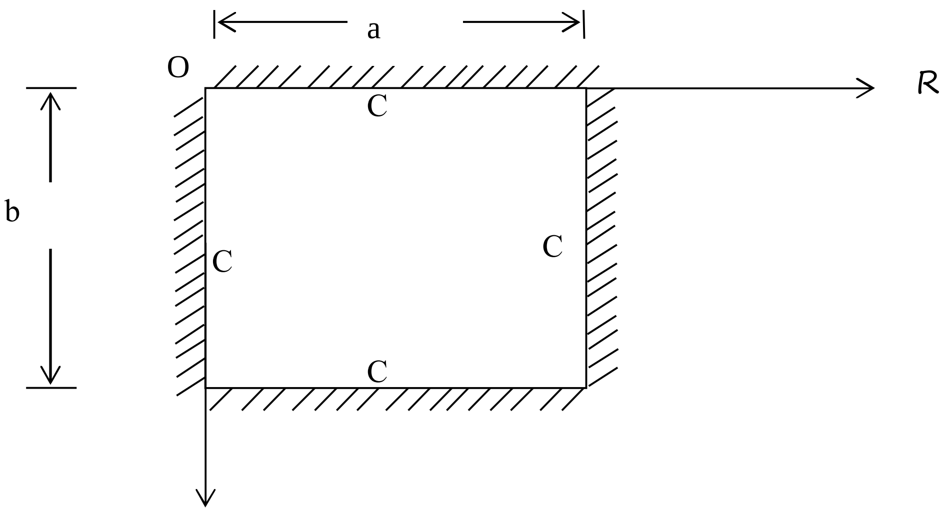
$$w(R,Q) = \sum_{m=1}^{4}\sum_{n=1}^{4} A_N B_n R^N Q^n = F(R).\,G(Q)$$

$$= (A_0 + A_1R + A_2R^2 + A_3R^3 + A_4R^4)(B_0 + B_1Q + B_2Q^2 + B_3Q^3 + B_4Q^4) \qquad (3.44)$$

$$= \begin{bmatrix} A_0 & A_0 & A_0 & A_0 & A_0 \\ A_1R & A_1R & A_1R & A_1R & A_1R \\ A_2R^2 & A_2R^2 & A_2R^2 & A_2R^2 & A_2R^2 \\ A_3R^3 & A_3R^3 & A_3R^3 & A_3R^3 & A_3R^3 \\ A_4R^4 & A_4R^4 & A_4R^4 & A_4R^4 & A_4R^4 \end{bmatrix} \times \begin{bmatrix} B_0 \\ B_1Q \\ B_2Q^2 \\ B_3Q^3 \\ B_4Q^4 \end{bmatrix}$$

Equation 3.44 is a 25 terms equation to maximum power of 8 with several unknowns. The coefficients A_m and B_n of the series are determined from the boundary conditions at the edges of the plate.

3.2.1 All Round Clamped Rectangular Plate



Q

Figure 3.1 Thin rectangular plate whose all edges R = 0, 1; and Q = 0, 1 are clamped

Boundary conditions

Deflections at all edges are zero

Slope at all edges are zero

For R – *directions*

At $R = 0$

From equation 3.44

$W_s = A_O = 0$

$\therefore \; A_o = 0 \qquad (3.45)$

(ii) $\frac{\partial W(R)}{a\partial R} = \frac{\partial}{a\partial R}(A_O + A_1 R + A_2 R^2 + A_3 R^3 + A_4 R^4)$

$$\frac{\partial W(R)}{a\partial R} = \frac{A_1 + 2A_2R + 3A_3R^2 + 4A_4R^3}{a} \qquad (3.46)$$

Then,

At $R = 0$

$\frac{\partial W(R)}{\partial R} = 0 = A_1 = 0$

$\therefore \; A_1 = 0 \qquad (3.47)$

At $R = 1$

(iii) $W_X = 0 = A_2 + A_3 + A_4$

$\therefore \; A_2 = -(A_3 + A_4) \qquad (3.48)$

From equation 3.46,

$$(iv)\ \frac{\&W_s}{\&R} = 2A_2 + 3A_3 + 4A_4 = 0$$

But from equation 3.48, we shall obtain

$$-2(A_3 + A_4) + 3A_3 + 4A_4 = 0$$

$$A_3 + 2A_4 = 0$$

$$\therefore\ A_3 = -2A_4 \qquad (3.49)$$

$$Thus,\quad A_2 = -(-2A_4 + A_4) = A_4 \qquad (3.50)$$

Putting the values of A_O, A_1, A_2, A_3, into 3.42, we have

$$W_s = F(R) = A_4R^2 + (-2A_4)R^3 + A_4R^4 = A_4(R^2 - 2R^3 + R^4) \quad (3.51)$$

For Q – *dircetion*

At $Q = 0$

From equation 3.43,

(i) $W_y = B_O = 0$

$$\therefore B_O = 0 \quad (3.52)$$

$$(ii)\ \frac{\delta W_y}{\delta_y} = \frac{\delta W_y}{b\delta_Q} = \frac{1}{b}(B_1 + 2B_2 Q + 3B_3 Q^2 + 4B_4 Q^3) \quad (3.53)$$

$$\frac{\delta W_y}{\delta_y} = 0 = \frac{1}{b} B_1 = 0$$

$$since \frac{1}{b} \neq 0$$

$$B_1 = 0 \quad (3.54)$$

At $Q = 1$

(iii) $W_y = G(Q = 1) = 0 = B_2 + B_3 + B_4 = 0$

$$\Rightarrow B_2 + B_3 + B_4 = 0 \quad (3.55)$$

$$B_2 = -(B_3 + B_4) \quad (3.56)$$

From Equation 3.53

$$\frac{\delta W_y}{4A} = 0 = \frac{1}{\ } (2A + 3A + \quad) = 0$$

$\&_y \qquad b \quad {}^2 \qquad {}^3 \qquad {}^4$

$$\text{since}\ \frac{1}{b} \neq 0$$

$$\Rightarrow (2B_2 + 3B_3 + 4B_4) = 0 \qquad (3.57)$$

Putting the value of B_2 of Equation 3.56 into equation 3.57, we obtain,

$-2(B_3 + B_4) + 3B_3 + 4B_4 = 0$

$\Rightarrow B_3 + 2B_4 = 0$

$$\text{Therefore, } B_3 = -2B_4 \qquad (3.58)$$

Putting the expression of 3.58 into 3.56, we obtain.

$B_2 = -(B_3 + B_4)$

$\Rightarrow B_2 = -(-2B_4 + B_4)$

$\text{Therefore, } B_2 = -(-B_4)$

$$B_2 = B_4 \qquad (3.59)$$

Putting the expression of B_O, B_1, B_2 and B_4 into Equation 3.43, we shall obtain;

$W_y = G(Q) = B_4Q^2 + (-2B_4)Q^3 + B_4Q^4$

$$= B_4(Q^2 - 2Q^3 + Q^4) \qquad (3.60)$$

Substituting 3.51 and 3.60 into 3.44, we will obtain the displacement function for a rectangular plate clamped all round as;

$W(x, y) = F(R) * G(Q) = W_s * W_y$

$W(x, y) = A_4(R^2 - 2R^3 + R^4) * B_4(Q^2 - 2Q^3 +$

$Q^4)$ $W(x, y) = A_4B_4(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 +$

$Q^4)$

$$W(x, y) = A(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \qquad (3.61)$$

Where A = $A_4 b_4$

3.2.2 Linearly Varying Distributed Load

Consider a plate fixed at all the edges, subjected to a uniformly distributed load, W(x) along x and a linearly varying distributed load, W(y) along y-axes as shown in Figure 3.2. Sections of the load are redrawn in Figure 3.2a and 3.2b respectively. For such a load, an assumed fourth power of displacement function would be adequate for its y – axis and fifth power of displacement function for its x – axis.

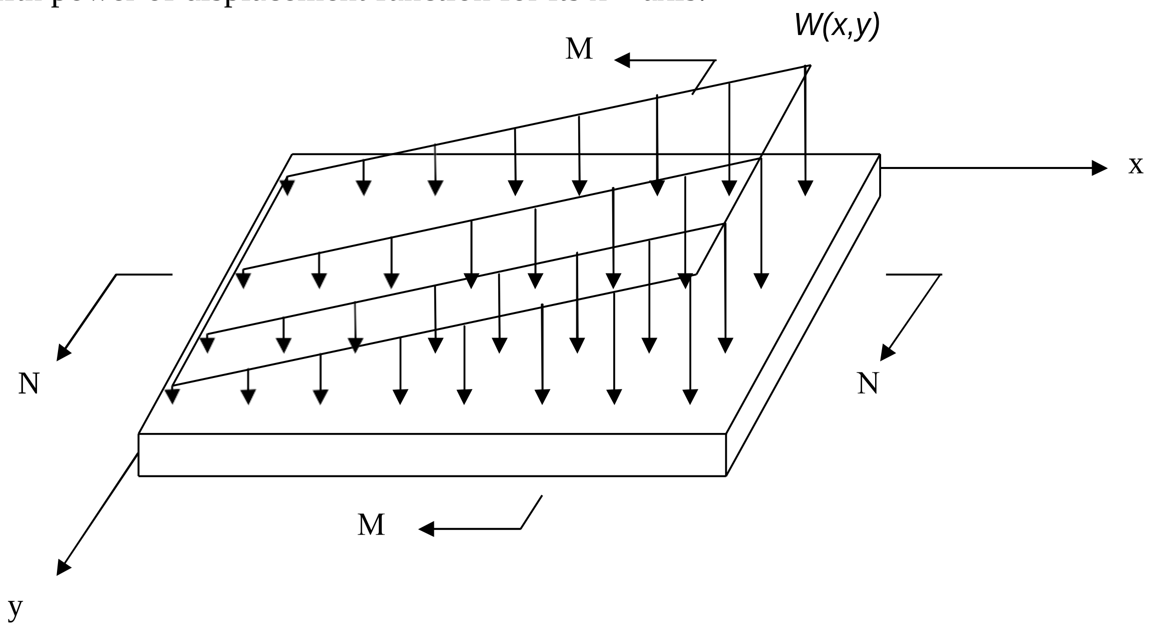


Figure 3.2: Plate clamped on all edges subjected to uniformly distributed loads

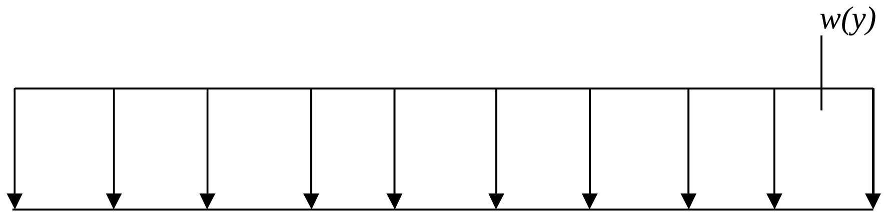


Figure 3.2a: Section M – M

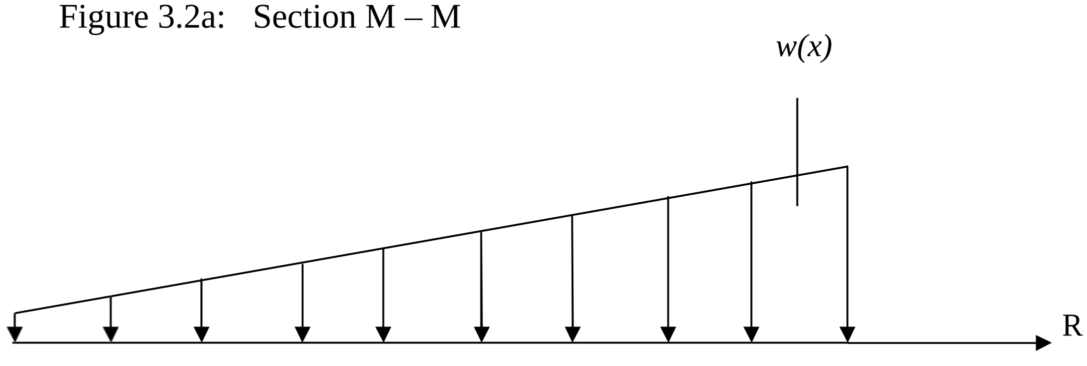


Figure 3.2b: Section N – N

Then, expanding Equation 3.42, and 3.43 to 5th and 4th series respectively, we shall obtain:

$$W_s = F(R) = \sum_{N=1}^{5} A_N \beta^N = (A_0 + A_1R + A_2R^2 + A_3R^3 + A_5R^5) \tag{3.62}$$

$$W_y = G(Q) = \sum_{n=1}^{\infty} B_nQ^n = (B_0 + B_1Q + B_2Q^2 + B_3Q^3 + B_4Q^4) \tag{3.63}$$

$$W(R, Q) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_N B_n R^N Q^n = F(R).\, G(Q)$$

$$W(R, Q) = (A_0 + A_1R + A_2R^2 + A_3R^3 + A_5R^5)(B_0 + B_1Q + B_2Q^2 + B_3Q^3 + B_4Q^4) \tag{3.64}$$

The coefficients A_m and $B_n$of the series are determined from the boundary conditions at the edges of the plate.

Boundary conditions

- Deflections at all edges are zero
- Slope at all edges are zero

For $R - directions$

At $R = 0$

From equation 3.62

(i) $W_s = A_0 = 0$

$$\therefore\ A_o = 0 \tag{3.65}$$

(ii) $\frac{\partial W_{(R)}}{\partial R} = \frac{\partial}{\partial R}(A_0 + A_1 R + A_2 R^2 + A_3 R^3 + A_5 R^5)$

$$\frac{\partial W_{(R)}}{\partial R} = \frac{A_1 + 2A_2R + 3A_3R^2 + 5A_5R^4}{a} \tag{3.66}$$

Then,

At $R = 0$ $\frac{\partial M_{(R)}}{\partial R} = 0 = A_1 = 0$

$\therefore \quad A_1 = 0 \qquad (3.67)$

At $R = 1$

(iii) $W_X = 0 = A_2 + A_3 + A_5$

$\therefore \quad A_2 = -(A_3 + A_5) \qquad (3.68)$

From equation 3.64,

(iv) $\frac{\delta W_s}{\delta R} = 2A_2 + 3A_3 + 5A_5 = 0$

But from equation 3.68, we shall obtain

$-2(A_3 + A_5) + 3A_3 + 5A_5 = 0$

$A_3 + 3A_5 = 0$

$\therefore \quad A_3 = -3A_5 \qquad (3.69)$

Putting Equation 3.69 into Equation 3.68:

$A_2 = -(-3A_5 + A_5)$

$A_2 = 2A_5 \qquad (3.70)$

Putting the values of A_O, A_1, A_2, A_3, A_5 into 3.62, we have

$W_s = F(R) = 2A_5R^2 - 3A_5R^3 + A_5R^5$

$W_s = (2R^2 - 3R^3 + R^5)A_5 \qquad (3.71)$

For Q – *dircetion*

At $Q = 0$

From equation 3.63,

(i) $W_y = B_O = 0$

$$\therefore B_0 = 0 \qquad (3.72)$$

$$(ii)\ \frac{\&W_y}{\&\varepsilon_y} = \frac{\&W_y}{b\&_Q} = \frac{1}{b}\left(B_1 + 2B_2 Q + 3B_3 Q^2 + 4B_4 Q^3\right) \qquad (3.73)$$

$$\frac{\&W_y}{\&\varepsilon_y} = 0 = \frac{1}{b} B_1 = 0$$

since $\frac{1}{b} \neq 0$

$$B_1 = 0 \tag{3.74}$$

At $Q = 1$

(iii) $W_y = G(Q = 1) = 0 = B_2 + B_3 + B_4 = 0$

$$\Rightarrow B_2 + B_3 + B_4 = 0 \tag{3.75}$$

$$B_2 = -(B_3 + B_4) \tag{3.76}$$

From Equation 3.73

$$\frac{\&W_y}{\&_y} = 0 = \frac{1}{b}(2A_2 + 3A_3 + 4A_4) = 0$$

since $\frac{1}{b} \neq 0$

$$\Rightarrow (2B_2 + 3B_3 + 4B_4) = 0 \tag{3.77}$$

Putting the value of B_2 of Equation 3.76 into equation 3.77, we obtain,

$-2(B_3 + B_4) + 3B_3 + 4B_4 = 0$

$\Rightarrow B_3 + 2B_4 = 0$

$$\therefore B_3 = -2B_4 \tag{3.78}$$

Putting the expression of 3.78 into 3.76, we obtain.

$B_2 = -(B_3 + B_4)$

$\Rightarrow B_2 = -(-2B_4 + B_4)$

$\therefore B_2 = -(-B_4)$

$$B_2 = B_4 \tag{3.79}$$

Putting the expression of B_0, B_1, B_2 and B_4 into Equation 3.63, we shall obtain;

$$W_y = G(Q) = B_4Q^2 + (-2B_4)Q^3 + B_4Q^4$$

$$= B_4(Q^2 - 2Q^3 + Q^4) \qquad (3.80)$$

Substituting 3.71 and 3.80 into 3.64, we will obtain the displacement function for a rectangular plate clamped all round as;

$$W(x, y) = F(R) \times G(Q) = W_s \times W_y$$

$$W(x, y) = (2R^2 - 3R^3 + R^5)A_5 \times B_4(Q^2 - 2Q^3 + Q^4)$$

$$W(x,y) = A_5B_4(2R^2 - 3R^3 + R^5)(Q^2 - 2Q^3 + Q^4) \quad (3.81)$$

Where $A = A_5B_4$

$$W(x,y) = A(2R^2 - 3R^3 + R^5)(Q^2 - 2Q^3 + Q^4) \quad (3.82)$$

3.3 Dynamic Regime of All Edges Built In Plate Subjected To Uniformly Distributed Load

From Equation 3.13, the shape function for CCCC in terms of coefficient of deflected plate is given as:

$$f_i = (R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \quad (3.83)$$

Putting Equation 3.83 into the total potential energy equation for a thin rectangular isotropic plate in vibration regime according to Galerkin's Method (Equation 3.17).

$$A = \frac{F(t) \iint_0^1 f_i dRdQ}{\frac{D}{b^4} \iint_0^1 \left[\frac{\partial^4 f_i}{p^4 \partial R^4} + 2\frac{\partial^4 f_i}{p^2 \partial R^2 \partial Q^2} + \frac{\partial^4 f_i}{\partial Q^4}\right] f_i dRdQ - qhm^2 \iint_0^1 f_i^2} \quad (3.84)$$

Where,

$$\frac{\partial^4 f_i}{\partial R^4} = \frac{\partial^4 (R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)}{\partial R^4} = 24(Q^2 - 2Q^3 + Q^4) \quad (3.85)$$

$$\frac{\partial^4 f_i}{\partial R^4} \cdot f_i = (R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \cdot 24(Q^2 - 2Q^3 + Q^4)$$

$$= 24(R^2 - 2R^3 + R^4)(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8) \quad (3.86)$$

$$\frac{D}{b^4} \int_0^1 \int_0^1 \frac{\partial^4 f_i}{p^4 \partial R^4} \cdot f_i \, dRdQ$$

$$= \frac{24D}{b^4 p^4} \int_0^1 \int_0^1 ((R^2 - 2R^3 + R^4)(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8)) \, dRdQ$$

24D 1 2 1 1 4 6 4 1

$$= \frac{}{b^4P^4}\left(\frac{}{3} - \frac{}{4} + \frac{}{5}\right)\left(\frac{}{5} - \frac{}{6} + \frac{}{7} - \frac{}{8} + \frac{}{9}\right)$$

$$= \frac{24D}{b^4P^4}\left(\frac{1}{30}\right)\left(\frac{1}{630}\right)$$

$$= \frac{0.00127D}{b^4P^4} \tag{3.87}$$

$$\frac{\partial^4 f_i}{\partial Q^4} = \frac{\partial^4 (R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)}{\partial Q^4} = 24(R^2 - 2R^3 + R^4) \qquad (3.88)$$

$$\frac{\partial^4 f_i}{\partial Q^4} \cdot f_i = (R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \cdot 24(R^2 - 2R^3 + R^4)$$

$$\frac{\partial^4 f_i}{\partial Q^4} \cdot f_i = 24(R^4 - 4R^5 + 6R^6 - 4R^7 + R^8)(Q^2 - 2Q^3 + Q^4) \qquad (3.89)$$

$$\frac{D}{b^4} \int_0^1 \int_0^1 \frac{\partial^4 f_i}{\partial Q^4} \cdot f_i \, dRdQ$$

$$= 24 \frac{D}{b^4} \int_0^1 \int_0^1 ((R^4 - 4R^5 + 6R^6 - 4R^7 + R^8)(Q^2 - 2Q^3 + Q^4)) \, dRdQ$$

$$= 24 \frac{D}{b^4} \left(\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9}\right)\left(\frac{1}{3} - \frac{2}{4} + \frac{1}{5}\right)$$

$$= 24 \frac{D_y}{b^4} \left(\frac{1}{630}\right)\left(\frac{1}{30}\right)$$

$$= 0.00127 \frac{D}{b^4} \qquad (3.90)$$

$$\frac{\partial^4 f_i}{\partial R^2 \partial Q^2} = (2 - 12R + 12R^2)(2 - 12Q + 12Q^2) \qquad (3.91)$$

$$2 \frac{\partial^4 f_i}{\partial R^2 \partial Q^2} \cdot f_i = 2(2R^2 - 16R^3 + 38R^4 - 36R^5 + 12R^6)(2Q^2 - 16Q^3 + 38Q^4 - 36Q^5 + 12Q^6) \qquad (3.92)$$

$$\frac{D}{b^4}\int_0^1\int_0^1 \frac{2\&^4 f_i}{P^2 dR^2 dQ^2}\cdot f_i\, dR\, dQ$$

$$= \frac{2D}{b^4P^2}\left(\frac{2}{3}-\frac{16}{4}+\frac{38}{5}-\frac{36}{6}+\frac{12}{7}\right)\left(\frac{2}{3}-\frac{16}{4}\;\frac{38}{5}-\frac{36}{6}+\frac{12}{7}\right)$$

$+$

$$= 2\frac{D}{b^4P^2}\left(\frac{-2}{105}\right)\left(\frac{-2}{105}\right)$$

$$= \frac{0.0007256D}{b^4P^2} \qquad (3.93)$$

$$f_i^2 = A(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)$$

$$= (R^2 - 2R^3 + R^4)^2(Q^2 - 2Q^3 + Q^4)^2$$

$$= (R^4 - 4R^5 + 6R^6 - 4R^7 + R^8)(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8) \qquad (3.94)$$

$$\rho h\omega^2 \int_0^1 f_i^2 \, dRdQ$$

$$= \rho h\omega^2 \int_{0,0}^{1,1} (R^4 - 4R^5 + 6R^6 - 4R^7 + R^8)(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8)dRdQ$$

$$= \rho h\omega^2 \left(\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9}\right)\left(\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9}\right)$$

$$= \rho h\omega^2 \left(\frac{1}{630}\right)\left(\frac{1}{630}\right)$$

$$= 0.000002520\rho h\omega^2 \qquad (3.95)$$

$$F(t) \int_0^1 f_i dRdQ = F(t) \int_{0,0}^{1,1} (R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)dRdQ$$

$$F(t) \int_0^1 f_i dRdQ = F(t)\left(\frac{1}{3} - \frac{2}{4} + \frac{1}{5}\right)^2 = 0.0011111F(t) \qquad (3.96)$$

Substituting expressions of 3.87, 3.90, 3.93, 3.95 and 3.96 into Equation 3.84:

$$A = \frac{0.0011111F(t)}{D(0.00127 + 0.0007256 + 0.00127)}$$

$- 0.000002520\rho h\omega^2$

(3.97)

b^4P^4 b^4P^2 b^4

In this section, dynamics of rectangular plates with approximate method (Galerkin method) are considered and analysed; using equations emphasized in the previous sections.

3.4 Dynamic Regime of All Edges Built In Plate Subjected To Varying Distributed Load

From Equation 3.13, the shape function for CCCC in terms of coefficient of deflected plate is given as:

$$f_i = (2R^2 - 3R^3 + R^5)(Q^2 - 2Q^3 + Q^4) \qquad (3.98)$$

Putting Equation 3.98 into the total potential energy equation for a thin rectangular isotropic plate in vibration regime according to Galerkin's Method (Equation 3.17).

$$A = \frac{F(t)\iint_0^1 f_i\,dRdQ}{\frac{D}{b^4}\iint_0^1\left[\frac{\partial^4 f_i}{p^4\partial R^4} + 2\frac{\partial^4 f_i}{p^2\partial R^2\partial Q^2} + \frac{\partial^4 f_i}{\partial Q^4}\right] f_i\,dRdQ - \rho h\omega^2\iint_0^1 f_i^2} \qquad (3.99)$$

Where,

$$\frac{\partial^4 f_i}{\partial R^4} = \frac{\partial^4(2R^2 - 3R^3 + R^5)(Q^2 - 2Q^3 + Q^4)}{\partial R^4} = 120R(Q^2 - 2Q^3 + Q^4) \qquad (3.100)$$

$$\frac{\partial^4 f_i}{\partial R^4}\cdot f_i = 120R(2R^2 - 3R^3 + R^5)(Q^2 - 2Q^3 + Q^4)(Q^2 - 2Q^3 + Q^4)$$

$$= 120(2R^3 - 3R^4 + R^6)(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8) \qquad (3.101)$$

$$\frac{D}{b^4}\int_0^1\int_0^1 \frac{\partial^4 f_i}{p^4\partial R^4}\cdot f_i\,dRdQ$$

$$= \frac{120D}{b^4p^4}\int_0^1\int_0^1((2R^3 - 3R^4 + R^6)(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8))\,dRdQ$$

$$= \frac{120D}{b^4p^4}\left(\frac{2}{4} - \frac{3}{5} + \frac{1}{7}\right)\left(\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9}\right)$$

$$= \frac{120D}{b^4p^4}\left(\frac{3}{70}\right)\left(\frac{1}{630}\right)$$

$0.0081633D$

$$= \frac{}{b^4 P^4} \quad (3.102)$$

$$\frac{\partial^4 f_i}{\partial Q^4} = \frac{\partial^4 (2R^2 - 3R^3 + R^5)(Q^2 - 2Q^3 + Q^4)}{\partial Q^4} = \quad - 3R^3 + R^5) \quad (3.103)$$

$$24(2R^2$$

$$\frac{\partial^4 f_i}{\partial Q^4} \cdot f_i = \quad - 3R^3 + R^5)^2 (Q^2 - 2Q^3 + Q^4)$$

$$24(2R^2$$

$$\frac{\partial^4 f_i}{\partial Q^4} \cdot f_i = \quad - \quad + 9R^6 + 4R^7 - 6R^8 + R^{10})(Q^2 - 2Q^3 + Q^4) \quad (3.104)$$

$$24(4R^4 \quad 12R^5$$

$$\frac{D}{b^4}\int_0^1\int_0^1 \frac{\partial^4 f_i}{\partial Q^4}\cdot f_i \, dR\,dQ$$

$$= 24\frac{D}{b^4}\int_0^1\int_0^1 \left((4R^4 - 12R^5 + 9R^6 + 4R^7 - 6R^8 + R^{10})(Q^2 - 2Q^3 + Q^4)\right) dR\,dQ$$

$$= 24\frac{D}{b^4}\left(\frac{4}{5}-\frac{12}{6}+\frac{9}{7}+\frac{4}{8}-\frac{6}{9}+\frac{1}{11}\right)\left(\frac{1}{3}-\frac{2}{4}+\frac{1}{5}\right)$$

+

$$= 24\frac{D_y}{b^4}\left(\frac{23}{2310}\right)\left(\frac{1}{30}\right)$$

$$= 0.00796537\frac{D}{b^4} \qquad (3.105)$$

$$\frac{\partial^4 f_i}{\partial R^2 \partial Q^2} = (4-18R+20R^3)(2-12Q+12Q^2) \qquad (3.106)$$

$$2\frac{\partial^4 f_i}{\partial R^2 \partial Q^2}\cdot f_i = 2(4-18R+20R^3)(2-12Q+12Q^2)(2R^2-3R^3+R^5)(Q^2-2Q^3+Q^4)$$

$$= 2(8R^2-48R^3+54R^4+44R^5-78R^6+20R^8)(2Q^2-16Q^3+38Q^4-36Q^5+12Q^6) \qquad (3.107)$$

$$\frac{D}{b^4}\int_0^1\int_0^1 \frac{2\partial^4 f_i}{P^2\partial R^2 \partial Q^2}\cdot f_i \, dR\,dQ$$

$$= \frac{2D}{b^4P^2}\left(\frac{8}{3}-\frac{48}{4}+\frac{54}{5}+\frac{44}{6}-\frac{78}{7}+\frac{20}{9}\right)\left(\frac{2}{3}-\frac{16}{4}+\frac{38}{5}-\frac{36}{6}+\frac{12}{7}\right)$$

+

$$= 2\frac{D}{b^4P^2}\left(\frac{-38}{315}\right)\left(\frac{-2}{105}\right)$$

$$= \frac{0.004595616D}{b^4 p^2} \qquad (3.108)$$

$$f_i^2 = (2R^2 - 3R^3 + R^5)(Q^2 - 2Q^3 + Q^4)(2R^2 - 3R^3 + R^5)(Q^2 - 2Q^3 + Q^4)$$

$$= (2R^2 - 3R^3 + R^5)^2(Q^2 - 2Q^3 + Q^4)^2$$

$$= (4R^4 - 12R^5 + 9R^6 + 4R^7 - 6R^8 + R^{10})(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8) \qquad (3.109)$$

$$\rho h \omega^2 \iint_0^1 f_i^2 \, dR dQ$$

$$= \rho h\omega^2 \int\int_{0,0}^{1,1} (4R^4 - 12R^5 + 9R^6 + 4R^7 - 6R^8 + R^{10})(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8)dRdQ$$

$$= \rho h\omega^2 \left(\frac{4}{5} - \frac{12}{6} + \frac{9}{7} + \frac{4}{8} - \frac{6}{9} + \frac{1}{11}\right)\left(\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9}\right)$$

$$= \rho h\omega^2 \left(\frac{23}{2310}\right)\left(\frac{1}{630}\right)$$

$$= 0.000015804\rho h\omega^2 \qquad (3.110)$$

$$F(t)\int\int_{0}^{1} f_i dRdQ = F(t)\int\int_{0,0}^{1,1}(2R^2 - 3R^3 + R^5)(Q^2 - 2Q^3 + Q^4)dRdQ$$

$$F(t)\int\int_{0}^{1} f_i dRdQ = F(t)\left(\frac{2}{3} - \frac{3}{4} + \frac{1}{6}\right)\left(\frac{1}{3} - \frac{2}{4} + \frac{1}{5}\right)$$

$$F(t)\int\int_{0}^{1} f_i dRdQ = F(t)\left(\frac{1}{12}\right)\left(\frac{1}{30}\right) = \frac{F(t)}{360} = 0.002777778F(t) \qquad (3.111)$$

Substituting expressions of 3.102, 3.105, 3.108, 3.110 and 3.111 into Equation 3.99:

$$A = \frac{0.002777778F(t)}{D\left(\frac{0.0081633}{b^4P^4} + \frac{0.004595616}{b^4P^2} + \frac{0.00796537}{b^4}\right) - 0.000015804\rho h\omega^2} \qquad (3.112)$$

3.5 Free Vibration of Plates

When the plate is exhibiting free vibration, the exciting force F(t) = 0, and the frequency changes from $\omega^2 = \lambda^2$. Expressions for the plate fundamental natural frequencies shall be examined. This loading which they are subjected depends on their position in the box parallelepiped.

3.5.1 Free Vibration of the Bottom Plate Subjected To Uniformly Distributed Load

Then, from Equation 3.97

$$A = \frac{0}{[D(\frac{0.00127}{b^4 p^4} + \frac{0.0007256}{b^4 p^2} + \frac{0.00127}{b^4}) - 0.000002520 q h h^2]} \qquad (3.113)$$

$$A\,[D(\frac{0.00127}{b^4 p^4} + \frac{0.0007256}{b^4 p^2} + \frac{0.00127}{b^4}) - 0.000002520 \rho h \lambda^2] = 0 \qquad (3.114)$$

$$\therefore A = 0 \text{ or } \left[D\left(\frac{0.00127}{b^4P^4} + \frac{0.0007256}{b^4P^2} + \frac{0.00127}{b^4}\right) - 0.000002520\rho h\lambda^2\right] = 0$$

$$D\left(\frac{0.00127}{b^4P^4} + \frac{0.0007256}{b^4P^2} + \frac{0.00127}{b^4}\right) - 0.000002520\rho h\lambda^2 = 0$$

$$\lambda^2 = \frac{D}{0.000002520\rho h}\left(\frac{0.00127}{b^4P^4} + \frac{0.0007256}{b^4P^2} + \frac{0.00127}{b^4}\right)$$

$$\lambda^2 = \frac{D}{0.000002520\rho h b^4}\left(\frac{0.00127}{P^4} + \frac{0.0007256}{P^2} + 0.00127\right) \qquad (3.115)$$

Let

$$\text{Let } \eta = \frac{1}{0.000002520}\left(\frac{0.00127}{P^4} + \frac{0.0007256}{P^2} + 0.00127\right) \qquad (3.116)$$

Then,

$$\lambda^2 = \frac{D\eta}{\rho h b^4} \qquad (3.117)$$

Table 3.1: Different values of 5 for different aspect ratios P

Aspect ratio, P	5 − Value
0.1	5068980.158730
0.2	322682.539683
0.3	65921.565746
0.4	21989.831349
0.5	9719.206349
0.6	5192.435822
0.7	3190.586925
0.8	2184.260293
0.9	1627.572621
1.0	1295.873016
1.1	1086.149409
1.2	946.964408
1.3	850.798180
1.4	782.061707
1.5	731.489320
1.6	693.342905
1.7	663.940568
1.8	640.845496
1.9	622.400360
2.0	607.450397

3.5.2 Free Vibration of the Bottom Plate Subjected to Linearly Varying Load

Then, from Equation 3.112

$$A = \frac{0}{[D(\frac{0.0081633}{b^4 P^4} + \frac{0.004595616}{b^4 P^2} + \frac{0.00796537}{b^4}) - 0.000015804qhh^2]} \quad (3.118)$$

$$A\,[D\,(\frac{0.0081633}{b^4 P^4} + \frac{0.004595616}{b^4 P^2} + \frac{0.00796537}{b^4}) - 0.000015804qhh^2] = 0 \quad (3.119)$$

$\therefore A = 0$

or $[D\,(\frac{0.0081633}{b^4 P^4} + \frac{0.004595616}{b^4 P^2} + \frac{0.00796537}{b^4}) - 0.000015804qhh^2] = 0$

$$D\,(\frac{0.0081633}{b^4 P^4} + \frac{0.004595616}{b^4 P^2} + \frac{0.00796537}{b^4}) - 0.000015804qhh^2 = 0$$

$$h^2 = \frac{D}{0.000015804qh}(\frac{0.0081633}{b^4 P^4} + \frac{0.004595616}{b^4 P^2} + \frac{0.00796537}{b^4})$$

$$h^2 = \frac{D}{0.000015804qhb^4}(\frac{0.0081633}{P^4} + \frac{0.004595616}{P^2} + 0.00796537) \quad (3.120)$$

Let

$$\text{Let } S = \frac{1}{0.000015804}(\frac{0.0081633}{P^4} + \frac{0.004595616}{P^2} + 0.00796537) \quad (3.121)$$

Then,

$$\lambda^2 = \frac{D5}{\rho h b^4} \qquad (3.122)$$

Table 3.2: Different values of 5 for different aspect ratios P

Aspect ratio, P	5 – Value
0.1	5194920.7144
0.2	330607.3317
0.3	67504.5928
0.4	22498.5368
0.5	9931.7030
0.6	5297.3548
0.7	3248.7827
0.8	2219.4351
0.9	1650.2865
1.0	1311.3317
1.1	1097.1301
1.2	955.0460
1.3	856.9266
1.4	786.8290
1.5	735.2803
1.6	696.4157
1.7	666.4733
1.8	642.9641
1.9	624.1960
2.0	608.9901

3.6 Forced Vibration of Plate

When a thin rectangular plate clamped at all edges plate is exhibiting forced vibration, the exciting force, F (t) ≠ 0, The forcing frequency symbol, $\omega^2 = \theta^2$.

Let the forcing frequency be expressed in terms of the natural frequency as shown in Table 3.3:

Table 3.3: θ^2 Values obtained from natural frequency.

S/No.	Expression of forcing frequency, θ^2	Values of forcing frequency, θ^2
1	$8^2 = 0\%h^2$	0
2	$8^2 = 20\%h^2$	$0.2h^2$
3	$8^2 = 40\%h^2$	$0.4h^2$
4	$8^2 = 60\%h^2$	$0.6h^2$
5	$8^2 = 80\%h^2$	$0.8h^2$
6	$8^2 = 100\%h^2$	h^2

Expressions for the coefficient, A of the plate behaviour under forcing force shall be examined. This loading, which they are subjected, depends on their position in the box parallelepiped.

3.6.1 Coefficient Of Forced Vibration, A of Plate At Different Aspect Ratio Under Uniformly Distributed Loads:

Then, from Equation 3.93

$$A = \frac{0.0011111F(t)}{D\left(\frac{0.00127}{b^4P^4} + \frac{0.0007256}{b^4P^2} + \frac{0.00127}{b^4}\right) - 0.000002520qh8^2} \qquad (3.123)$$

Where $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$

$$A = \frac{0.0011111F(t)}{D\left(\frac{0.00127}{b^4p^4} + \frac{0.0007256}{b^4p^2} + \frac{0.00127}{b^4}\right) - (0.000002520qh\alpha h^2)} \tag{3.124}$$

$$A = \frac{0.0011111F(t)}{D(\frac{0.00127}{b^4P^4} + \frac{0.0007256}{b^4P^2} + \frac{0.00127}{b^4}) - (0.000002520qh\alpha(\frac{Dy}{qhb^4}))} \tag{3.125}$$

$$A = \frac{0.0011111F(t)}{D(\frac{0.00127}{b^4P^4} + \frac{0.0007256}{b^4P^2} + \frac{0.00127}{b^4}) - (0.0000025200\alpha(\frac{Dy}{b^4}))}$$

$$A = \frac{0.0011111F(t)}{\frac{D}{b^4}[(\frac{0.00127}{P^4} + \frac{0.0007256}{P^2} + 0.00127) - (0.0000025205\alpha)]} \tag{3.126}$$

$$A = \frac{b^4F(t)}{D}\chi \tag{3.127}$$

Where,

$$3 = \frac{0.0011111}{[(\frac{0.00127}{P^4} + \frac{0.0007256}{P^2} + 0.00127) - (0.0000025205\alpha)]} \tag{3.128}$$

for $\alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and5isgivenas 3.116

The values of z for different aspect ratio of the plate, P is given in Table 3.4.

Table 3.4: χ values for different aspect ratios of the plate

ASPECT RATIO, P	h^2 Coefficient of free vibration	χ – Values for different, α 0.0	0.2	0.4	0.6	0.8	1.0
0.1	5068980.1587	0.00009	0.00011	0.00014	0.00022	0.00043	∞
0.2	322682.5397	0.00137	0.00171	0.00228	0.00342	0.00683	∞
0.3	65921.5657	0.00669	0.00836	0.01115	0.01672	0.03344	∞
0.4	21989.8313	0.02005	0.02506	0.03342	0.05013	0.10025	∞
0.5	9719.2063	0.04537	0.05671	0.07561	0.11341	0.22683	∞
0.6	5192.4358	0.08491	0.10614	0.14152	0.21229	0.42457	∞
0.7	3190.5869	0.13819	0.17274	0.23032	0.34548	0.69096	∞
0.8	2184.2603	0.20186	0.25232	0.33643	0.50465	1.00930	∞
0.9	1627.5726	0.27090	0.33863	0.45150	0.67726	1.35451	∞
1.0	1295.8730	0.34024	0.42530	0.56707	0.85061	1.70122	∞
1.1	1086.1494	0.40594	0.50743	0.67657	1.01485	2.02971	∞
1.2	946.9644	0.46561	0.58201	0.77601	1.16402	2.32803	∞
1.3	850.7982	0.96976	0.64779	0.86372	1.29559	2.59117	∞
1.4	782.0617	0.56378	0.70473	0.93964	1.40946	2.81891	∞
1.5	731.4893	0.60276	0.75345	1.00460	1.50690	3.01380	∞
1.6	693.3429	0.63592	0.79490	1.05987	1.58981	3.17961	∞
1.7	663.9406	0.66408	0.83011	1.10681	1.66021	3.32042	∞
1.8	640.8455	0.68802	0.86002	1.14670	1.72004	3.44009	∞
1.9	622.4004	0.70841	0.88551	1.18068	1.77102	3.54203	∞
2.0	607.4504	0.72584	0.90730	1.20974	1.81460	3.62921	∞

3.6.2 Coefficient Of Forced Vibration, A of Plate At Different Aspect Ratio Under Linearly Varying Loads:

Then, from Equation 3.108

$$A = \frac{0.002777778F(t)}{D\left(\frac{0.0081633}{b^4P^4} + \frac{0.004595616}{b^4P^2} + \frac{0.00796537}{b^4}\right) - 0.000015804\rho h h^2} \qquad (3.129)$$

Where $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$

$$A = \frac{0.002777778F(t)}{D\left(\frac{0.0081633}{b^4P^4} + \frac{0.004595616}{b^4P^2} + \frac{0.00796537}{b^4}\right) - 0.000015804\rho h\alpha h^2}$$

$$A = \frac{0.002777778F(t)}{D\left(\frac{0.0081633}{b^4P^4} + \frac{0.004595616}{b^4P^2} + \frac{0.00796537}{b^4}\right) - \left(0.000015804qh\alpha\left(\frac{Dy}{qhb^4}\right)\right)}$$

$$A = \frac{0.002777778F(t)}{D\left(\frac{0.0081633}{b^4P^4} + \frac{0.004595616}{b^4P^2} + \frac{0.00796537}{b^4}\right) - \left(0.000015804\alpha\left(\frac{Dy}{b^4}\right)\right)}$$

$$A = \frac{0.002777778F(t)}{\frac{D}{b^4}\left[\left(\frac{0.0081633}{P^4} + \frac{0.004595616}{P^2} + 0.00796537\right) - (0.0000158045\alpha)\right]} \qquad (3.130)$$

$$A = \frac{b^4F(t)}{D}3 \qquad (3.131)$$

Where,

$$3 = \frac{0.002777778}{\left[\left(\frac{0.0081633}{P^4} + \frac{0.004595616}{P^2} + 0.00796537\right) - (0.0000158045\alpha)\right]} \qquad (3.132)$$

for $\alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and 5 is given as 3.116

The values of z for different aspect ratio of the plate, P is given in Table 3.5.

Table 3.5: χ values for different aspect ratios of the plate

ASPECT RATIO, P	h^2	χ – *Values* for different, α					
	Coefficient of free vibration	0.0	0.2	0.4	0.6	0.8	1.0
0.1	5194920.7144	0.00003	0.00004	0.00006	0.00008	0.00017	∞
0.2	330607.3317	0.00053	0.00066	0.00089	0.00133	0.00266	∞
0.3	67504.5928	0.00260	0.00325	0.00434	0.00651	0.01302	∞
0.4	22498.5368	0.00781	0.00977	0.01302	0.01953	0.03906	∞
0.5	9931.7030	0.01770	0.02212	0.02950	0.04424	0.08849	∞
0.6	5297.3548	0.03318	0.04147	0.05530	0.08295	0.16590	∞
0.7	3248.7827	0.05410	0.06763	0.09017	0.13525	0.27051	∞
0.8	2219.4351	0.07919	0.09899	0.13199	0.19798	0.39597	∞
0.9	1650.2865	0.10651	0.13313	0.17751	0.26626	0.53253	∞
1.0	1311.3317	0.13403	0.16754	0.22339	0.33509	0.67017	∞
1.1	1097.1301	0.16020	0.20025	0.26701	0.40051	0.80102	∞
1.2	955.0460	0.18404	0.23005	0.30673	0.46009	0.92019	∞
1.3	856.9266	0.20511	0.25639	0.34185	0.51278	1.02555	∞
1.4	786.8290	0.22338	0.27923	0.37231	0.55846	1.11692	∞
1.5	735.2803	0.23904	0.29880	0.39841	0.59761	1.19522	∞
1.6	696.4157	0.25238	0.31548	0.42064	0.63096	1.26192	∞
1.7	666.4733	0.26372	0.32965	0.43954	0.65931	1.31861	∞
1.8	642.9641	0.27337	0.34171	0.45561	0.68341	1.36683	∞
1.9	624.1960	0.28159	0.35198	0.46931	0.70396	1.40793	∞
2.0	608.9901	0.28862	0.36077	0.48103	0.72154	1.44308	∞

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Comparative analysis of free vibration of CCCC rectangular plate by different studies:

The fundamental natural frequency of the CCCC plate is generally given as:

$$\lambda^2 = \frac{D5}{\rho h b^4}$$

Where, the amplitude coefficient 5 varies for different studies:

$$5 = 512\left(\frac{1}{P^4} + \frac{256}{441 b^4 P^2} + 1\right) \quad \text{(Ventsel \& KrauthaNNer, 2001)} \qquad (4.1)$$

$$5 = 504\left(\frac{1}{P^4} + \frac{4}{7P^2} + 1\right) \quad \text{(Galin, 1947)} \qquad (4.2)$$

$$5 = \frac{\pi^4}{2.25}\left(\frac{12}{P^4} + \frac{8}{P^2} + 12\right) \quad \text{(Janich, 1962)} \qquad (4.3)$$

$$\eta = \frac{1}{0.000002520}\left(\frac{0.00127}{P^4} + \frac{0.0007256}{P^2} + 0.00127\right) \quad \text{(Present Study)} \qquad (4.4)$$

Tables 4.1 and 4.2 show the comparative relationship between the square of fundamental frequency coefficient, η and fundamental frequency coefficient, $\bar{f}y$ respectively.

Table 4.1: Comparative relationship of the square of fundamental frequency coefficients η for CCCC Rectangular thin Plate from various studies.

P= a/b	ς			
	Present study	Ventsel& Krauthammer (2001)	Galin (1947)	Janich (1962)
0.1	5068980.159	5150233.5420	5069304.0000	5238731.2457
0.2	322682.540	327942.3855	322704.0000	334412.9339
0.3	65921.566	67024.2701	65926.2222	68615.8089
0.4	21989.831	22369.5964	21991.5000	23014.7387
0.5	9719.206	9892.8617	9720.0000	10233.5908
0.6	5192.436	5288.2157	5192.8889	5499.0293
0.7	3190.587	3251.0069	3190.8805	3395.5447
0.8	2184.260	2226.3991	2184.4688	2332.7763
0.9	1627.573	1659.3015	1627.7311	1741.7241
1.0	1295.873	1321.2154	1296.0000	1387.6055
1.1	1086.149	1107.3355	1086.2553	1162.4549
1.2	946.964	965.3132	947.0556	1012.1972
1.3	850.798	867.1325	850.8786	907.8089
1.4	782.062	796.9183	782.1341	832.7945
1.5	731.489	745.2315	731.5556	777.3161
1.6	693.343	706.2248	693.4043	735.2599
1.7	663.941	676.1447	663.9981	702.6891
1.8	640.845	652.5062	640.8999	676.9891
1.9	622.400	633.6187	622.4521	656.3751
2.0	607.450	618.3039	607.5000	639.5994

Table 4.2: Comparative relationship of the fundamental frequency coefficient, $\bar{f}5$ for CCCC Rectangular thin Plate from various studies

P= a/b	Fundamental Frequency coefficient, $\bar{f}5$			
	Present study	Ventsel& Krauthammer(2001)	Galin (1947)	Janich (1962)
0.1	2251.4396	2269.4126	2251.5115	2288.8275
0.2	568.0515	572.6625	568.0704	578.2845
0.3	256.7520	258.8905	256.7610	261.9462
0.4	148.2897	149.5647	148.2953	151.7061
0.5	98.5860	99.4629	98.5901	101.1612
0.6	72.0586	72.7201	72.0617	74.1554
0.7	56.4853	57.0176	56.4879	58.2713
0.8	46.7361	47.1847	46.7383	48.2988
0.9	40.3432	40.7345	40.3452	41.7340
1.0	**35.9982**	**36.3485**	**36.0000**	**37.2506**
1.1	32.9568	33.2767	32.9584	34.0948
1.2	30.7728	31.0695	30.7743	31.8150
1.3	29.1684	29.4471	29.1698	30.1299
1.4	27.9654	28.2297	27.9667	28.8582
1.5	27.0461	27.2989	27.0473	27.8804
1.6	26.3314	26.5749	26.3326	27.1157
1.7	25.7670	26.0028	25.7682	26.5083
1.8	25.3149	25.5442	25.3160	26.0190
1.9	24.9480	25.1718	24.9490	25.6198
2.0	24.6465	24.8657	24.6475	25.2903

All the four results were obtained from energy variational principle. From Table 4.1 and Table 4.2, the results of the present study shows very perfect correlation when compared with the results from other previous work from Galin (1947), Ventsel and Krauthammer (2001) and Janich (1962). This justifies the shape function formulated for the plate by the characteristic orthogonal polynomials. Whereas, others except Ventsel and Krauthammer (2001) sought the solution of the governing differential equations from the Bessel trigonometric function.

4.2 Deflections, Shear Force and Moments for Various Values of Coefficient A, With Respect To Different Aspect Ratio of Thin Rectangular Plate Clamped On All Edges Subjected To Uniformly Distributed Load

A section of a thin rectangular plate whose all edges are clamped is shown in Figure 4.1. The plate is subjected to uniformly distributed load causing the forced excitation of the plate.

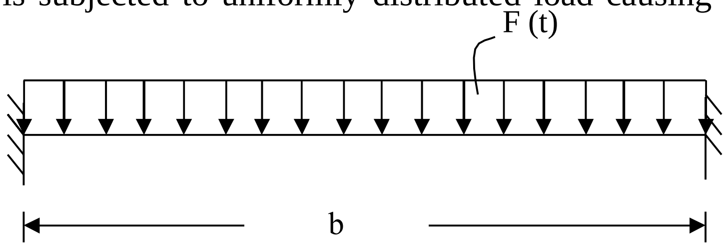


Figure 4.1 Section through a rectangular plate whose all edges are clamped subjected to uniformly distributed load causing the forced excitation of the plate.

Recall, from Equation 3.127

$$A = \frac{b^4 F(t)}{D}\chi$$

Where the values of χ at different aspect ratios and forcing frequencies are tabulated in Table 3.3.

4.2.1 Deflection

From Equation 3.61:

$$W(x, y) = A(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)$$

Where, at the midspan, R = Q = ½

$$W_{Nas} = A\left(\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4\right)\left(\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4\right) = \frac{A}{256} \qquad (4.5)$$

$$W_{Nas} = \frac{1}{256}\frac{b^4F(t)}{D}\chi$$

$$W_{Nas} = \frac{\chi}{256}\left(\frac{b^4F(t)}{D}\right)$$

Where,

$$W_{Nas} = K\frac{}{} \qquad \chi$$

$$K = \frac{256\, b^4 F(t)}{D}$$

Table 4.3 shows the $K - values$ at different aspect ratios and forcing frequencies

Table 4.3: K – $values$ at different aspect ratios and forcing frequencies for Maximum deflection

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	K-Values for different, α					
0.1	4.34913E-05	5.44E-05	7.25E-05	0.000109	0.000217	∞
0.2	0.000683199	0.000854	0.001139	0.001708	0.003416	∞
0.3	0.003344222	0.00418	0.005574	0.008361	0.016721	∞
0.4	0.010025377	0.012532	0.016709	0.025063	0.050127	∞
0.5	0.022682546	0.028353	0.037804	0.056706	0.113413	∞
0.6	0.042457212	0.053072	0.070762	0.106143	0.212286	∞
0.7	0.069095861	0.08637	0.11516	0.17274	0.345479	∞
0.8	0.100929523	0.126162	0.168216	0.252324	0.504648	∞
0.9	0.135451006	0.169314	0.225752	0.338628	0.677255	∞
1.0	0.170121877	0.212652	0.283536	0.425305	0.850609	∞
1.1	0.202970556	0.253713	0.338284	0.507426	1.014853	∞
1.2	0.232803205	0.291004	0.388005	0.582008	1.164016	∞
1.3	0.259117091	0.323896	0.431862	0.647793	1.295585	∞
1.4	0.281891246	0.352364	0.469819	0.704728	1.409456	∞
1.5	0.301380134	0.376725	0.50230	0.75345	1.506901	∞
1.6	0.317961499	0.397452	0.529936	0.794904	1.589807	∞
1.7	0.332042294	0.415053	0.553404	0.830106	1.660211	∞
1.8	0.34400858	0.430011	0.573348	0.860021	1.720043	∞
1.9	0.354203441	0.442754	0.590339	0.885509	1.771017	∞
2.0	0.362920743	0.453651	0.604868	0.907302	1.814604	∞

Deflection and slope at the edge of the plate equal to zero (R=Q=0).But at the mid-span (R = Q = ½) as the forcing frequency is gradually increased i.e.$8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$and approaches the fundamental natural frequency, the deflection also increases until the forcing frequency coincides with the natural frequency. At this point, the deflection, turn to infinity.

4.2.2 Moment

$$M_x = -D\left[\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right]$$

$$M_y = -D\left[\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right] \text{ and} \qquad (4.6)$$

$$M_{xy} = M_{yx} = -D[1 - \mu]\frac{\partial^2 w}{\partial x dy}$$

Expressing the moments in terms of non – dimensional parameters R and Q and aspect ratio, p.

Where, x = aR, y = bQ, a/b = P

$$M_x = -D\left[\frac{\partial^2 w}{a^2\,\partial R^2} + \mu\frac{\partial^2 w}{b^2\,\partial Q^2}\right]$$

$$M_x = -D\left[\frac{\partial^2 w}{(bP)^2\,\partial R^2} + \mu\frac{\partial^2 w}{b^2\,\partial Q^2}\right]$$

$$M_x = \frac{-D}{b^2}\left[\frac{\partial^2 w}{P^2\,\partial R^2} + \quad \frac{\partial^2 w}{\partial Q^2}\right] \qquad (4.7)$$

μ

Similarly,

$$M_y = \frac{-D}{b^2}\left[\mu\frac{\partial^2 w}{P^2\,\partial R^2} \quad \frac{\partial^2 w}{\partial Q^2}\right] \qquad (4.8)$$

+

$$M_{xy} = M_{yx} = \frac{-D[1-\mu]}{ab}\frac{\partial^2 w}{\partial R dQ}$$

$$M_{xy} = M_{yx} = \frac{-D[1-\mu]}{Pb^2}\frac{\partial^2 w}{\partial R dQ} \qquad (4.9)$$

Substituting shape function for a plate with all edges clamped and subjected to uniformly load as given in Equation 3.61 into Equations 4.7, 4.8 and 4.9:

$$M_x = \frac{-AD}{b^2}\left[\frac{\partial^2}{P^2\,\partial R^2} + \mu\frac{\partial^2}{\partial Q^2}\right](R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)$$

$$M_x = \frac{-AD}{b^2}\left[\frac{(2-12R+12R^2)(Q^2-2Q^3+Q^4)}{P^2} + \mu(R^2-2R^3+R^4)(2-12Q+12Q^2)\right] \quad (4.10)$$

Similarly,

$$M_y = \frac{-AD}{b^2}\left[\mu\frac{(2-12R+12R^2)(Q^2-2Q^3+Q^4)}{P^2} + (R^2-2R^3+R^4)(2-12Q+12Q^2)\right] \qquad (4.11)$$

$$M_{xy} = M_{yx} = \frac{-DA[1-\mu]}{Pb^2}\frac{\partial^2((R^2-2R^3+R^4)(Q^2-2Q^3+Q^4))}{\partial R dQ} \qquad (4.12)$$

$$M_{xy} = M_{yx} = \frac{-DA[1-\mu]}{Pb^2}(2R-6R^2+4R^3)(2Q-6Q^2+4Q^3) \qquad (4.13)$$

Where,

$$A = \frac{0.0011111b^4F(t)}{D\left[\left(\frac{0.00127}{P^4}+\frac{0.0007256}{P^2}+0.00127\right)-(0.0000025205\alpha)\right]}$$

$$A = \frac{b^4F(t)}{D}3$$

$$3 = \frac{0.0011111}{\left[\left(\frac{0.00127}{P^4}+\frac{0.0007256}{P^2}+0.00127\right)-(0.0000025205\alpha)\right]}$$

for $\alpha = 0,\ 0.2,\ 0.4,\ 0.6,\ 0.8,\ 1.0$

$$M_x = -b^2F(t)\left[\frac{(2-12R+12R^2)(Q^2-2Q^3+Q^4)}{P^2} + \mu(R^2-2R^3+R^4)(2-12Q+12Q^2)\right]\chi$$

(4.14

$$) \; M_x = -b^2 F(t)\, \beta_x$$

(4.15)

$$\beta_x = [\frac{(2-12R+12R^2)(Q^2-2Q^3+Q^4)}{P^2} + \mu\,(R^2 - 2R^3 + R^4)(2 - 12Q + 12Q^2)]\,\chi \qquad (4.16)$$

Similarly,

$$M_y = -b^2 \; F(t) \; [\mu \; \frac{(2-12R+12R^2)(Q^2-2Q^3+Q^4)}{P^2}$$

$$+ (R^2 - 2R^3 + R^4)(2 - 12Q + 12Q^2)]\,\chi$$

(4.17

$$) \; M_y = -b^2 F(t)\, \beta_y$$

(4.18

)

$$\beta_y = [\mu \frac{(2-12R+12R^2)(Q^2-2Q^3+Q^4)}{P^2} + (R^2 - 2R^3 + R^4)(2 - 12Q + 12Q^2)]\chi \qquad (4.19)$$

$$M_{xy} = M_{yx} = \frac{-b^2F(t)[1-\mu]}{P}(2R - \quad + 4R^3)(2Q - \quad + 4Q^3)z \qquad (4.20)$$

$$6R^2 \qquad 6Q^2$$

$$M_{xy} = M_{yx} = -b^2F(t)\ \beta_{xy}$$

$$= -b^2F(t)\,\beta_{yx} \qquad (4.21)$$

$$\beta_{xy} = \beta_{yx} = [\frac{[1-\mu]}{)\,P}(2R - 6R^2 + 4R^3)(2Q - 6Q^2 + 4Q^3)]z \qquad (4.22$$

The values of moment coefficients β_x *and* β_y for different plate aspect ratios and forcing frequencies are tabulated in Tables 4.4, through 4.9b.

Table 4.4: Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the centre of the plate

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_s					
0.1	-0.00055	-0.00068	-0.00091	-0.00136	-0.00273	∞
0.2	-0.00216	-0.00270	-0.00360	-0.00540	-0.01080	∞
0.3	-0.00477	-0.00596	-0.00795	-0.01193	-0.02385	∞
0.4	-0.00821	-0.01026	-0.01368	-0.02052	-0.04104	∞
0.5	-0.01219	-0.01524	-0.02032	-0.03048	-0.06096	∞
0.6	-0.01633	-0.02042	-0.02722	-0.04084	-0.08167	∞
0.7	-0.02022	-0.02527	-0.03370	-0.05054	-0.10109	∞
0.8	-0.02350	-0.02937	-0.03916	-0.05874	-0.11749	∞
0.9	-0.02598	-0.03248	-0.04330	-0.06496	-0.12991	∞
1	-0.02764	-0.03456	-0.04607	-0.06911	-0.13822	∞
1.1	-0.02858	-0.03572	-0.04763	-0.07145	-0.14290	∞
1.2	-0.02894	-0.03617	-0.04823	-0.07235	-0.14469	∞
1.3	-0.02888	-0.03610	-0.04814	-0.07221	-0.14441	∞
1.4	-0.02855	-0.03569	-0.04758	-0.07137	-0.14274	∞
1.5	-0.02805	-0.03506	-0.04674	-0.07011	-0.14023	∞
1.6	-0.02745	-0.03431	-0.04575	-0.06862	-0.13725	∞
1.7	-0.02681	-0.03352	-0.04469	-0.06703	-0.13407	∞
1.8	-0.02617	-0.03272	-0.04362	-0.06543	-0.13086	∞
1.9	-0.02555	-0.03193	-0.04258	-0.06387	-0.12774	∞
2	-0.02495	-0.03119	-0.04158	-0.06238	-0.12475	∞

Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the centre of the plate R= Q =1/2. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural

frequency, Moment coefficients β_x also increases until the forcing frequency coincides with the natural frequency. At this point, Moment coefficients β_x turn to infinity.

Table 4.5: Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 0

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_s					
0.1	-0.00000	-0.00000	-0.00001	-0.00001	-0.00002	∞
0.2	-0.00005	-0.00006	-0.00009	-0.00013	-0.00026	∞
0.3	-0.00025	-0.00031	-0.00042	-0.00063	-0.00125	∞
0.4	-0.00075	-0.00094	-0.00125	-0.00188	-0.00376	∞
0.5	-0.00170	-0.00213	-0.00284	-0.00425	-0.00851	∞
0.6	-0.00318	-0.00398	-0.00531	-0.00796	-0.01592	∞
0.7	-0.00518	-0.00648	-0.00864	-0.01296	-0.02591	∞
0.8	-0.00757	-0.00946	-0.01262	-0.01892	-0.03785	∞
0.9	-0.01016	-0.01270	-0.01693	-0.02540	-0.05079	∞
1	-0.01276	-0.01595	-0.02127	-0.03190	0.06380	∞
1.1	-0.01522	-0.01903	-0.02537	-0.03806	-0.07611	∞
1.2	-0.01746	-0.02183	-0.02910	-0.04365	-0.08730	∞
1.3	-0.01943	-0.02429	-0.03239	-0.04858	-0.09717	∞
1.4	-0.02114	-0.02643	-0.03524	-0.05285	-0.10571	∞
1.5	-0.02260	-0.02825	-0.03767	-0.05651	--0.11302	∞
1.6	-0.02385	-0.02981	-0.03975	-0.05962	-0.11924	∞
1.7	-0.02490	-0.03113	-0.04151	-0.06226	-0.12452	∞
1.8	-0.02580	-0.03225	-0.04300	-0.06450	-0.12900	∞
1.9	-0.02657	-0.03321	-0.04428	-0.06641	-0.13283	∞
2	-0.02722	-0.03402	-0.04537	-0.06805	-0.13610	∞

Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 0. As the forcing frequency is gradually increased i.e. $8^2 =$ $, \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural

frequency, Moment coefficients β_x also increases until the forcing frequency coincides with the natural frequency. At this point, Moment coefficients β_x turn to infinity.

Table 4.6: Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = 0 and Q = ½

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_s					
0.1	-0.00109	-0.00136	-0.00181	-0.00272	-0.00544	∞
0.2	-0.00427	-0.00534	-0.00712	-0.01067	-0.02135	∞
0.3	-0.00929	-0.01161	-0.01548	-0.02322	-0.04645	∞
0.4	-0.01566	-0.01958	-0.02611	-0.03916	-0.07832	∞
0.5	-0.02268	-0.02835	-0.03780	-0.05671	-0.11341	∞
0.6	-0.02948	-0.03686	-0.04914	-0.07371	0.14742	∞
0.7	-0.03525	-0.04407	-0.05875	-0.08813	-0.17626	∞
0.8	-0.03943	-0.04928	-0.06571	-0.09856	-0.19713	∞
0.9	-0.04181	-0.05226	-0.06968	-0.10451	-0.20903	∞
1	-0.04253	-0.05316	-0.07088	-0.10633	-0.21265	∞
1.1	-0.04194	-0.05242	-0.06989	-0.10484	-0.20968	∞
1.2	-0.04042	-0.05052	-0.06736	-0.10104	-0.20209	∞
1.3	-0.03833	-0.04791	-0.06388	-0.09583	-0.19165	∞
1.4	-0.03596	-0.04494	-0.05993	-0.08989	-0.17978	∞
1.5	-0.03349	-0.04186	-0.05581	-0.08372	-0.16743	∞
1.6	-0.03105	-0.03881	-0.05175	-0.07763	-0.15525	∞
1.7	-0.02872	-0.03590	-0.04787	-0.07181	-0.14362	∞
1.8	-0.02654	-0.03318	-0.04424	-0.06636	-0.13272	∞
1.9	-0.02453	-0.03066	-0.04088	-0.06132	-0.12265	∞
2	-0.02268	-0.02835	-0.03780	-0.05671	-0.11341	∞

Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = 0 and Q = ½. As the forcing frequency is gradually increased i.e. $8^2 =$ $, \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural

frequency, Moment coefficients β_x also increases until the forcing frequency coincides with the natural frequency. At this point, Moment coefficients β_x turn to infinity.

Table 4.6a: Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = 1 and Q = ½

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_s					
0.1	-0.00109	-0.00136	-0.00181	-0.00272	-0.00544	∞
0.2	-0.00427	-0.00534	-0.00712	-0.01067	-0.02135	∞
0.3	-0.00929	-0.01161	-0.01548	-0.02322	-0.04645	∞
0.4	-0.01566	-0.01958	-0.02611	-0.03916	-0.07832	∞
0.5	-0.02268	-0.02835	-0.03780	-0.05671	-0.11341	∞
0.6	-0.02948	-0.03686	-0.04914	-0.07371	-0.14742	∞
0.7	-0.03525	-0.04407	-0.05875	-0.08813	-0.17626	∞
0.8	-0.03943	-0.04928	-0.06571	-0.09856	-0.19713	∞
0.9	-0.04181	-0.05226	-0.06968	-0.10451	-0.20903	∞
1	-0.04253	-0.05316	-0.07088	-0.10633	-0.21265	∞
1.1	-0.04194	-0.05242	-0.06989	-0.10484	-0.20968	∞
1.2	-0.04042	-0.05052	-0.06736	-0.10104	-0.20209	∞
1.3	-0.03833	-0.04791	-0.06388	-0.09583	-0.19165	∞
1.4	-0.03596	-0.04494	-0.05993	-0.08989	-0.17978	∞
1.5	0.03349	-0.04186	-0.05581	0.08372	-0.16743	∞
1.6	-0.03105	-0.03881	-0.05175	-0.07763	-0.15525	∞
1.7	-0.02872	-0.03590	-0.04787	-0.07181	-0.14362	∞
1.8	-0.02654	-0.03318	-0.04424	-0.06636	-0.13272	∞
1.9	-0.02453	-0.03066	-0.04088	-0.06132	-0.12265	∞
2	-0.02268	-0.02835	-0.03780	-0.05671	-0.11341	∞

Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = 1 and Q = ½. As the forcing frequency is gradually increased i.e. $8^2 =$, αh^2, $\alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural

frequency, Moment coefficients β_x also increases until the forcing frequency coincides with the natural frequency. At this point, Moment coefficients β_x turn to infinity.

Table 4.6b: Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 1

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_s					
0.1	-0.00000	-0.00000	-0.00001	-0.00001	-0.00002	∞
0.2	-0.00005	-0.00006	-0.00009	-0.00013	-0.00026	∞
0.3	-0.00025	-0.00031	-0.00042	-0.00063	-0.00125	∞
0.4	-0.00075	-0.00094	-0.00125	-0.00188	-0.00376	∞
0.5	-0.00170	-0.00213	-0.00284	-0.00425	-0.00851	∞
0.6	-0.00318	-0.00398	-0.00531	-0.00796	-0.01592	∞
0.7	-0.00518	-0.00648	-0.00864	-0.01296	-0.02591	∞
0.8	-0.00757	-0.00946	-0.01262	-0.01892	-0.03785	∞
0.9	-0.01016	-0.01270	-0.01693	-0.02540	-0.05079	∞
1	-0.01276	-0.01595	-0.02127	-0.03190	-0.06380	∞
1.1	-0.01522	-0.01903	-0.02537	-0.03806	-0.07611	∞
1.2	-0.01746	-0.02183	-0.02910	-0.04365	-0.08730	∞
1.3	-0.01943	-0.02429	-0.03239	-0.04858	-0.09717	∞
1.4	-0.02114	-0.02643	-0.03524	-0.05285	-0.10571	∞
1.5	-0.02260	-0.02825	-0.03767	-0.05651	-0.11302	∞
1.6	-0.02385	-0.02981	-0.03975	-0.05962	-0.11924	∞
1.7	-0.02490	-0.03113	-0.04151	-0.06226	-0.12452	∞
1.8	-0.02580	-0.03225	-0.04300	-0.06450	-0.12900	∞
1.9	-0.02657	-0.03321	-0.04428	-0.06641	-0.13283	∞
2	-0.02722	-0.03402	-0.04537	-0.06805	-0.13610	∞

Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 1. As the forcing frequency is gradually increased i.e. $8^2 =$ $, \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural

frequency, Moment coefficients β_x also increases until the forcing frequency coincides with the natural frequency. At this point, Moment coefficients β_x turn to infinity.

Table 4.7: Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the centre of the plate

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_y					
0.1	-0.00017	-0.00021	-0.00028	-0.00042	-0.00084	∞
0.2	-0.00073	-0.00091	-0.00121	-0.00181	-0.00363	∞
0.3	-0.00181	-0.00226	-0.00302	-0.00453	-0.00906	∞
0.4	-0.00360	-0.00450	-0.00600	-0.00901	-0.01801	∞
0.5	-0.00624	-0.00780	-0.01040	-0.01559	-0.03119	∞
0.6	-0.00973	-0.01216	-0.01622	-0.02432	-0.04865	∞
0.7	-0.01392	-0.01741	-0.02321	-0.03481	-0.06962	∞
0.8	-0.01853	-0.02316	-0.03088	-0.04633	-0.09265	∞
0.9	-0.02320	-0.02900	-0.03867	-0.05801	-0.11601	∞
1	-0.02764	-0.03456	-0.04607	-0.06911	-0.13822	∞
1.1	-0.03166	-0.03958	-0.05277	-0.07915	-0.15831	∞
1.2	-0.03516	-0.04395	-0.05860	-0.08791	-0.17581	∞
1.3	-0.03814	-0.04767	-0.06357	-0.09535	-0.19070	∞
1.4	-0.04063	-0.05079	-0.06772	-0.10157	-0.20315	∞
1.5	-0.04270	-0.05337	-0.07116	-0.10674	-0.21348	∞
1.6	-0.04440	-0.05550	-0.07400	-0.11101	-0.22201	∞
1.7	-0.04581	-0.05727	-0.07636	-0.11453	-0.22907	∞
1.8	-0.04698	-0.05873	-0.07830	-0.11746	-0.23491	∞
1.9	-0.04795	-0.05994	-0.07992	-0.11989	-0.23977	∞
2	-0.04877	-0.06096	-0.08128	-0.12192	-0.24384	∞

Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the centre of the plate, R= Q =1/2. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Moment coefficients β_y also increases until the forcing frequency coincides with the natural frequency. At this point, Moment coefficients β_y turn to infinity.

Table 4.8: Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 0

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_y					
0.1	-0.00001	-0.00001	-0.00002	-0.00003	-0.00005	∞
0.2	-0.00017	-0.00021	-0.00028	-0.00043	-0.00085	∞
0.3	-0.00084	-0.00105	-0.00139	-0.00209	-0.00418	∞
0.4	-0.00251	-0.00313	-0.00418	-0.00627	-0.01253	∞
0.5	-0.00567	-0.00709	-0.00945	-0.01418	-0.02835	∞
0.6	-0.01061	-0.01327	-0.01769	-0.02654	-0.05307	∞
0.7	-0.01727	-0.02159	-0.02879	-0.04318	-0.08637	∞
0.8	-0.02523	-0.03154	-0.04205	-0.06308	-0.12616	∞
0.9	-0.03386	-0.04233	-0.05644	-0.08466	-0.16931	∞
1	-0.04253	-0.05316	-0.07088	-0.10633	-0.21265	∞
1.1	-0.05074	-0.06343	-0.08457	-0.12686	-0.25371	∞
1.2	-0.05820	-0.07275	-0.09700	-0.14550	-0.29100	∞
1.3	-0.06478	-0.08097	-0.10797	-0.16195	-0.32390	∞
1.4	-0.07047	-0.08809	-0.11745	-0.17618	-0.35236	∞
1.5	-0.07535	-0.09418	-0.12558	-0.18836	-0.37673	∞
1.6	-0.07949	-0.09936	-0.13248	-0.19873	-0.39745	∞
1.7	-0.08301	-0.10376	-0.13835	-0.20753	-0.41505	∞
1.8	-0.08600	-0.10750	-0.14334	-0.21501	-0.43001	∞
1.9	-0.08855	-0.11069	-0.14758	-0.22138	-0.44275	∞
2	-0.09073	-0.11341	-0.15122	-0.22683	-0.45365	∞

Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 0. As the forcing frequency is gradually increased i.e. $8^2 = ,\alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Moment coefficients β_y also increases until the forcing frequency coincides with the

natural frequency. At this point, Moment coefficients β_y turn to infinity.

Table 4.9: Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = 0 and Q = ½

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P			β_y			
0.1	-0.00033	-0.00041	-0.00054	-0.00082	-0.00163	∞
0.2	-0.00128	-0.00160	-0.00213	-0.00320	-0.00640	∞
0.3	-0.00279	-0.00348	-0.00464	-0.00697	-0.01393	∞
0.4	-0.00470	-0.00587	-0.00783	-0.01175	-0.02350	∞
0.5	-0.00680	-0.00851	-0.01134	-0.01701	-0.03402	∞
0.6	-0.00885	-0.01106	-0.01474	-0.02211	-0.04423	∞
0.7	-0.01058	-0.01322	-0.01763	-0.02644	-0.05288	∞
0.8	-0.01183	-0.01478	-0.01971	-0.02957	-0.05914	∞
0.9	-0.01254	-0.01568	-0.02090	-0.03135	-0.06271	∞
1	-0.01276	-0.01595	-0.02127	-0.03190	-0.06380	∞
1.1	-0.01258	-0.01573	-0.02097	-0.03145	-0.06290	∞
1.2	-0.01213	-0.01516	-0.02021	-0.03031	-0.06063	∞
1.3	-0.01150	-0.01437	-0.01917	-0.02875	-0.05750	∞
1.4	-0.01079	-0.01348	-0.01798	-0.02697	-0.05393	∞
1.5	-0.01005	-0.01256	-0.01674	-0.02512	-0.05023	∞
1.6	-0.00932	-0.01164	-0.01553	-0.02329	-0.04658	∞
1.7	-0.00862	-0.01077	-0.01436	-0.02154	-0.04309	∞
1.8	-0.00796	-0.00995	-0.01327	-0.01991	-0.03982	∞
1.9	-0.00736	-0.00920	-0.01226	-0.01840	-0.03679	∞
2	-0.00680	-0.00851	-0.01134	-0.01701	-0.03402	∞

Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = 0 and Q = ½. As the forcing frequency is gradually increased i.e. $8^2 = ,\alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Moment coefficients β_y also increases until the forcing frequency coincides with the

natural frequency. At this point, Moment coefficients β_y turn to infinity.

Table 4.9a: Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = 1 and Q = ½

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_y					
0.1	-0.00033	-0.00041	-0.00054	-0.00082	-0.00163	∞
0.2	-0.00128	-0.00160	-0.00213	-0.00320	-0.00640	∞
0.3	-0.00279	-0.00348	-0.00464	-0.00697	-0.01393	∞
0.4	-0.00470	-0.00587	-0.00783	-0.01175	-0.02350	∞
0.5	-0.00680	-0.00851	-0.01134	-0.01701	-0.03402	∞
0.6	-0.00885	-0.01106	-0.01474	-0.02211	-0.04423	∞
0.7	-0.01058	-0.01322	-0.01763	-0.02644	-0.05288	∞
0.8	-0.01183	-0.01478	-0.01971	-0.02957	-0.05914	∞
0.9	-0.01254	-0.01568	-0.02090	-0.03135	-0.06271	∞
1	-0.01276	-0.01595	-0.02127	-0.03190	-0.06380	∞
1.1	-0.01258	-0.01573	-0.02097	-0.03145	-0.06290	∞
1.2	-0.01213	-0.01516	-0.02021	-0.03031	-0.06063	∞
1.3	-0.01150	-0.01437	-0.01917	-0.02875	-0.05750	∞
1.4	-0.01079	-0.01348	-0.01798	-0.02697	-0.05393	∞
1.5	-0.01005	-0.01256	-0.01674	-0.02512	-0.05023	∞
1.6	-0.00932	-0.01164	-0.01553	-0.02329	-0.04658	∞
1.7	-0.00862	-0.01077	-0.01436	-0.02154	-0.04309	∞
1.8	-0.00796	-0.00995	-0.01327	-0.01991	-0.03982	∞
1.9	-0.00736	-0.00920	-0.01226	-0.01840	-0.03679	∞
2	-0.00680	-0.00851	-0.01134	-0.01701	-0.03402	∞

Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = 1 and Q = ½. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Moment coefficients β_y also increases until the forcing frequency coincides with the

natural frequency. At this point, Moment coefficients β_y turn to infinity.

Table 4.9b: Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 1

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_y					
0.1	-0.00001	-0.00001	-0.00002	-0.00003	-0.00005	∞
0.2	-0.00017	-0.00021	-0.00028	-0.00043	-0.00085	∞
0.3	-0.00084	-0.00105	-0.00139	-0.00209	-0.00418	∞
0.4	-0.00251	-0.00313	-0.00418	-0.00627	-0.01253	∞
0.5	-0.00567	-0.00709	-0.00945	-0.01418	-0.02835	∞
0.6	-0.01061	-0.01327	-0.01769	-0.02654	-0.05307	∞
0.7	-0.01727	-0.02159	-0.02879	-0.04318	-0.08637	∞
0.8	-0.02523	-0.03154	-0.04205	-0.06308	-0.12616	∞
0.9	-0.03386	-0.04233	-0.05644	-0.08466	-0.16931	∞
1	-0.04253	-0.05316	-0.07088	-0.10633	-0.21265	∞
1.1	-0.05074	-0.06343	-0.08457	-0.12686	-0.25371	∞
1.2	-0.05820	-0.07275	-0.09700	-0.14550	-0.29100	∞
1.3	-0.06478	-0.08097	-0.10797	-0.16195	-0.32390	∞
1.4	-0.07047	-0.08809	-0.11745	-0.17618	-0.35236	∞
1.5	-0.07535	-0.09418	-0.12558	-0.18836	-0.37673	∞
1.6	-0.07949	-0.09936	-0.13248	-0.19873	-0.39745	∞
1.7	-0.08301	-0.10376	-0.13835	-0.20753	-0.41505	∞
1.8	-0.08600	-0.10750	-0.14334	-0.21501	-0.43001	∞
1.9	-0.08855	-0.11069	-0.14758	-0.22138	-0.44275	∞
2	-0.09073	-0.11341	-0.15122	-0.22683	-0.45365	∞

Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 1. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Moment coefficients β_x also increases until the forcing frequency coincides with the natural frequency. At this point, Moment coefficients β_x turn to infinity.

4.2.3 Shear Forces

Shear forces acting on the plate are given by the expressions:

$$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y}$$

$$Q_y = \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y}$$

Expressing these Equations in non – dimensional parameters R and Q and aspect ratio, p.

Where, x = aR, y = bQ, a/b = P:

$$Q_x = \frac{\partial M_x}{a\,\partial R} + \frac{\partial M_{xy}}{b\,\partial Q}$$

$$Q_x = \frac{\partial M_x}{Pb\,\partial R} + \frac{\partial M_{xy}}{b\,\partial Q}$$

$$Q_x = \frac{1}{b}\left(\frac{\partial M_x}{P\,\partial R} + \frac{\partial M_{xy}}{\partial Q}\right) \qquad (4.23)$$

Similarly,

$$Q_y = \frac{1}{b}\left(\frac{\partial M_{xy}}{P\,\partial R} + \frac{\partial M_y}{\partial Q}\right) \qquad (4.24)$$

Putting Equations 4.14, 4.17 and 4.20 into Equations 4.23 and 4.24:

$$Q_x = \frac{1}{bP}\left(\frac{\partial\, -b^2F(t)\left[\frac{(2-12R+12R^2)(Q^2-2Q^3+Q^4)}{P^2} + \mu\,(R^2-2R^3+R^4)(2-12Q+12Q^2)\right]\chi}{\partial R}\right)$$

$$+\frac{1}{b}\left(\frac{\partial\, \frac{-b^2F(t)[1-\mu]}{P}(2R-6R^2+4R^3)(2Q-6Q^2+4Q^3)z}{\partial Q}\right)$$

$$Q_x = -\frac{bF(t)}{P}\left[\frac{(-12R+24R)(Q^2-2Q^3+Q^4)}{P^2} + \mu\,(2R-6R^2+4R^3)(2-12Q+12Q^2)\right]\chi$$

$$-\frac{bF(t)}{12Q^2))z)\ P}(([1-\mu](2R-6R^2+4R^3)(2-12Q+$$

$$Q_x = -bF(t)\frac{z}{P}[[\frac{(-12R+24R)(Q^2-2Q^3+Q^4)}{P^2}+\mu(2R-6R^2+4R^3)(2-12Q+12Q^2)]$$

$$+([1-\mu](2R-6R^2+4R^3)(2-12Q+12Q^2))] \tag{4.25}$$

$$Q_x = -bF(t)\,\eta_x \quad (4.26)$$

Where,

$$\eta_x = \frac{z}{P^2}\Big[\Big[\frac{(-12R + 24R)(Q^2 - 2Q^3 + Q^4)}{P^2} + \mu(2R - 6R^2 + 4R^3)(2 - 12Q + 12Q^2)\Big]$$

$$+ ([1-\mu](2R - 6R^2 + 4R^3)(2 - 12Q + 12Q^2))\Big] \quad (4.27)$$

$$Q_y = \frac{1}{b}\Big(\frac{\partial M_{xy}}{P\,\partial R} + \frac{\partial M_y}{\partial Q}\Big)$$

$$Q_y = -\frac{bF(t)}{P^2} z([1-\mu](2 - 12R + 12R^2)(2Q - 6Q^2 + 4Q^3))$$

$$-bF(t)\chi\Big(\mu\frac{(2 - 12R + 12R^2)(2Q - 6Q^2 + 4Q^3)}{P^2} + (R^2 - 2R^3 + R^4)(-12 + 24Q)\Big)$$

$$Q_y = -bF(t)z\Big[\Big(\frac{[1-\mu]}{P^2}(2 - 12R + 12R^2)(2Q - 6Q^2 + 4Q^3)\Big)$$

$$+ \Big(\mu\frac{(2 - 12R + 12R^2)(2Q - 6Q^2 + 4Q^3)}{P^2} + (R^2 - 2R^3 + R^4)(-12 + 24Q)\Big)\Big] \quad (4.28)$$

$$Q_y = -bF(t)\,\eta_y \quad (4.29)$$

Where,

$$\eta_y = z\Big[\Big(\frac{[1-\mu]}{P^2}(2 - 12R + 12R^2)(2Q - 6Q^2 + 4Q^3)\Big)$$

$$+ \Big(\mu\frac{(2 - 12R + 12R^2)(2Q - 6Q^2 + 4Q^3)}{P^2} + (R^2 - 2R^3 + R^4)(-12 + 24Q)\Big)\Big] \quad (4.30)$$

Table 4.10: Shear force coefficients η_x for different plate aspect ratios and forcing Frequencies at the Centre of the plate

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	p_s					
0.1	-0.03262	-0.04077	-0.05436	-0.08155	-0.16309	∞
0.2	-0.06405	-0.08006	-0.10675	-0.16012	-0.32025	∞
0.3	-0.09290	-0.11612	-0.15483	-0.23224	-0.46448	∞
0.4	-0.11748	-0.14686	-0.19581	-0.29371	-0.58742	∞
0.5	-0.13610	-0.17012	-0.22683	-0.34024	-0.68048	∞
0.6	-0.14742	-0.18428	-0.24570	-0.36855	-0.73710	∞
0.7	-0.15108	-0.18886	-0.25181	-0.37771	-0.75542	∞
0.8	-0.14785	-0.18481	-0.24641	-0.36961	-0.73923	∞
0.9	-0.13935	-0.17419	-0.23225	-0.34838	-0.69676	∞
1	-0.12759	-0.15949	-0.21265	-0.31898	-0.63796	∞
1.1	-0.11437	-0.14296	-0.19062	-0.28593	-0.57186	∞
1.2	-0.10104	-0.12630	-0.16841	-0.25261	-0.50522	∞
1.3	-0.08846	-0.11057	-0.14743	-0.22114	-0.44228	∞
1.4	-0.07705	-0.09631	-0.12841	-0.19262	-0.38524	∞
1.5	-0.06697	-0.08372	-0.11162	-0.16743	-0.33487	∞
1.6	-0.05822	-0.07278	-0.09703	-0.14555	-0.29110	∞
1.7	-0.05069	-0.06336	-0.08448	-0.12672	-0.25344	∞
1.8	-0.04424	-0.05530	-0.07373	-0.11060	-0.22120	∞
1.9	-0.03873	-0.04841	-0.06455	-0.09683	-0.19365	∞
2	-0.03402	-0.04253	-0.05671	-0.08506	-0.17012	∞

Shear force coefficients η_x for different plate aspect ratios and forcing frequencies at the centre of the plate R= Q =1/2. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Shear force coefficients η_x also increases until the forcing frequency coincides

with the natural frequency. At this point, Shear force coefficients η_x turn to infinity.

Table 4.11: Shear force coefficients η_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = 1 and Q = ½

$Q_x = -bF(t)\ \eta_x$ at the egde R = 1 and Q = ½)						
α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	η_s					
0.1	-0.06524	-0.08155	-0.10873	-0.16309	-0.32618	∞
0.2	-0.12810	-0.16012	-0.21350	-0.32025	-0.64050	∞
0.3	-0.18579	-0.23224	-0.30965	-0.46448	-0.92895	∞
0.4	-0.23497	-0.29371	-0.39162	-0.58742	-1.17485	∞
0.5	-0.27219	-0.34024	-0.45365	-0.68048	-1.36095	∞
0.6	-0.29484	-0.36855	-0.49140	-0.73710	-1.47421	∞
0.7	-0.30217	-0.37771	-0.50361	-0.75542	-1.51084	∞
0.8	-0.29569	-0.36961	-0.49282	-0.73923	-1.47846	∞
0.9	-0.27871	-0.34838	-0.46451	-0.69676	-1.39353	∞
1	-0.25518	-0.31898	-0.42530	-0.63796	-1.27591	∞
1.1	-0.22874	-0.28593	-0.38124	-0.57186	-1.14371	∞
1.2	-0.20209	-0.25261	-0.33681	-0.50522	-1.01043	∞
1.3	-0.17691	-0.22114	-0.29485	-0.44228	-0.88456	∞
1.4	-0.15410	-0.19262	-0.25683	-0.38524	-0.77048	∞
1.5	-0.13395	-0.16743	-0.22324	-0.33487	-0.66973	∞
1.6	-0.11644	-0.14555	-0.19407	-0.29110	-0.58220	∞
1.7	-0.10138	-0.12672	-0.16896	-0.25344	-0.50688	∞
1.8	-0.08848	-0.11060	-0.14747	-0.22120	-0.44240	∞
1.9	-0.07746	-0.09683	-0.12910	-0.19365	-0.38731	∞
2	-0.06805	-0.08506	-0.11341	-0.17012	-0.34024	∞

Shear force coefficients η_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = 1 and Q = ½. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Shear force coefficients η_x also increases until the forcing frequency coincides with the natural frequency. At this point, Shear force coefficients η_x turn to infinity.

Table 4.12: Shear force coefficients η_y for different plate aspect ratios and forcing frequencies at the center of plate, R = ½ and Q = ½

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	η_y					
0.1	-0.00003	-0.00004	-0.00005	-0.00008	-0.00016	∞
0.2	-0.00051	-0.00064	-0.00085	-0.00128	-0.00256	∞
0.3	-0.00251	-0.00314	-0.00418	-0.00627	-0.01254	∞
0.4	-0.00752	-0.00940	-0.01253	-0.01880	-0.03760	∞
0.5	-0.01701	-0.02126	-0.02835	-0.04253	-0.08506	∞
0.6	-0.03184	-0.03980	-0.05307	-0.07961	-0.15921	∞
0.7	-0.05182	-0.06478	-0.08637	-0.12955	-0.25911	∞
0.8	-0.07570	-0.09462	-0.12616	-0.18924	-0.37849	∞
0.9	-0.10159	-0.12699	-0.16931	-0.25397	-0.50794	∞
1	-0.12759	-0.15949	-0.21265	-0.31898	-0.63796	∞
1.1	-0.15223	-0.19028	-0.25371	-0.38057	-0.76114	∞
1.2	-0.17460	-0.21825	-0.29100	-0.43651	-0.87301	∞
1.3	-0.19434	-0.24292	-0.32390	-0.48584	-0.97169	∞
1.4	-0.21142	-0.26427	-0.35236	-0.52855	-1.05709	∞
1.5	-0.22604	-0.28254	-0.37673	-0.56509	-1.13018	∞
1.6	-0.23847	-0.29809	-0.39745	-0.59618	-1.19236	∞
1.7	-0.24903	-0.31129	-0.41505	-0.62258	-1.24516	∞
1.8	-0.25801	-0.32251	-0.43001	-0.64502	-1.29003	∞
1.9	-0.26565	-0.33207	-0.44275	-0.66413	-1.32826	∞
2	-0.27219	-0.34024	-0.45365	-0.68048	-1.36095	∞

Shear force coefficients η_y for different plate aspect ratios and forcing frequencies at the center of plate, R = ½ and Q = ½. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Shear force coefficients η_y also increases until the forcing frequency coincides with the natural frequency. At this point, Shear force coefficients η_y turn to infinity.

Table 4.13: Shear force coefficients η_y for different plate aspect ratios and forcing frequencies at plate edge (R = ½ Q = 1)

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	η_y					
0.1	-0.00007	-0.00008	-0.00011	-0.00016	-0.00033	∞
0.2	-0.00102	-0.00128	-0.00171	-0.00256	-0.00512	∞
0.3	-0.00502	-0.00627	-0.00836	-0.01254	-0.02508	∞
0.4	-0.01504	-0.01880	-0.02506	-0.03760	-0.07519	∞
0.5	-0.03402	-0.04253	-0.05671	-0.08506	-0.17012	∞
0.6	-0.06369	-0.07961	-0.10614	-0.15921	-0.31843	∞
0.7	-0.10364	-0.12955	-0.17274	-0.25911	-0.51822	∞
0.8	-0.15139	-0.18924	-0.25232	-0.37849	-0.75697	∞
0.9	-0.20318	-0.25397	-0.33863	-0.50794	-1.01588	∞
1	-0.25518	-0.31898	-0.42530	-0.63796	-1.27591	∞
1.1	-0.30446	-0.38057	-0.50743	-0.76114	-1.52228	∞
1.2	-0.34920	-0.43651	-0.58201	-0.87301	-1.74602	∞
1.3	-0.38868	--0.48584	-0.64779	-0.97169	-1.94338	∞
1.4	-0.42284	-0.52855	-0.70473	-1.05709	-2.11418	∞
1.5	-0.45207	-0.56509	-0.75345	-1.13018	-2.26035	∞
1.6	-0.47694	-0.59618	-0.79490	-1.19236	-2.38471	∞
1.7	-0.49806	-0.62258	-0.83011	-1.24516	-2.49032	∞
1.8	-0.51601	-0.64502	-0.86002	-1.29003	-2.58006	∞
1.9	-0.53131	-0.66413	-0.88551	-1.32826	-2.65653	∞
2	-0.54438	-0.68048	-0.90730	-1.36095	-2.72191	∞

Shear force coefficients η_y for different plate aspect ratios and forcing frequencies at plate edge (R = ½ Q = 1). As the forcing frequency is gradually increased i.e. $8^2 = ,$ $\alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Shear force coefficients η_y also increases until the forcing frequency coincides with the natural frequency. At this point, Shear force coefficients η_y turn to infinity.

4.3 Deflections, Shear Force and Moments for Various Values of Coefficient A, With Respect To Different Aspect Ratio of Thin Rectangular Plate Clamped On All Edges Subjected To Uniformly Varying Load

A section of a thin rectangular plate whose all edges are clamped is shown in Figure 4.2. The plate is subjected to uniformly varying load causing the forced excitation of the plate.

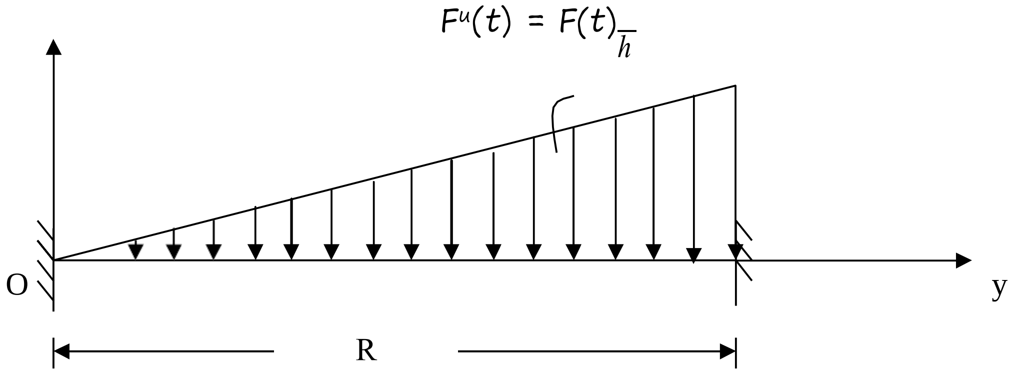


Figure 4.2 Section through of rectangular plate whose all edges are clamped subjected to uniformly varying load causing the forced excitation of the plate.

Recall, from Equation 3.131

$$A = \frac{b^4 F(t)}{D} \chi$$

Where the values of χ at different aspect ratios and forcing frequencies are tabulated in Table 3.4.

4.3.1 Deflection

From Equation 3.82:

$$W(x, y) = A(2R^2 - 3R^3 + R^5)(Q^2 - 2Q^3 + Q^4)$$

Where,

At the mid-span, R = Q = ½

$$W_{Nas} = A\left(2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5\right)\left(\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4\right)$$

$$W_{Nas} = A\left(\frac{5}{32}\right)\frac{1}{16} = \frac{5A}{(16)} \quad (4.31)$$

512

$$W_{Nas} = \frac{5}{51} \frac{b^4 F(t)}{D} \chi$$

2

$$W_{Nas} = \left(\frac{5\chi}{512}\frac{b^4F(t)}{D}\right)$$

$$W_{Nas} = K\frac{b^4F(t)}{D}$$

Where,

$$K = \frac{5\chi}{512} \tag{4.32}$$

Table 4.14 shows the $K - values$ at different aspect ratios and forcing frequencies

Table 4.14: $K - values$ at different aspect ratios and forcing frequencies for Maximum deflection at the centre of the plate

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	K-Values for different, α					
0.1	3.30409E-07	4.13E-07	5.51E-07	8.26E-07	1.65E-06	∞
0.2	5.1918E-06	6.49E-06	8.65E-06	1.3E-05	2.6E-05	∞
0.3	2.54271E-05	3.18E-05	4.24E-05	6.36E-05	0.000127	∞
0.4	7.62915E-05	9.54E-05	0.000127	0.000191	0.000381	∞
0.5	0.000172825	0.000216	0.000288	0.000432	0.000864	∞
0.6	0.00032402	0.000405	0.00054	0.00081	0.00162	∞
0.7	0.000528336	0.00066	0.000881	0.001321	0.002642	∞
0.8	0.000773371	0.000967	0.001289	0.001933	0.003867	∞
0.9	0.001040091	0.0013	0.001733	0.0026	0.0052	∞
1.0	0.001308935	0.001636	0.002182	0.003272	0.006545	∞
1.1	0.001564489	0.001956	0.002607	0.003911	0.007822	∞
1.2	0.001797241	0.002247	0.002995	0.004493	0.008986	∞
1.3	0.002003028	0.002504	0.003338	0.005008	0.010015	∞
1.4	0.002181475	0.002727	0.003636	0.005454	0.010907	∞
1.5	0.002334413	0.002918	0.003891	0.005836	0.011672	∞
1.6	0.002464688	0.003081	0.004108	0.006162	0.012323	∞
1.7	0.002575419	0.003219	0.004292	0.006439	0.012877	∞
1.8	0.002669585	0.003337	0.004449	0.006674	0.013348	∞
1.9	0.002749854	0.003437	0.004583	0.006875	0.013749	∞
2.0	0.002818515	0.003523	0.004698	0.007046	0.014093	∞

At maximum deflection and slope at the edge of the plate equal to zero (R=Q=0) but at the mid-span R = Q = ½ .As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$and approaches the fundamental natural frequency, the deflection also increases until the forcing frequency coincides with the natural frequency.

At this point, the deflection, turn to infinity.

4.3.2 Moment

From Equations 4.7, 4.8 and 4.9;

$$M_x = \frac{-D}{b^2}\left[\frac{\partial^2 w}{P^2\,\partial R^2} + \mu\frac{\partial^2 w}{\partial Q^2}\right]$$

$$M_y = \frac{-D}{b^2}\left[\mu\frac{\partial^2 w}{P^2\,\partial R^2} + \frac{\partial^2 w}{\partial Q^2}\right]$$

$$M_{xy} = M_{yx} = \frac{-D[1-\mu]}{Pb^2}\frac{\partial^2 w}{\partial R dQ}$$

Substituting the shape function for a plate with all edges clamped subjected to varying uniformly load as given in Equation 3.82:

$$M_x = \frac{-AD}{b^2}\left[\frac{\partial^2}{P^2\,\partial R^2} + \mu\frac{\partial^2}{\partial Q^2}\right] - 3R^3 + R^5)(Q^2 - 2Q^3 + Q^4)$$

$$M_x = \frac{-AD}{b^2}\left[\frac{(4-18R+20R^3)(Q^2-2Q^3+Q^4)}{P^2} + \mu(2R^2-3R^3+R^5)(2-12Q+12Q^2)\right] \quad (4.33)$$

Similarly,

$$M_y = \frac{-AD}{b^2}\left[\mu\frac{(4-18R+20R^3)(Q^2-2Q^3+Q^4)}{P^2} + (2R^2-3R^3+R^5)(2-12Q+12Q^2)\right] \quad (4.34)$$

$$M_{xy} = M_{yx} = \frac{-DA[1-\mu]}{Pb^2}\frac{\partial^2((2R^2-3R^3+R^5)(Q^2-2Q^3+Q^4))}{\partial R dQ}$$

$$M_{xy} = M_{yx} = \frac{-DA[1-\mu]}{Pb^2}(4R-9R^2+5R^4)(2Q-6Q^2+4Q^3)$$

Where,

$$A = \frac{0.002777778F(t)}{\frac{D}{b^4}[(\frac{0.0081633}{P^4} + \frac{0.004595616}{P^2} + 0.00796537) - (0.0000158045\alpha)]}$$

$$A = \frac{b^4F(t)}{D}3$$

$$M_{xy} = M_{yx} = \frac{[1-\mu]}{)P}(4R - 9R^2 + 5R^4)(2Q - 6Q^2 + 4Q^3)b^2F(t)\chi \qquad (4.35$$

$$\chi = \frac{0.002777778}{\left[\left(\frac{0.0081633}{P^4} + \frac{0.004595616}{P^2} + 0.00796537\right) - (0.0000158045\alpha)\right]}$$

for $\alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$

$$M_x = -b^2 F(t)\left(\frac{(4 - 18R + 20R^3)(Q^2 - 2Q^3 + Q^4)}{P^2} + \mu\,(2R^2 - 3R^3 + R^5)(2 - 12Q + 12Q^2)\right)$$

$$\times \left[\frac{0.002777778}{\left[\left(\frac{0.0081633}{P^4} + \frac{0.004595616}{P^2} + 0.00796537\right) - (0.0000158045\alpha)\right]}\right]$$

$$M_x = -b^2 F(t)\left(\frac{(4 - 18R + 20R^3)(Q^2 - 2Q^3 + Q^4)}{P^2} + \mu\,(2R^2 - 3R^3 + R^5)(2 - 12Q + 12Q^2)\right)\chi$$

$$M_x = -b^2 F(t)\,\beta_x \qquad (4.36)$$

$$\beta_x = \left(\frac{(4 - 18R + 20R^3)(Q^2 - 2Q^3 + Q^4)}{P^2} + \mu\,(2R^2 - 3R^3 + R^5)(2 - 12Q + 12Q^2)\right)$$

$$\times \frac{0.002777778}{\left[\left(\frac{0.0081633}{P^4} + \frac{0.004595616}{P^2} + 0.00796537\right) - (0.0000158045\alpha)\right]}$$

$$\beta_x = \left(\frac{(4 - 18R + 20R^3)(Q^2 - 2Q^3 + Q^4)}{P^2} + \mu\,(2R^2 - 3R^3 + R^5)\right)\chi$$

$$\chi = \frac{0.002777778}{\left[\left(\frac{0.0081633}{P^4} + \frac{0.004595616}{P^2} + 0.00796537\right) - (0.0000158045\alpha)\right]} \qquad (4.37)$$

Similarly, $M_y + = -b^2 F(t)\left(\mu\,\frac{(4-18R+20R^3)(Q^2-2Q^3+Q^4)}{P^2} + (2R^2 - 3R^3 + R^5)(2 - 12Q\right.$

$12Q2\times$ $0.0027777780.0081633P4+$

$0.004595616P2+0.00796537-0.0000158045\alpha$

$$M_y = -b^2 F(t) \left(\mu \frac{(4-18R+20R^3)(Q^2-2Q^3+Q^4)}{P^2} + (2R^2-3R^3+R^5)(2-12Q+12Q^2)\right)\chi$$

$$M_y = -b^2 F(t)\beta_y \qquad (4.38)$$

$$\beta_y = \left(\mu \frac{(4-18R+20R^3)(Q^2-2Q^3+Q^4)}{P^2} + (2R^2-3R^3+R^5)(2-12Q+12Q^2)\right)\chi \qquad (4.39)$$

$$M_{xy} = M_{yx} = \frac{-b^2F(t)[1-\mu]}{P}(4R-9R^2+5R^4)(2Q-6Q^2+4Q^3)$$

$$\times \left[\frac{0.002777778}{\left[\left(\frac{0.0081633}{P^4}+\frac{0.0045956 16}{P^2}+0.00796537\right)-(0.000015804 5\alpha)\right]}\right] \qquad (4.40)$$

$$M_{xy} = M_{yx} = \frac{-b^2F(t)[1-\mu]}{P}(4R-9R^2+5R^4)(2Q-6Q^2+4Q^3)\chi$$

$$M_{xy} = M_{yx} = -b^2F(t)\beta_{xy} = -b^2F(t)\beta_{yx} \qquad (4.41)$$

$$\beta_{xy} = \beta_{yx} = [\frac{[1-\mu]}{)P}(4R - 9R^2 + 5R^4)(2Q - 6Q^2 + 4Q^3)]\chi \quad (4.42$$

The values of moment coefficients β_x *and* β_y for different plate aspect ratios and forcing frequencies are tabulated in Tables 4.15, through 4.19.

Table 4.15: Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the centre of the plate

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	p_s					
0.1	-0.00053	-0.00066	-0.00088	-0.00133	-0.00265	∞
0.2	-0.0021	-0.00263	-0.0035	-0.00525	-0.01051	∞
0.3	-0.00464	-0.0058	-0.00774	-0.01161	-0.02321	∞
0.4	-0.008	-0.00999	-0.01333	-0.01999	-0.03998	∞
0.5	-0.01189	-0.01486	-0.01982	-0.02973	-0.05945	∞
0.6	-0.01596	-0.01995	-0.02659	-0.03989	-0.07978	∞
0.7	-0.01979	-0.02473	-0.03298	-0.04947	-0.09894	∞
0.8	-0.02305	-0.02881	-0.03841	-0.05762	-0.11523	∞
0.9	-0.02554	-0.03192	-0.04256	-0.06384	-0.12769	∞
1	-0.02723	-0.03403	-0.04538	-0.06806	-0.13613	∞
1.1	-0.0282	-0.03525	-0.047	-0.07049	-0.14099	∞
1.2	-0.0286	-0.03575	-0.04766	-0.07149	-0.14298	∞
1.3	-0.02858	-0.03572	-0.04763	-0.07145	-0.14289	∞
1.4	-0.02828	-0.03535	-0.04713	-0.0707	-0.1414	∞
1.5	-0.02781	-0.03476	-0.04634	-0.06951	-0.13903	∞
1.6	-0.02723	-0.03404	-0.04539	-0.06809	-0.13617	∞
1.7	-0.02662	-0.03328	-0.04437	-0.06655	-0.1331	∞
1.8	-0.026	-0.0325	-0.04333	-0.06499	-0.12999	∞
1.9	-0.02539	-0.03173	-0.04231	-0.06347	-0.12694	∞
2	-0.0248	-0.031	-0.04134	-0.06201	-0.12401	∞

Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the centre of the plate, R = Q = ½. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Moment coefficients β_X also increases until the forcing frequency coincides with the natural frequency. At this point, Moment coefficients β_X turn to infinity.

Table 4.16: Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 0

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_s					
0.1	-3.17E-06	-3.96E-06	-5.29E-06	-7.93E-06	-1.59E-05	∞
0.2	-.98E-05	-6.23E-05	-8.31E-05	-0.000125	-0.000249	∞
0.3	-0.000244	-0.000305	-0.000407	-0.00061	-0.001221	∞
0.4	-0.000732	-0.000915	-0.001221	-0.001831	-0.003662	∞
0.5	-0.001659	-0.002074	-0.002765	-0.004148	-0.008296	∞
0.6	-0.003111	-0.003888	-0.005184	-0.007776	-0.015553	∞
0.7	-0.005072	-0.00634	-0.008453	-0.01268	-0.02536	∞
0.8	-0.007424	-0.00928	-0.012374	-0.018561	-0.037122	∞
0.9	-0.009985	-0.012481	-0.016641	-0.024962	-0.049924	∞
1	-0.012566	-0.015707	-0.020943	-0.031414	-0.062829	∞
1.1	-0.015019	-0.018774	-0.025032	-0.037548	-0.075095	∞
1.2	-0.017254	-0.021567	-0.028756	-0.043134	-0.086268	∞
1.3	-0.019229	-0.024036	-0.032048	-0.048073	-0.096145	∞
1.4	-0.020942	-0.026178	-0.034904	-0.052355	-0.104711	∞
1.5	-0.02241	-0.028013	-0.037351	-0.056026	-0.112052	∞
1.6	-0.023661	-0.029576	-0.039435	-0.059153	-0.118305	∞
1.7	-0.024724	-0.030905	-0.041207	-0.06181	-0.12362	∞
1.8	-0.025628	-0.032035	-0.042713	-0.06407	-0.12814	∞
1.9	-0.026399	-0.032998	-0.043998	-0.065996	-0.131993	∞
2	-0.027058	-0.033822	-0.045096	-0.067644	-0.135289	∞

Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 0. As the forcing frequency is gradually increased i.e. $8^2 =$ $, \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural

frequency, Moment coefficients β_X also increases until the forcing frequency coincides with the natural frequency. At this point, Moment coefficients $\beta_X$turn to infinity.

Table 4.17: Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = 0 and Q = ½

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_s					
0.1	-0.000846	-0.001057	-0.00141	-0.002115	-0.004229	∞
0.2	-0.003323	-0.004153	-0.005538	-0.008307	-0.016614	∞
0.3	-0.007233	-0.009041	-0.012054	-0.018082	-0.036163	∞
0.4	-0.012207	-0.015258	-0.020344	-0.030517	-0.061033	∞
0.5	-0.017697	-0.022122	-0.029495	-0.044243	-0.088486	∞
0.6	-0.023041	-0.028802	-0.038402	-0.057604	-0.115207	∞
0.7	-0.027603	-0.034504	-0.046005	-0.069007	-0.138014	∞
0.8	-0.030935	-0.038669	-0.051558	-0.077337	-0.154674	∞
0.9	-0.032872	-0.04109	-0.054787	-0.08218	-0.16436	∞
1	-0.033509	-0.041886	-0.055848	-0.083772	-0.167544	∞
1.1	-0.0331	-0.041375	-0.055167	-0.08275	-0.1655	∞
1.2	-0.031951	-0.039939	-0.053252	-0.079877	-0.159755	∞
1.3	-0.030342	-0.037927	-0.05057	-0.075854	-0.151709	∞
1.4	-0.028493	-0.035616	-0.047488	-0.071232	-0.142464	∞
1.5	-0.02656	-0.033201	-0.044267	-0.066401	-0.132802	∞
1.6	-0.024647	-0.030809	-0.041078	-0.061617	-0.123234	∞
1.7	-0.022813	-0.028517	-0.038022	-0.057033	-0.114067	∞
1.8	-0.021093	-0.026366	-0.035155	-0.052733	-0.105465	∞
1.9	-0.0195	-0.024375	-0.032501	-0.048751	-0.097502	∞
2	-0.018038	-0.022548	-0.030064	-0.045096	-0.090192	∞

Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = 0 and Q = ½. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural

frequency, Moment coefficients β_X also increases until the forcing frequency coincides with the natural frequency. At this point, Moment coefficients $\beta_X$turn to infinity.

Table 4.17a: Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = 1 and Q = ½

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_s					
0.1	-0.001269	-0.001586	-0.002115	-0.003172	-0.006344	∞
0.2	-0.004984	-0.00623	-0.008307	-0.01246	-0.024921	∞
0.3	-0.010849	-0.013561	-0.018082	-0.027122	-0.054245	∞
0.4	-0.01831	-0.022887	-0.030517	-0.045775	-0.09155	∞
0.5	-0.026546	-0.033182	-0.044243	-0.066365	-0.13273	∞
0.6	-0.034562	-0.043203	-0.057604	-0.086405	-0.172811	∞
0.7	-0.041404	-0.051755	-0.069007	-0.103511	-0.207021	∞
0.8	-0.046402	-0.058003	-0.077337	-0.116006	-0.232011	∞
0.9	-0.049308	-0.061635	-0.08218	-0.12327	-0.24654	∞
1	-0.050263	-0.062829	-0.083772	-0.125658	-0.251315	∞
1.1	-0.04965	-0.062062	-0.08275	-0.124125	-0.248249	∞
1.2	-0.047926	-0.059908	-0.079877	-0.119816	-0.239632	∞
1.3	-0.045513	-0.056891	-0.075854	-0.113781	-0.227563	∞
1.4	-0.042739	-0.053424	-0.071232	-0.106848	-0.213695	∞
1.5	-0.039841	-0.049801	-0.066401	-0.099602	-0.199203	∞
1.6	-0.03697	-0.046213	-0.061617	-0.092426	-0.184852	∞
1.7	-0.03422	-0.042775	-0.057033	-0.08555	-0.1711	∞
1.8	-0.03164	-0.039549	-0.052733	-0.079099	-0.158198	∞
1.9	-0.029251	-0.036563	-0.048751	-0.073126	-0.146253	∞
2	-0.027058	-0.033822	-0.045096	-0.067644	-0.135289	∞

Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = 1 and Q = ½. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural

frequency, Moment coefficients β_X also increases until the forcing frequency coincides with the natural frequency. At this point, Moment coefficients $\beta_X$turn to infinity.

Table 4.17b: Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 1

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_s					
0.1	-3.17E-06	-3.96E-06	5.29E-06	-7.93E-06	-1.59E-05	∞
0.2	-4.98E-05	-6.23E-05	8.31E-05	-0.000125	-0.000249	∞
0.3	-0.000244	-0.000305	0.000407	-0.00061	-0.001221	∞
0.4	-0.000732	-0.000915	0.001221	-0.001831	-0.003662	∞
0.5	-0.001659	-0.002074	-0.002765	-0.004148	-0.008296	∞
0.6	-0.003111	-0.003888	-0.005184	-0.007776	-0.015553	∞
0.7	-0.005072	-0.00634	-0.008453	-0.01268	-0.02536	∞
0.8	-0.007424	-0.00928	-0.012374	-0.018561	-0.037122	∞
0.9	-0.009985	-0.012481	-0.016641	-0.024962	-0.049924	∞
1	-0.012566	-0.015707	-0.020943	-0.031414	-0.062829	∞
1.1	-0.015019	-0.018774	-0.025032	-0.037548	-0.075095	∞
1.2	-0.017254	-0.021567	-0.028756	-0.043134	-0.086268	∞
1.3	-0.019229	-0.024036	-0.032048	-0.048073	-0.096145	∞
1.4	-0.020942	-0.026178	-0.034904	-0.052355	-0.104711	∞
1.5	-0.02241	-0.028013	-0.037351	-0.056026	-0.112052	∞
1.6	-0.023661	-0.029576	-0.039435	-0.059153	-0.118305	∞
1.7	-0.024724	-0.030905	-0.041207	-0.06181	-0.12362	∞
1.8	-0.025628	-0.032035	-0.042713	-0.06407	-0.12814	∞
1.9	-0.026399	-0.032998	-0.043998	-0.065996	-0.131993	∞
2	-0.027058	-0.033822	-0.045096	-0.067644	-0.135289	∞

Moment coefficients β_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 1. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural

frequency, Moment coefficients β_X also increases until the forcing frequency coincides with the natural frequency. At this point, Moment coefficients $\beta_X$turn to infinity.

Table 4.18: Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the centre of the plate

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_y					
0.1	-0.00016	-0.0002	-0.00027	-0.00041	-0.00082	∞
0.2	-0.00071	-0.00088	-0.00118	-0.00177	-0.00353	∞
0.3	-0.00176	-0.0022	-0.00294	-0.00441	-0.00881	∞
0.4	-0.00351	-0.00439	-0.00585	-0.00877	-0.01755	∞
0.5	-0.00608	-0.0076	-0.01014	-0.01521	-0.03042	∞
0.6	-0.0095	-0.01188	-0.01584	-0.02376	-0.04752	∞
0.7	-0.01363	-0.01704	-0.02271	-0.03407	-0.06814	∞
0.8	-0.01817	-0.02272	-0.03029	-0.04544	-0.09087	∞
0.9	-0.0228	-0.02851	-0.03801	-0.05701	-0.11402	∞
1	-0.02723	-0.03403	-0.04538	-0.06806	-0.13613	∞
1.1	-0.03124	-0.03905	-0.05206	-0.0781	-0.15619	∞
1.2	-0.03475	-0.04343	-0.05791	-0.08687	-0.17373	∞
1.3	-0.03774	-0.04717	-0.0629	-0.09434	-0.18869	∞
1.4	-0.04025	-0.05031	-0.06708	-0.10061	-0.20123	∞
1.5	-0.04233	-0.05291	-0.07055	-0.10583	-0.21165	∞
1.6	-0.04406	-0.05507	-0.07343	-0.11014	-0.22028	∞
1.7	-0.04548	-0.05686	-0.07581	-0.11371	-0.22742	∞
1.8	-0.04667	-0.05834	-0.07778	-0.11667	-0.23334	∞
1.9	-0.04765	-0.05957	-0.07942	-0.11913	-0.23827	∞
2	-0.04848	-0.0606	-0.0808	-0.1212	-0.24239	∞

Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the centre of the plate, R = Q = ½. As the forcing frequency is gradually increased i.e. $8^2 = ,\alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Moment coefficients β_y also increases until the forcing frequency coincides with the natural frequency. At this point, Moment coefficients β_y turn to infinity.

Table 4.19: Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 1

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_y					
0.1	-1.06E-05	-1.32E-05	-1.76E-05	-2.64E-05	-5.29E-05	∞
0.2	-0.000166	-0.000208	-0.000277	-0.000415	-0.000831	∞
0.3	-0.000814	-0.001017	-0.001356	-0.002034	-0.004068	∞
0.4	-0.002441	-0.003052	-0.004069	-0.006103	-0.012207	∞
0.5	-0.00553	-0.006913	-0.009217	-0.013826	-0.027652	∞
0.6	-0.010369	-0.012961	-0.017281	-0.025922	-0.051843	∞
0.7	-0.016907	-0.021133	-0.028178	-0.042267	-0.084534	∞
0.8	-0.024748	-0.030935	-0.041246	-0.06187	-0.123739	∞
0.9	-0.033283	-0.041604	-0.055472	-0.083207	-0.166415	∞
1	-0.041886	-0.052357	-0.06981	-0.104715	-0.20943	∞
1.1	-0.050064	-0.06258	-0.083439	-0.125159	-0.250318	∞
1.2	-0.057512	-0.07189	-0.095853	-0.143779	-0.287559	∞
1.3	-0.064097	-0.080121	-0.106828	-0.160242	-0.320484	∞
1.4	-0.069807	-0.087259	-0.116345	-0.174518	-0.349036	∞
1.5	-0.074701	-0.093377	-0.124502	-0.186753	-0.373506	∞
1.6	-0.07887	-0.098588	-0.13145	-0.197175	-0.39435	∞
1.7	-0.082413	-0.103017	-0.137356	-0.206033	-0.412067	∞
1.8	-0.085427	-0.106783	-0.142378	-0.213567	-0.427134	∞
1.9	-0.087995	-0.109994	-0.146659	-0.219988	-0.439977	∞
2	-0.090192	-0.112741	-0.150321	-0.225481	-0.450962	∞

Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 1. As the forcing frequency is gradually increased i.e. $8^2 = ,\alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Moment coefficients β_y also increases until the forcing frequency coincides with the

natural frequency. At this point, Moment coefficients β_y turn to infinity.

Table 4.20: Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = 1 and Q = ½

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_y					
0.1	-0.000381	-0.000476	-0.000634	-0.000952	-0.001903	∞
0.2	-0.001495	-0.001869	-0.002492	-0.003738	-0.007476	∞
0.3	-0.003255	-0.004068	-0.005424	-0.008137	-0.016273	∞
0.4	-0.005493	-0.006866	-0.009155	-0.013732	-0.027465	∞
0.5	-0.007964	-0.009955	-0.013273	-0.019909	-0.039819	∞
0.6	-0.010369	-0.012961	-0.017281	-0.025922	-0.051843	∞
0.7	-0.012421	-0.015527	-0.020702	-0.031053	-0.062106	∞
0.8	-0.013921	-0.017401	-0.023201	-0.034802	-0.069603	∞
0.9	-0.014792	-0.018491	-0.024654	-0.036981	-0.073962	∞
1	-0.015079	-0.018849	-0.025132	-0.037697	-0.075395	∞
1.1	-0.014895	-0.018619	-0.024825	-0.037237	-0.074475	∞
1.2	-0.014378	-0.017972	-0.023963	-0.035945	-0.07189	∞
1.3	-0.013654	-0.017067	-0.022756	-0.034134	-0.068269	∞
1.4	-0.012822	-0.016027	-0.02137	-0.032054	-0.064109	∞
1.5	-0.011952	-0.01494	-0.01992	-0.02988	-0.059761	∞
1.6	-0.011091	-0.013864	-0.018485	-0.027728	-0.055455	∞
1.7	-0.010266	-0.012833	-0.01711	-0.025665	-0.05133	∞
1.8	-0.009492	-0.011865	-0.01582	-0.02373	-0.047459	∞
1.9	-0.008775	-0.010969	-0.014625	-0.021938	-0.043876	∞
2	-0.008117	-0.010147	-0.013529	-0.020293	-0.040587	∞

Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = 1 and Q = ½. As the forcing frequency is gradually increased i.e. $8^2 = ,\alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Moment coefficients β_y also increases until the forcing frequency coincides with the

natural frequency. At this point, Moment coefficients β_y turn to infinity.

Table 4.20a: Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 0

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_y					
0.1	-1.06E-05	-1.32E-05	-1.76E-05	-2.64E-05	-5.29E-05	∞
0.2	-0.000166	-0.000208	-0.000277	-0.000415	-0.000831	∞
0.3	-0.000814	-0.001017	-0.001356	-0.002034	-0.004068	∞
0.4	-0.002441	-0.003052	-0.004069	-0.006103	-0.012207	∞
0.5	-0.00553	-0.006913	-0.009217	-0.013826	-0.027652	∞
0.6	-0.010369	-0.012961	-0.017281	-0.025922	-0.051843	∞
0.7	-0.016907	-0.021133	-0.028178	-0.042267	-0.084534	∞
0.8	-0.024748	-0.030935	-0.041246	-0.06187	-0.123739	∞
0.9	-0.033283	-0.041604	-0.055472	-0.083207	-0.166415	∞
1	-0.041886	-0.052357	-0.06981	-0.104715	-0.20943	∞
1.1	-0.050064	-0.06258	-0.083439	-0.125159	-0.250318	∞
1.2	-0.057512	-0.07189	-0.095853	-0.143779	-0.287559	∞
1.3	-0.064097	-0.080121	-0.106828	-0.160242	-0.320484	∞
1.4	-0.069807	-0.087259	-0.116345	-0.174518	-0.349036	∞
1.5	-0.074701	-0.093377	-0.124502	-0.186753	-0.373506	∞
1.6	-0.07887	-0.098588	-0.13145	-0.197175	-0.39435	∞
1.7	-0.082413	-0.103017	-0.137356	-0.206033	-0.412067	∞
1.8	-0.085427	-0.106783	-0.142378	-0.213567	-0.427134	∞
1.9	-0.087995	-0.109994	-0.146659	-0.219988	-0.439977	∞
2	-0.090192	-0.112741	-0.150321	-0.225481	-0.450962	∞

Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 0. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Moment coefficients β_y also increases until the forcing frequency coincides with the

natural frequency. At this point, Moment coefficients β_y turn to infinity.

Table 4.20b: Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = 0 and Q = ½

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	β_y					
0.1	-0.000254	-0.000317	-0.000423	-0.000634	-0.001269	∞
0.2	-0.000997	-0.001246	-0.001661	-0.002492	-0.004984	∞
0.3	-0.00217	-0.002712	-0.003616	-0.005424	-0.010849	∞
0.4	-0.003662	-0.004577	-0.006103	-0.009155	-0.01831	∞
0.5	-0.005309	-0.006636	-0.008849	-0.013273	-0.026546	∞
0.6	-0.006912	-0.008641	-0.011521	-0.017281	-0.034562	∞
0.7	-0.008281	-0.010351	-0.013801	-0.020702	-0.041404	∞
0.8	-0.00928	-0.011601	-0.015467	-0.023201	-0.046402	∞
0.9	-0.009862	-0.012327	-0.016436	-0.024654	-0.049308	∞
1	-0.010053	-0.012566	-0.016754	-0.025132	-0.050263	∞
1.1	-0.00993	-0.012412	-0.01655	-0.024825	-0.04965	∞
1.2	-0.009585	-0.011982	-0.015975	-0.023963	-0.047926	∞
1.3	-0.009103	-0.011378	-0.015171	-0.022756	-0.045513	∞
1.4	-0.008548	-0.010685	-0.014246	-0.02137	-0.042739	∞
1.5	-0.007968	-0.00996	-0.01328	-0.01992	-0.039841	∞
1.6	-0.007394	-0.009243	-0.012323	-0.018485	-0.03697	∞
1.7	-0.006844	-0.008555	-0.011407	-0.01711	-0.03422	∞
1.8	-0.006328	-0.00791	-0.010547	-0.01582	-0.03164	∞
1.9	-0.00585	-0.007313	-0.00975	-0.014625	-0.029251	∞
2	-0.005412	-0.006764	-0.009019	-0.013529	-0.027058	∞

Moment coefficients β_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = 0 and Q = ½. As the forcing frequency is gradually increased i.e. $8^2 = ,\alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Moment coefficients β_y also increases until the forcing frequency coincides with the

natural frequency. At this point, Moment coefficients β_y turn to infinity.

4.3.3 Shear forces

Shear forces acting on the plate are given by the expressions 4.23 and 4.24:

$$Q_x = \frac{1}{b}\left(\frac{\partial M_x}{P\,\partial R} + \frac{\partial M_{xy}}{\partial Q}\right)$$

$$Q_y = \frac{1}{b}\left(\frac{\partial M_{xy}}{P\,\partial R} + \frac{\partial M_y}{\partial Q}\right)$$

Substituting the shape function for a plate with all edges clamped subjected to varying uniformly load as given in Equation 3.98:

$$Q_x = \frac{1}{bP}\left(\frac{\partial\left(-b^2F(t)\left(\frac{(4-18R+20R^3)(Q^2-2Q^3+Q^4)}{P^2} + \mu(2R^2-3R^3+R^5)(2-12Q+12Q^2)\right)\chi\right)}{\partial R}\right) + \frac{1}{b}\left(\frac{\partial\left(\frac{-b^2F(t)[1-\mu]}{P}(4R-9R^2+5R^4)(2Q-6Q^2+4Q^3)\chi\right)}{\partial Q}\right)$$

$$Q_x = \frac{-b}{P}F(t)\chi\left(\frac{\partial\left(\left(\frac{(4-18R+20R^3)(Q^2-2Q^3+Q^4)}{P^2} + \mu(2R^2-3R^3+R^5)(2-12Q+12Q^2)\right)\right)}{\partial R}\right) - b\frac{[1-\mu]}{P}F(t)\chi\left(\frac{\partial((4R-9R^2+5R^4)(2Q-6Q^2+4Q^3))}{\partial Q}\right)$$

$$Q_x = \frac{-b}{P}F(t)\chi\left(\frac{(-18+60R^2)(Q^2-2Q^3+Q^4)}{P^2} + \mu(4R-9R^2+5R^4)(2-12Q+12Q^2)\right)$$

$$-\frac{[1-\mu]}{12Q^2))\ P} b F(t)\chi((4R-9R^2+5R^4)(2-12Q+$$

$$Q_x = \frac{\frac{-b}{P} F(t)\chi\,[((-18+60R^2)(Q^2-2Q^3+}{Q^4)\ P2}$$

$$+\ \mu\ (4R - 9R^2 + 5R^4)(2 - 12Q + 12Q^2))$$

$$+([1-\mu]((4R-9R^2+5R^4)(2-12Q+12Q^2)))] \qquad (4.43)$$

$$Q_x = -bF(t)\eta_x \qquad (4.44)$$

Where,

$$\eta_x = \frac{\chi}{P}\left[\left(\frac{(-18+60R^2)(Q^2-2Q^3+Q^4)}{P^2} + \mu(4R - 9R^2 + 5R^4)(2-12Q+12Q^2)\right)\right.$$

$$+([-\mu]((4R-9R^2+5R^4)(2-12Q+12Q^2)))] \qquad (4.45)$$

$$Q_y = \frac{1}{bP}\left(\frac{\partial\left(\frac{-b^2F(t)[1-\mu]}{P}(4R-9R^2+5R^4)(2Q-6Q^2+4Q^3)\chi\right)}{\partial R}\right)$$

$$+\frac{1}{b}\left(\frac{\partial\left(-b^2F(t)\left(\mu\frac{(4-18R+20R^3)(Q^2-2Q^3+Q^4)}{P^2}+(2R^2-3R^3+R^5)(2-12Q+12Q^2)\right)\chi\right)}{\partial Q}\right)$$

$$Q_y = -b\left(\frac{[1-\mu]}{P^2}F(t)\chi\frac{\partial((4R-9R^2+5R^4)(2Q-6Q^2+4Q^3))}{\partial R}\right)$$

$$-bF(t)\chi\left(\frac{\partial\left(\left(\mu\frac{(4-18R+20R^3)(Q^2-2Q^3+Q^4)}{P^2}+(2R^2-3R^3+R^5)(2-12Q+12Q^2)\right)\right)}{\partial Q}\right)$$

$$[1-\mu]\,Q_y = -\frac{b}{P^2}F(t)\chi((4-18R+20R^3)(2Q-6Q^2+4Q^3))$$

$$-bF(t)\chi\left(\mu\frac{(4-18R+20R^3)(2Q-6Q^2+4Q^3)}{P^2}s+(2R^2-3R^3+R^5)(-12+24Q^2)\right)$$

$$Q_y = -bF(t)\chi\ [(\frac{[1-\mu]}{P^2}((4-18R+20R^3)(2Q-6Q^2+4Q^3)))$$

$$+ (\mu\frac{(4-18R+20R^3)(2Q-6Q^2+4Q^3)}{P^2}$$

$$+ (2R^2-3R^3+R^5)(-12+24Q^2))] \quad (4.46)$$

$$Q_y = -bF(t)\eta_y \quad (4.47)$$

Where,

$$\eta_y = \chi\ [(\frac{[1-\mu]}{P^2}((4-18R+20R^3)(2Q-6Q^2+4Q^3)))$$

$$+ (\mu\frac{(4-18R+20R^3)(2Q-6Q^2+4Q^3)}{P^2} + (2R^2-3R^3+R^5)(-12+24Q^2))] \quad (4.48)$$

Table 4.21: Shear force coefficients η_x for different plate aspect ratios and forcing frequencies at the centre of the plate

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	η_x					
0.1	-0.00636	-0.00796	-0.01061	-0.01591	-0.03182	∞
0.2	-0.01263	-0.01578	-0.02104	-0.03157	-0.06313	∞
0.3	-0.01862	-0.02328	-0.03104	-0.04656	-0.09312	∞
0.4	-0.02411	-0.03014	-0.04018	-0.06027	-0.12054	∞
0.5	-0.02876	-0.03595	-0.04793	-0.0719	-0.14379	∞
0.6	-0.03226	-0.04032	-0.05376	-0.08064	-0.16129	∞
0.7	-0.0344	-0.04301	-0.05734	-0.08601	-0.17202	∞
0.8	-0.03519	-0.04399	-0.05865	-0.08797	-0.17594	∞
0.9	-0.03479	-0.04349	-0.05798	-0.08697	-0.17395	∞
1	-0.03351	-0.04189	-0.05585	-0.08377	-0.16754	∞
1.1	-0.03167	-0.03959	-0.05278	-0.07918	-0.15835	∞
1.2	-0.02955	-0.03694	-0.04926	-0.07389	-0.14777	∞
1.3	-0.02737	-0.03421	-0.04561	-0.06841	-0.13683	∞
1.4	-0.02524	-0.03155	-0.04206	-0.06309	-0.12618	∞
1.5	-0.02324	-0.02905	-0.03873	-0.0581	-0.1162	∞
1.6	-0.02141	-0.02676	-0.03569	-0.05353	-0.10706	∞
1.7	-0.01976	-0.0247	-0.03293	-0.0494	-0.0988	∞
1.8	-0.01828	-0.02285	-0.03047	-0.0457	-0.0914	∞
1.9	-0.01696	-0.0212	-0.02827	-0.0424	-0.0848	∞
2	-0.01578	-0.01973	-0.02631	-0.03946	-0.07892	∞

Shear force coefficients η_x for different plate aspect ratios and forcing frequencies at the centre of the plate, R= Q =1/2. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Shear force coefficients η_x also increases until the forcing frequency coincides

with the natural frequency. At this point, Shear force coefficients η_x turn to infinity.

Table 4.22: Shear force coefficients η_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = 1 and Q = ½

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	η_s					
0.1	-0.088814	-0.111017	-0.148023	-0.222035	-0.444069	∞
0.2	-0.174445	-0.218056	-0.290741	-0.436111	-0.872223	∞
0.3	-0.253141	-0.316426	-0.421902	-0.632853	-1.265706	∞
0.4	-0.320424	-0.40053	-0.534041	-0.801061	-1.602122	∞
0.5	-0.371643	-0.464554	-0.619405	-0.929108	-1.858216	∞
0.6	-0.403225	-0.504031	-0.672041	-1.008061	-2.016123	∞
0.7	-0.414043	-0.517553	-0.690071	-1.035107	-2.070213	∞
0.8	-0.40602	-0.507525	-0.6767	-1.01505	-2.0301	∞
0.9	-0.383507	-0.479383	-0.639178	-0.958767	-1.917533	∞
1	-0.351842	-0.439802	-0.586403	-0.879604	-1.759208	∞
1.1	-0.315954	-0.394942	-0.52659	-0.789885	-1.579769	∞
1.2	-0.279571	-0.349463	-0.465951	-0.698927	-1.397854	∞
1.3	-0.245068	-0.306335	-0.408446	-0.612669	-1.225339	∞
1.4	-0.213695	-0.267119	-0.356159	-0.534239	-1.068477	∞
1.5	-0.185923	-0.232404	-0.309872	-0.464808	-0.929615	∞
1.6	-0.161745	-0.202181	-0.269575	-0.404363	-0.808726	∞
1.7	-0.140906	-0.176133	-0.234844	-0.352266	-0.704531	∞
1.8	-0.123043	-0.153803	-0.205071	-0.307607	-0.615213	∞
1.9	-0.107765	-0.134706	-0.179608	-0.269413	-0.538825	∞
2	-0.094702	-0.118378	-0.157837	-0.236755	-0.47351	∞

Shear force coefficients η_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = 1 and Q = ½. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Shear force coefficients η_X also increases until the forcing frequency coincides with the natural frequency. At this point, Shear force coefficients $\eta_X$turn to infinity.

Table 4.22a: Shear force coefficients η_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 0

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	η_s					
0.1	-4.23E-05	-5.29E-05	-7.05E-05	-0.000106	-0.000211	∞
0.2	-0.000332	-0.000415	-0.000554	-0.000831	-0.001661	∞
0.3	-0.001085	-0.001356	-0.001808	-0.002712	-0.005424	∞
0.4	-0.002441	-0.003052	-0.004069	-0.006103	-0.012207	∞
0.5	-0.004424	-0.00553	-0.007374	-0.011061	-0.022122	∞
0.6	-0.006912	-0.008641	-0.011521	-0.017281	-0.034562	∞
0.7	-0.009661	-0.012076	-0.016102	-0.024152	-0.048305	∞
0.8	-0.012374	-0.015467	-0.020623	-0.030935	-0.06187	∞
0.9	-0.014792	-0.018491	-0.024654	-0.036981	-0.073962	∞
1	-0.016754	-0.020943	-0.027924	-0.041886	-0.083772	∞
1.1	-0.018205	-0.022756	-0.030342	-0.045512	-0.091025	∞
1.2	-0.019171	-0.023963	-0.031951	-0.047926	-0.095853	∞
1.3	-0.019722	-0.024653	-0.03287	-0.049305	-0.098611	∞
1.4	-0.019945	-0.024931	-0.033242	-0.049862	-0.099725	∞
1.5	-0.01992	-0.0249	-0.033201	-0.049801	-0.099602	∞
1.6	-0.019718	-0.024647	-0.032863	-0.049294	-0.098588	∞
1.7	-0.019391	-0.024239	-0.032319	-0.048478	-0.096957	∞
1.8	-0.018984	-0.02373	-0.03164	-0.047459	-0.094919	∞
1.9	-0.018525	-0.023157	-0.030876	-0.046313	-0.092627	∞
2	-0.018038	-0.022548	-0.030064	-0.045096	-0.090192	∞

Shear force coefficients η_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 0. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Shear force coefficients η_X also increases until the forcing frequency coincides

with the natural frequency. At this point, Shear force coefficients η_X turn to infinity.

Table 4.22b: Shear force coefficients η_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 0

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	η_s					
0.1	-4.23E-05	-5.29E-05	-7.05E-05	-0.000106	-0.000211	∞
0.2	-0.000332	-0.000415	-0.000554	-0.000831	-0.001661	∞
0.3	-0.001085	-0.001356	-0.001808	-0.002712	-0.005424	∞
0.4	-0.002441	-0.003052	-0.004069	-0.006103	-0.012207	∞
0.5	-0.004424	-0.00553	-0.007374	-0.011061	-0.022122	∞
0.6	-0.006912	-0.008641	-0.011521	-0.017281	-0.034562	∞
0.7	-0.009661	-0.012076	-0.016102	-0.024152	-0.048305	∞
0.8	-0.012374	-0.015467	-0.020623	-0.030935	-0.06187	∞
0.9	-0.014792	-0.018491	-0.024654	-0.036981	-0.073962	∞
1	-0.016754	-0.020943	-0.027924	-0.041886	-0.083772	∞
1.1	-0.018205	-0.022756	-0.030342	-0.045512	-0.091025	∞
1.2	-0.019171	-0.023963	-0.031951	-0.047926	-0.095853	∞
1.3	-0.019722	-0.024653	-0.03287	-0.049305	-0.098611	∞
1.4	-0.019945	-0.024931	-0.033242	-0.049862	-0.099725	∞
1.5	-0.01992	-0.0249	-0.033201	-0.049801	-0.099602	∞
1.6	-0.019718	-0.024647	-0.032863	-0.049294	-0.098588	∞
1.7	-0.019391	-0.024239	-0.032319	-0.048478	-0.096957	∞
1.8	-0.018984	-0.02373	-0.03164	-0.047459	-0.094919	∞
1.9	-0.018525	-0.023157	-0.030876	-0.046313	-0.092627	∞
2	-0.018038	-0.022548	-0.030064	-0.045096	-0.090192	∞

Shear force coefficients η_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 0. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Shear force coefficients η_X also increases until the forcing frequency coincides

with the natural frequency. At this point, Shear force coefficients η_X turn to infinity.

Table 4.23: Shear force coefficients η_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = 0 and Q = ½

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	η_s					
0.1	-0.03806	-0.04758	-0.06344	-0.09516	-0.19032	∞
0.2	-0.07476	-0.09345	-0.1246	-0.1869	-0.37381	∞
0.3	-0.10849	-0.13561	-0.18082	-0.27122	-0.54245	∞
0.4	-0.13732	-0.17166	-0.22887	-0.34331	-0.68662	∞
0.5	-0.15928	-0.19909	-0.26546	-0.39819	-0.79638	∞
0.6	-0.17281	-0.21601	-0.28802	-0.43203	-0.86405	∞
0.7	-0.17745	-0.22181	-0.29574	-0.44362	-0.88723	∞
0.8	-0.17401	-0.21751	-0.29001	-0.43502	-0.87004	∞
0.9	-0.16436	-0.20545	-0.27393	-0.4109	-0.8218	∞
1	-0.15079	-0.18849	-0.25132	-0.37697	-0.75395	∞
1.1	-0.13541	-0.16926	-0.22568	-0.33852	-0.67704	∞
1.2	-0.11982	-0.14977	-0.19969	-0.29954	-0.59908	∞
1.3	-0.10503	-0.13129	-0.17505	-0.26257	-0.52515	∞
1.4	-0.09158	-0.11448	-0.15264	-0.22896	-0.45792	∞
1.5	-0.07968	-0.0996	-0.1328	-0.1992	-0.39841	∞
1.6	-0.06932	-0.08665	-0.11553	-0.1733	-0.3466	∞
1.7	-0.06039	-0.07549	-0.10065	-0.15097	-0.30194	∞
1.8	-0.05273	-0.06592	-0.08789	-0.13183	-0.26366	∞
1.9	-0.04619	-0.05773	-0.07698	-0.11546	-0.23093	∞
2	-0.04059	-0.05073	-0.06764	-0.10147	-0.20293	∞

Shear force coefficients η_x for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 0. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Shear force coefficients η_x also increases until the forcing frequency coincides

with the natural frequency. At this point, Shear force coefficients $\eta_X$turn to infinity.

Table 4.24: Shear force coefficients η_y for different plate aspect ratios and forcing frequencies at the center of plate, R = ½ and Q = ½

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	η_y					
0.1	-4.8E-05	-5.94736E-05	-7.92981E-05	-0.00012	-0.00024	∞
0.2	-0.00075	-0.000934524	-0.001246032	-0.00187	-0.00374	∞
0.3	-0.00366	-0.004576882	-0.00610251	-0.00915	-0.01831	∞
0.4	-0.01099	-0.013732474	-0.018309965	-0.02746	-0.05493	∞
0.5	-0.02489	-0.031108519	-0.041478026	-0.06222	-0.12443	∞
0.6	-0.04666	-0.058323556	-0.077764742	-0.11665	-0.23329	∞
0.7	-0.07608	-0.095100412	-0.126800549	-0.1902	-0.3804	∞
0.8	-0.11137	-0.139206854	-0.185609139	-0.27841	-0.55683	∞
0.9	-0.14977	-0.187216322	-0.249621762	-0.37443	-0.74887	∞
1	-0.18849	-0.235608256	-0.314144342	-0.47122	-0.94243	∞
1.1	-0.22529	-0.281607973	-0.375477297	-0.56322	-1.12643	∞
1.2	-0.2588	-0.32350335	-0.4313378	-0.64701	-1.29401	∞
1.3	-0.28844	-0.360544976	-0.480726635	-0.72109	-1.44218	∞
1.4	-0.31413	-0.392665446	-0.523553928	-0.78533	-1.57066	∞
1.5	-0.33616	-0.420194281	-0.560259041	-0.84039	-1.68078	∞
1.6	-0.35492	-0.443643914	-0.591525218	-0.88729	-1.77458	∞
1.7	-0.37086	-0.463575345	-0.618100459	-0.92715	-1.8543	∞
1.8	-0.38442	-0.480525375	-0.6407005	-0.96105	-1.9221	∞
1.9	-0.39598	-0.494973667	-0.659964889	-0.98995	-1.97989	∞
2	-0.40587	-0.507332631	-0.676443508	-1.01467	-2.02933	∞

Shear force coefficients η_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = ½. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Shear force coefficients η_y also increases until the forcing frequency coincides with the natural frequency. At this point, Shear force coefficients η_y turn to infinity.

Table 4.25: Shear force coefficients η_y for different plate aspect ratios and forcing frequencies at plate edge (R = ½ Q = 0)

α	0	0.2	0.4	0.6	0.8	1
Aspect ratio, P	η_y					
0.1	-6.3438E-05	-7.92981E-05	-0.000106	-0.000159	-0.000317	∞
0.2	-0.000997	-0.001246	-0.001661	-0.002492	-0.004984	∞
0.3	-0.004882	-0.006103	-0.008137	-0.012205	-0.024410	∞
0.4	-0.014648	-0.018310	-0.024413	-0.036620	-0.073240	∞
0.5	-0.033182	-0.041478	-0.055304	-0.082956	-0.165912	∞
0.6	-0.062212	-0.077765	-0.103686	-0.155529	-0.311059	∞
0.7	-0.101440	-0.126801	-0.169067	-0.253601	-0.507202	∞
0.8	-0.148487	-0.185609	-0.247479	-0.371218	-0.742437	∞
0.9	-0.199697	-0.249622	-0.332829	-0.499244	-0.998487	∞
1	-0.251315	-0.314144	-0.418859	-0.628289	-1.256577	∞
1.1	-0.300382	-0.375477	-0.500636	-0.750955	-1.501909	∞
1.2	-0.345070	-0.431338	-0.575117	-0.862676	-1.725351	∞
1.3	-0.384581	-0.480727	-0.640969	-0.961453	-1.922907	∞
1.4	-0.418843	-0.523554	-0.698072	-1.047108	-2.094216	∞
1.5	-0.448207	-0.560259	-0.747012	-1.120518	-2.241036	∞
1.6	-0.473220	-0.591525	-0.788700	-1.183050	-2.366101	∞
1.7	-0.494480	-0.618100	-0.824134	-1.236201	-2.472402	∞
1.8	-0.512560	-0.640700	-0.854267	-1.281401	-2.562802	∞
1.9	-0.527972	-0.659965	-0.879953	-1.319930	-2.639860	∞
2	-0.541155	-0.676444	-0.901925	-1.352887	-2.705774	∞

Shear force coefficients η_y for different plate aspect ratios and forcing frequencies at the edge of plate, R = ½ and Q = 0. As the forcing frequency is gradually increased i.e. $8^2 = , \alpha h^2, \alpha = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and approaches the fundamental natural frequency, Shear force coefficients η_y also increases until the forcing frequency coincides with the natural frequency. At this point, Shear force coefficients η_y turn to infinity.

This study has analyzed the dynamic regime of the independent thin rectangular plate components of RLCS during its recharge using Galerkin's indirect variational method. The shape function, which compares favorably best from the litany of trigonometric and polynomial shape functions from the literature was sought from the characteristic orthogonal polynomials approach. From the litany of literature reviewed, for a square rectangular CCCC plate (P = 1.0), the accurate value of the square of the coefficient for the fundamental natural frequency, $\sqrt{\bar{n}}$ is 35.9866. The present study gave the square of the coefficient for the fundamental natural frequency, $\sqrt{\bar{n}}$ as 35.9982, whereas results from the literature gave values $\sqrt{n}$ 36.3485. This shows that the shape function used in the present study is almost exact and can be used in utmost confidence with any of the energy method to analyze the free vibration of the plate system, as well as the its forced vibration regime. Thus, the functional was use to examine the effect of reservoir rectangular tank during recharge (forced dynamic regime) on the deflections, stresses, moments and shear forces on the walls of the tanks; which is made up of thin elastic plate systems. The details of the results are given in Tables in above.

Owing to scarce literature in the areas of numerical plate-forced vibration, comparative analysis study of the results from the present work is handicapped. However, the justification of the results from the forced vibration presented was sought from the free vibrations rectangular plate problems, which have enjoyed numerous attentions by researchers in the past. In which, the present study compares favorably with the accurate results.

CHAPTER FIVE

CONCLUSIONAND RECOMMENDATIONS

5.1 Conclusion

The following conclusions were drawn based on the results of the study.

a) Characteristic orthogonal polynomial shape functions for rectangular plates used are satisfactory in approximating the deformed shape of thin rectangular clamped plates.
b) An indirect variation principle (based on Galerkin's method) could be used in confidence to satisfactorily analyze forced vibration of rectangular thin clamped plate's problems.
c) The results obtained herein for free vibrations of the plate are very closer to the exact results than the one by previous research works that used different modal shape in analysis.

5.3 Recommendations

The following recommendations were made:

a) It is recommended that the shape functions developed for CCCC rectangular plate be applied by engineers in the design of rectangular tank.
b) Future research work should consider using the characteristic orthogonal polynomial derived shape functions and apply it to Rayleigh – Ritz direct variational method to compare their results the present results.
c) Future studies should use the characteristic orthogonal polynomial theorem used herein and Galerkin's method to analyze plate of different support conditions under force vibration regime.
d) Future studies should use the characteristic orthogonal polynomial theorem used herein and Galerkin's method to analyze rectangular plate on elastic foundation.

5.2 Contribution to Knowledge

This research has:

a) Provided reliable approximate solution for the fundamental natural frequency of CCCC plate, which compares favorably with the exact solution, through the characteristic orthogonal polynomial approach.
b) Provided comprehensive design data: deflections solution for the forced vibration problems of CCCC plate due to uniformly and varying dynamic loads.

c) Provided comprehensive design data: shear forces solution for the forced vibration problems of CCCC plate due to uniformly and varying dynamic loads.

d) Provided comprehensive design data: moments solution for the forced vibration problems of CCCC plate due to uniformly and varying dynamic loads.

REFERENCES

Bangash, M. Y. (1993). Impact and Explosion: Analysis and Design. Oxford: Blackwell Scientific Publication.

Bao, S.Y. and Deng, Z.C. (2005). A general solution of free vibration for rectangular thin plates in Hamilton Systems. Journal of Dynamics and Control, Vol. 3(2), pp.10–16 (in Chinese)

Bhat, R. B. (1985a). Natural Frequencies of Rectangular Plates using Characteristic Orthogonal Polynomials in Rayleigh - Ritz Method. Journal of Sound and Vibration, Vol. 102 (4), pp. 493 - 499.

Bhat, R. B. (1985b). Plate Deflections using Orthogonal Polynomials. Journal of Engineering.

Brown, R. E. and stone, M. A. (1997). On the use of polynomial series with the Rayleigh Ritz method. Composite structure, Vol. 39(3-4), pp. 191-196.

Chen, W. Q. and Lüe, C. F. (2005). 3D free vibration analysis of cross-ply laminated plates with one pair of opposite edges simply supported. Composite Structure. Vol. 69, pp. 77–87.

Courant, R. and Hilbert, D. (1953). Method of Mathematical Physics. Vol. I, Interscience Publishers, Inc., New York, N. Y., MR 0065391.

Chen, J. Z., Ghaemmaghami, A. R. and Kianoush, M. R. (2008). Dynamic Analysis of Concrete Rectangular Liquid Storage Tanks. The 14th World Conference on Earthquake Engineering. Beijing. pp. 12-17

Ezeh, J. C., Ibearugbulem, O. M. and Ebirim, S. I. (2014). Fundamental natural frequency for isotopic rectangular plate simply supported on three edges with one edge free of support (SSSF plate): International Journal of Research in Engineering & Technology ISSN (E): 2321-8843; ISSN (P): 2347-4599; Vol. 2, Issue 2, pp. 67-74.

Filipich, C. P. and Rosales, M. B. (2000). Arbitrary precision frequencies of a free rectangular thin plate. Journal of Sound and Vibration Vol. 230(3), pp. 521–539.

Gutierrez, R. H. and Laura, P. A. (2000). Transverse Vibrations of a Circular Plate of polar Anisotropy with a Concentric Circular Support. Journal of Sound and Vibration, Vol. 231 (4), pp. 1175 -1178.

Gupta, U. S., Lal, R. and Sharma, S. (2006). Vibration Analysis of Non-homogeneous Circular Plate of Nonlinear Thickness Variation by Differential Quadrature Method. Journal of Sound and Vibration, Vol. 298 (4 - 5), pp. 892-906.

Galin, M. P. (1947). On the Transverse Vibrations of Plates. Prikl. Mat. Mekh , Vol. 12 (3), pp. 387-388.

Mindlin, R. D. (1951). Influence of rotatory inertia and shear on flexural motion of isotropic, elastic plates. ASME journal of applied Mechanics, Vol. 18, pp. 31-38.

Gorman, D. J. (1982). Free Vibration Analysis of Rectangular Plates. North Holland: Elsevier.

Hsu, M. (2003). Vibration Analysis of Isotropic and Orthotropic Plates with Mixed Boundary Conditions with Tamkang Journal of Science and Engineering, Vol. 6 (No 4), pp. 217 – 226.

Hedrih, K. (2006). Transversal vibrations of double-plate systems. Acta Mechanical Sinica Vol. 22, pp. 487–501

Ibearugbulem, O. M. (2012). Application of a direct variational principle in elastic stability of rectangular flat thin plates. Ph. D. thesis submitted to Postgraduate School, Federal University of Technology, Owerri.

Iyengar, N. G. R. (1988). Structural Stability of Columns and Plates.Chichester: Ellis Horwood Limited.

Janich, R. (1962). Die näherungsweiseBerechnung der Eigenfrequenzen von rechteckigenPlattenbeiverschiedenenRandbedingungen. Die Bautechnik, vol. 3, pp. 93-99.

Kirchoff, G. R. (1885). Vorlesungen Uber Mathematische Physic, Mechanik (2nd).

Kshirsagar, S. and Bhaskar, K. (2008). Accurate and elegant free vibration and buckling studies of orthotropic rectangular plates using untruncated infinite series. Journal of Sound and Vibration, 314, pp. 37–50.

Lal, R., Kumar, Y. and Gupta, U. S. (2009). Transverse vibrations of non-homogeneous rectangular plates of uniform thickness using boundary characteristic orthogonal polynomials. Int. J. of Appl. Math and Mech., Vol. 6(14), pp. 93 – 109.

Laura, P. A. and Gutierrez, R. H. (1995). Analysis of Vibrating Circular Plates of Nonuniform Thickness by the Method of Differential Quadrature. Ocean Engineering , Vol. 22 (1), pp. 97 - 100.

Laura, P. P. and Sanchez, M. D. (1998). Vibrations of Circular Plates of Rectangular Orthotropy. Journal of Sound and Vibration, Vol. 209 (1), pp. 199 - 202.

Lee, S. J. (2004). Free vibration analysis of plates by using a four – node finite element formulated with assumed natural transverse shear strain. Journal of Sound and Vibration, 278, pp. 657 – 684.

Leissa, A. W. (1973). The Free Vibration of Rectangular Plates. Journal of Sound and vibration, Vol.31 (3), pp. 257 – 293.

Love, A. E. H. (1888). On the Small Free Vibrations and Deformations of Elastic Shells, Philosophical Trans. Of The Royal Society (London), Vol. Series A, N^O 17 pp. 491-549.

Meirovitch, L. (1967). Analytical Methods in Vibrations. New York: Macmillan Co.

Misra, R. K. (2012). Static and dynamic analysis of rectangular isotropic plate using multiquadric radial Basis function. International journal of Management, I.T and Engineering, Vol. 2(8), pp. 166 – 178.

Mitchell, G. R. (1972). Loadings on Buildings. Proceedings of International Conference on Planning and Design of Tall Buildings, Vol 1b, No 2A, pp. 19 – 33.

Navier, L. (1823). Bull. Soc. Philomath., Paris, 75, pp.177 – 183.

Njoku, K. O. (2013). Dynamic Analysis of Thin Rectangular Flat Isotropic Plates by Galerkin's Method. M.Eng. Thesis Submitted to Postgraduate School, Federal University of Technology, Owerri.

Onyeyili, I. O. (2012). Lecture Notes on Advanced Theory of Plates and Shells. FUTO: SPGS.

Ouyang, H. J. and Zhong, W.X. (1993). Hamiltonian system and symplectic geometry in mechanics of materials (III) — Flexure and free vibration of plates. Applied Mathematics and Mechanics, 14(1), (in Chinese).

Reddy, J. N. (2007), Theory and Analysis of Elastic Plates and Shells, CRS Press, Taylor and Francis.

Reissner, E. (1945), the effect of Transverse shear deformation on the bending of elastic plates, ASME journal of applied Mechanics, Vol. 12, pp. A68-77.

Sakata, T., Takahashi, K. and Bhat, R. B. (1996). Natural Frequencies of Orthotropic Rectangular Plates Obtained by Iterative Reduction of the Partial Differential Equation. Journal of Sound Vibration, 189, pp. 89 – 101.

Seok, J.W., Tiersten, H.F. and Scarto, H.A. (2004). Free vibrations of rectangular cantilever plates. Part 1: out-of-plane motion. Journal of Sound and Vibration, 271, pp. 131–146.

Shi, G. (1990). Flexural vibration and buckling analysis of orthotropic plates by the boundary element method. International journal of solids structures, Vol. 2(2), pp. 59 – 67.

Shu, C., Wu, W. X., Ding, H. and Wang, C. M. (2007). Free vibration analysis of plates using least square based finite difference method. Computer Methods Appl. Mech. Engr., 196, pp. 1330-1343.

Timoshenko, S. P. and Woinowsky - Krieger, S. (1970). Theory of Plates and Shells (2nd edition.). Auckland: McGraw-Hill Inc.

Topalian, V., Larrondo, H. A., Avalos, D. R. and Laura, P. A. (1997). Journal of Sound and Vibration. Vol. 202 (1), pp. 125 - 133.

Ventsel, E. and Krauthammer, T. (2001). Thin Plates and Shell (Theory, Analysis and Applications). New York: Marcel Dekker.

Vera, S. A., Laura, P. A. and Vega, D. A. (1999). Transverse Vibrations of a free-free Circular Annular Plate. Journal of Sound and Vibration, Vol. 224 (2), pp. 379 - 88.

Wang, C.M., Lim, G.T., Reddy, J. N. and Lee, K. H. (2001), Relationships between bending solutions of Reissner and Mindlin Plate theories, Engineering structures, Vol. 23, pp. 838- 849.

Werfalli, N. M. and Karaid, A. A. (2005). Free vibration analysis of rectangular plates using Galerkin based finite element method. International Journal of Mechanical Engineering, Vol. 2(2), PP. 59-67.

Wu, T. Y., Wang, Y. Y. and Liu, G. R. (2002). Free Vibration Analysis of Circular Plates using Generalized Differential Quadrature Rule. Computer Methods in Applied Mechanics and Engineering, Vol. 191 (46), pp. 5365 - 5380.

Zhong, Y. and Zhang, Y. S. (2006). Free vibration of rectangular thin plate on elastic foundation with four edges free. Journal of Vibration Engineering, Vol. 19 (4), S (in Chinese).