

**PREDICTIVE MODELING AND ANALYSIS OF NIGERIAN
PETROLEUM DEPLETION, USING A COMPOSITE
UNDERGROUND RESERVIOR**

BY

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A PhD DISSERTATION

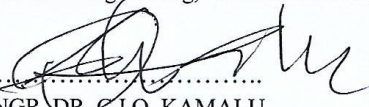
**SUBMITTED TO THE POSTGRADUATE SCHOOL
FEDERAL UNIVERSITY OF TECHNOLOGY, OWERRI**

**IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR
THE AWARD OF DOCTOR OF PHILOSOPHY (PhD)
DEGREE IN CHEMICAL ENGINEERING**


FEBRUARY, 2021

CERTIFICATION


This is to certify that this work: **'Predictive Modeling and Analysis of Nigerian Petroleum Depletion, Using a Composite Underground Reservoir'**, was carried out by **OKERE, PRINCEWILL CHIBUZOR** with Registration Number **20114770308**, in partial fulfillment of the requirements for the award of the Degree of Doctor of Philosophy (PhD) in Chemical Engineering, in the Department of Chemical Engineering, Federal University of Technology, Owerri.


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DEDICATION

This research work is dedicated to Almighty God, my maker and heavenly Father, who is the Author of Life.

ACKNOWLEDGEMENTS

My profound gratitude goes to Almighty God, the giver of life for his infinite kindness, love, grace, guidance, divine inspiration and protections throughout the duration of this program.

In no small measure, I greatly appreciate the intellectual input and encouragements given to me by my amiable supervisor, Engr. Dr. C.I.O. Kamulu. His continuous support, patience, motivation and immense knowledge served as necessary catalyst that drove this study to a successful completion. Sir, you literally walked me through all the stages of this research and writing of this thesis, I could not have imagined having a better supervisor and mentor for the program other than you.

My sincere appreciation goes to my awesome co-supervisors; firstly to Engr. Prof. E.E. Anyanwu for painstakingly reviewing this work from cover to cover, going through all the drafts and proffering suggestions for improvements, your office was ever open to me whenever I ran into a troubled spot begging for clarification to navigate through. You consistently steered me in the right direction thereby building me up confidently. Secondly Engr. Prof. M.S. Nwakaudu; Sir, you brought out the salient questions and the technical writing format of this work by making sure my readers are not left in doubt. Your words of motivation have remained a boost for the completion of this work. Thirdly, I am also grateful to Engr. Dr. O.E. Onyelucheya for his valuable and insightful comments upon this work, indeed you are a friend and a brother. I am also grateful to the internal reviewer of this thesis, Engr. Dr. E.C. Osoka, your criticism and corrections also improved the work and made it worthwhile.

My profound thanks also go to the staff and Management of Department of Petroleum Resources, Ministry of Petroleum and Mineral Resources Lagos Nigeria, mostly Mr. Eddy Nyokwa for making available the Nigerian Petroleum Data used for this work.

My profound appreciation also goes to my professional mentor Engr. Dr. Grace C. Akujobi-Emetuche, an astute professional who have always believed in me, Ma your words of encouragement really spur me to the completion of this program. Engr. Mohammed Bash and Engr Jackreece Enefa, of Nigeria Petroleum Development Company (NPDC) Benin Nigeria for taking me through various oil fields, Oil Mining Lease (OML), the stages of Petroleum Drilling and Production processes.

My utmost gratitude also goes to my amiable Lecturers, Engr. Prof. (Mrs.) K.B. Oyoh for her motherly advice and for taking me through some courses, Prof. O.C. Ndukwe for his forthrightness and elderly

advice, not forgetting my able Head of Department Engr. Dr. I.S. Ike for his effort towards this work by encouraging me to see to the end of it. I sincerely appreciate your moral support.

My sincere appreciation goes to Engr. Prof. J.A. Akpa and Engr. K. Kurotamunoye both of Department of Chemical Engineering, Rivers State University, Nigeria, for their outstanding academic prowess in tackling the sensitivity analysis aspect of this work. My humble appreciation also goes to Engr. J.C. Obijiaku of Chemical Engineering Department, Federal University of Technology Owerri (FUTO) Nigeria, who handled most of the draft editing and typesetting of this work, and for his salient technical inputs too.

I wish to express my sincere and candid gratitude to Engr. (Mrs) Ugonna Betty for her motherly counsel and encouragement. Ma, your task on project assignments trusted to me has in no small measure sharpened the quality of this work in terms of report writing. Engr. Dudu Rowland, Mr. Nyobar Friday, Chief Meme Godfrey, I thank you immensely for your necessary support. To my immediate boss Mr. Madumere Jonathan your show of understanding and kind heartedness is overwhelming and second to none.

To my loving and caring wife, Mrs Priscilla Ekelemchi Okere, and my wonderful and loving kids, Jeremiah Chidozie Princewill, Favour Chioma Princewill, and Jessica Ihunayachi Princewill, your patience, understanding, sacrifice, unflinching support and continuous encouragement throughout my years of study and thesis writing gladdens my heart. This accomplishment could not have been possible without you all, indeed you are all my stars.

Also many thanks to my loving mother, Madam Grace Nkechinyere Okere, My brothers, Mr. Okereh ThankGod, Pastor Israel Okeren, and my loving sisters, Mrs. Peace Jumbo, Mrs. Blessing Okere and Pastor (Mrs.) Comfort Shadrack for their support and prayer throughout the course of this PhD program and my life in general.

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NOMENCLATURE

Abbreviation	Description
Q	Cumulative
D	Dumb-bell
P _o	Oil production
d _o	Oil discovery
D _g	Gasdiscovery
D _Q	Dumb-bell made out of sigmoidal profile
D _o	Dumb-bell Oil
D _{oI}	Dumb-bell Oil intersection
D _G	Dumb-bell Gas
D _{GI}	Dumb-bell Gas intersection
Q _R	Cumulative reserve
Q _D	Cumulative discovery
Q _P	Cumulative production
V _o	Initial reserve
R, F	Arbitrary constants
Q	Volumetric flow rate of oil
q _o	Volumetric outflow rate of the Tank (Reservoir)
V	Volume of oil reserve
T	Depletion time
ρ	Density of the crude o

ABSTRACT

The thrust of this study is predictive modeling and analysis of Nigeria's petroleum resources depletion, using a composite underground reservoir. The Nigerian Petroleum Data from 1957 to 2014, were obtained from the Department of Petroleum Resources (DPR), of the Ministry of Petroleum and Minerals Resources, Lagos Nigeria, which were applied as the experimental data. Predictive models were developed from material balance of Nigerian petroleum resources around a composite underground reservoir. Petroleum depletion models were developed using Hubbert concept, as the input functions were varied, and scatter diagrams of oil and gas production in Nigeria from 1957 to 2014 were plotted using MATLAB 7.9. The models were validated by superimposing them on the scatter diagram profiles of the cumulative data to determine their goodness of fit which were declared by the R-squared (R^2) produced by the computer software. Hubbert oil depletion concept was employed for the peak determination, dumbbell intersection and establishment of petroleum depletion. Plots of these models were also made to find which models gave almost identical curves as the curves of the plots made from raw data for oil and gas. The models so obtained are all nonlinear equations which have volume as a function of time, and the best chosen to be the forecasting/predictive tool so desired. For the cumulative production and discovery of oil/gas resources, the chosen model 5 obtained is given as $V_{(t)} = R^3(1 - e^{-\frac{1}{R}t}) + (V_0 - R^2t - \frac{Rt^2}{2}) e^{-\frac{1}{R}t}$, and for the annual production and discovery of oil/gas resources, the chosen model 5 obtained is given as $P_{(t)} = (\frac{V_0}{R} - \frac{t^2}{2})e^{-\frac{1}{R}t}$. Based on the peak determinations and dumbbell intersections for the various models, it was found that the Nigerian oil reserve peaked in the year 2008AD (with volume = 28,018.9MMB), and the gas reserve will peak in the year 2065AD (with volume = 2.546×10^6 bscf). The Nigerian oil reserve will be exhausted (vol. = 0.136817MMB) in the year 2682AD, while the gas follows suit in the year 3151AD (vol. = 1.62447bscf). The accuracies of these results are based on R^2 of 0.9955 - 0.9963 for oil and 0.9979 - 0.9983 for gas. These results clearly show the efficacy of the models so developed, and their reliability in the prediction of Nigeria's petroleum resources depletion. This result can be used by Nigerian government for planning, diversification of the economy and international bargaining and positioning in OPEC.

Key Words: Nigerian petroleum depletion, composite underground reservoir, predictive modeling and analysis, transfer and input functions, curve fitting, intersection of dumbbell profiles.

CHAPTER ONE

INTRODUCTION

1.1 Background Information

Today, most countries of the world are importers of energy for their various needs. The accumulated fossil fuels over aeons of geological activities are irreversibly consumed at a rate which is more than a million times faster than the rate at which they were formed. This left us in a precarious position due to our dependence on fossil fuels for energy, especially for petroleum resources and its products. Then the hike in price of petroleum and its products, both at the national and international scenes, happens for two simple reasons: the geometrically increasing demands and fast depletion of the reserves. The importance of petroleum resources in our present day civilization is ever increasing as a result of its unmatched contribution to our energy requirements in lubrication, petrochemical fields and other applications. Hence, its competence to serve the energy needs of mankind, is very unquestionable and unique too.

The oil boom of the early 1970s somehow led Nigeria to neglect its strong agricultural and light manufacturing bases in preference over much dependence on crude oil. Nigeria joined the organization of petroleum exporting countries (OPEC) in July 1971 as the eleventh member (Sayne, Glies, & Katsouris, 2015), and the first serious increase in price of crude oil was from 1973-1974 due to the then oil embargo by Arab countries. This yielded monumental financial benefits to the country (Esekumemu, 2016). Subsequently, crude oil have continued to play a predominant and strategic role in the economy of the country.

The impression was, thus, imprinted on the minds of nearly every Nigerian that no other area of our natural resources deserves to be further developed and nurtured aside petroleum resources. Farm produce like the groundnut pyramid in the North, cocoa in the West and palm oil in the East, that used to be the main sources of revenue before the discovery of petroleum were all neglected as Nigerians found easier and less laborious means of foreign exchange. The rural

work force was as a result depleted as youths migrated to urban centres such that the production of both food and cash crops fell drastically, yet the demands on the limited amenities and commodities outstripped the rate of supply.

By 1973 when the Middle-East war broke out, this aided Nigeria to make more money from her petroleum production. And with the nation's desire for rapid development coupled with our rising bills for imported food and industrial machinery, Nigeria became increasingly dependent on petroleum. As it was said, the question was not how to make the money but how and on what to spend it. That could be the reason the then Head of State was quoted to have said that: the problem of Nigeria was not money but how to spend it, either to spend it on people-oriented projects or to spend it on gigantic white elephant project schemes (Nlerum, 2011).

Oil is a wasting asset. It cannot be replenished. Between 1974 and 1978 production remained stable on an average of 2,005 million barrels per day, and since this means non replenishable, it means depletion.

Crude oil production in Nigeria as at today is 1573Bbl/day. Average production from 1973 to 2018 is 1884.68Bbl/day. The highest production Nigeria has produced is 2475Bbl/day in 2005 and the lowest production rate is 675Bbl/day in 2016. Crude oil price as of today is being sold at \$65/barrel. As at 1979, Nigeria was producing about 2.5 million barrels of oil per day, which was sold at about \$40 (US) dollars per barrel. This gave us a daily receipt of about \$100 million (US) dollars. Technical problems and shutdowns for routine maintenance reduced local production. Nigeria is the world's sixth largest oil exporter and the leader in sub-Saharan Africa. Nonetheless Nigeria experienced an external trade surplus only from 1973 to 1975 and 1979 to 1980, during two oil price peaks (Okogu, 2007).

The oil glut came and suddenly Nigerians realized that they had left undone essential things which they ought to have done. Global inflation was running into over 400 percent with the devaluation of our currency while the high rate of smuggling of our products especially

petroleum products out of the country, gained sway. Unemployment became a common occurrence.

Beside oil, Nigeria had substantial reserves of natural gas. Although the consumption of natural gas increased steadily in the late 1970s, 1980s, and in 1990s, it constituted more than 20 percent of Nigeria's total energy from commercial sources, the quantity of gas used was only a fraction of what was available. In 1988, with the largest natural gas reserves in Africa, Nigeria produced 21.2 billion cubic meters per day, with 2.9 billion cubic meters used by the National Electric Power PLC (NEP, PLC) and other domestic customers. About 2.6 billion cubic meters are used by foreign oil companies, and 15.7 billion cubic meters (77%) wasted through flaring Nigeria Oil and Gas Company (NOAG, 2000).

Between 1980 and 1988 the average annual production of oil stood at about 0.50 billion barrels, and between 1989 and 1995 the annual production increased slightly to 0.70 billion barrels to the extent by 2003 it had come up to about 0.85 billion barrels per year, This means that by year 2000 oil as the main export accounted for more than 98% of the nation's export earnings and about 83% of Federal Government Revenue (NOAG, 2000). By year 2000 Nigeria's per capita income had plunged to about one quarter below the level at independence (Okogu, 2007).

If all the world's resources were easily accessible and development were not complicated by changes in demand, prices, costs and technology, the production (or depletion) of the world's resources would resemble a simple decline curve and would be mathematically simple to model. Obviously, depletion is considerably more complicated than this, however, production from oil and gas wells will generally follow a pattern of hyperbolic decline (Kamalu, 2010).

Hubbert (1956) earlier proposed a model for the extrapolation of finite resource production curves into the future. The model assumes that production levels generally begin at zero, before the production has started, and equally ends at zero, when the resource has been exhausted. In

between, the production curve normally passes through one or more maxima, and the actual shape of the production curves may vary but they are ultimately limited by the recoverable amounts of the finite resource. However, the recoverability is influenced by factors such as technology, accessibility, restrictions and economics, as well as geology. It is not possible to extract more petroleum than what is geologically available. And constraints imposed by technology, economics and other factors will in practice make far less than all the geologically available volumes available for production. Hubbert (1956) had initially proposed a bell shaped curve for idealized production behaviours, and representing various stages of maturity, without giving any exact mathematical description for it. Moreover, Bardi (2005) subsequently showed that mineral production will always result in a generally bell shaped curve except for very special assumption, but the shape was not necessarily symmetric. However, asymmetric models have been used for petroleum forecasts in many cases (Kamalu, 2010).

With regards to the reserve depletion phenomenon, this work proposes to provide predictive models and analysis of Nigeria's petroleum resources depletion. This was achieved by formulating several mathematical production and exhaustion (i.e. depletion) models, followed by detailed computational engineering analysis of the models by way of curve-fitting, evaluation of the constants, optimization considerations for the developed models and the computer-based calculation of the areas under the curves as the Nigeria total petroleum reserves.

1.2 Problem Statement

Following a global recession on world petroleum reserves, several scholars ((Hubbert, 1956); (Ivanhoe, 1995); (Laherrere, 2001); (Campbell, 2005); etc.) have at one time or the other attempted to alert the world or their nation of origin that danger looms in the future fossil energy supplies as this form of energy depletes exhaustibly. They have also attempted to forecast different countries and continents reserves limits, peaking and exhaustion periods. Strangely, the records of such data for Nigeria are limited.

The Nigerian economy as at 2015 is about 90% dependent on petroleum, to the detriment of other sectors because of false assumptions that petroleum will be forever, and that its depletion theory is unreal. However, contrary to this erroneous view, Nigerian petroleum is fast depleting at the rate much greater than its rate of accumulation.

Even government, without benefit of informed data on petroleum depletion, has commenced diversification of our economy from its oil dependence to non-oil sources and renewable energy.

The depletion crisis will affect, first the conventional and then the non-conventional petroleum. This is because when the conventional is exhausted, the world will need very advanced technology which many nations may not afford, to tap the non-conventional petroleum. And even if they do, the cost of production may be more than the cost of the petroleum itself and so uneconomical.

Against this background, the problem is to find when Nigerian petroleum will peak or had peaked in the past years, so that we can monitor the downward bumps of plateau to exhaustion. Since oil formation will give rise to gas formation or vice versa, the exhaustion of one will lead to the eventual exhaustion of the other with time. And in this lite, this study is poised to model and forecast the Nigerian petroleum depletion profiles, with the peaking and exhaustion dates.

1.3 Objectives of Study

The main objective of this research work is to predictively model and analyse the Nigerian petroleum resources depletion, using a composite underground reservoir.

The specific objectives are:

- (a) To develop mathematical model for the depletion, peaking and reserve of Nigeria petroleum resources,
- (b) To determine the solution of the developed model equations for predicting and forecasting Nigerian petroleum resources depletion using a composite underground reservoir,

- (c) To determine the peak and exhaustion periods for Nigeria's petroleum reserves from the established models,
- (d) To validate the models and carry out sensitivity analysis of the selected model,
- (e) To compare the developed models with existing petroleum resources depletion models from literature.

1.4 Justification of the Study

Oil is accounting for about 90% of Nigeria's annual budget, so it is important that such resource is studied as to guide us in planning and budgeting, and to know how long this unique resource can last. Also, in addition to existing knowledge on fossil fuel utilization and the investigation of the natural conditions which lead to oil decline, this study will help to know when to diversify to other natural resources such as non-fossil fuels, solid minerals and the like, to prevent the nation's economy from getting into ruin. Even for academics, the knowledge of the depletion, peaking, exhaustion dates and volumes will help guide further researchers in their quest in providing adequate information about this fossil fuel utilization and the future.

1.5 Scope of the Study

This work is limited to the evaluation of the performance of models developed to predictively model and analyze the Nigerian petroleum depletion by the use of a composite underground reservoir of Nigeria's conventional petroleum reserve both offshore and onshore.

The Nigeria petroleum ultimate reserve is dynamic, as more explorations and discoveries are always adding to the known ultimately recoverable pool, hence the new discoveries after 2014 (for both oil and gas) are not considered. Also excluded are other forms of natural resources like coal, wood and unconventional oil.

CHAPTER TWO

LITERATURE REVIEW

2.1 Research Concept

2.1.1 Oil Reserves, Origin and Formation

2.1.1.1 Nigerian oil and world oil reserves

The petroleum resources reserves held by a country usually constitute a strong determinant in the assessment of the economic potentials of that country and the rate of investment and long term economic planning. Oil reserve is the much oil known to be producible, within a known time with known techniques at known costs and in known fields. But it is not just the much oil in the rock in the earth; this is because the oil is categorized into conventional and non-conventional oil.

(1) Conventional oil: This is the amount of oil that rushes to the surface on its own. It has a characteristic depletion profile with production starting at zero and rising rapidly to one or more peaks before declining exponentially.

(2) Non-conventional oil: This is the amount of oil that cannot rush to the surface on its own. It is made up of heavy oil and tars. This oil is dependent on enhanced recovery techniques that change its fluid properties in the reservoir by such methods as steam injection. Example of non-conventional type of oil includes oil in hostile environments, and oil in accumulations, too small to be viable for exploration targets. It has a different depletion profile, rising only slowly to a long low plateau before eventually declining in the far future. Although the boundary between the two categories may be blurred, it is very important to make the fundamental distinction. Natural Gas Liquids (NGL) are separated from oil because they relate to the gas domain, but in practice are often lumped together, being in some cases pumped through the same pipeline and metered together (Merem *et al.*, 2017).

By the year 2000, Nigeria's proven crude oil and natural gas reserves were estimated at 25 billion barrels and well over 28×10^4 bscf respectively. Being OPEC member, in mid-2001 its crude oil production averaged around 2.2 million barrels per day. And as the reserves continued to increase, both due to new discoveries and politics, Nigeria's OPEC quota continues to rise to the fifth largest OPEC producer of petroleum in the world.

But new discoveries both offshore and onshore do not necessarily mean new petroleum formation in the earth. It simply means that either another small 'old' reserve has been detected or the 'former' large reserve has another place where it is closest to the earth's surface (Kamalu, 2010). However, Nigerians may not know that their petroleum recoverable reserves is seriously depleting; and any serious-minded patriot knows that if a large pool is constantly being scooped away without any replenishment, depletion and eventual exhaustion inevitably follows. The amount of oil and gas we get from the ground is about to reach the point of diminishing returns, that is getting to its peak and starts coming down forever, no matter how much ingenuity or determination to make it rise (Kamalu, 2010).

In most countries other than the USA and Federal Soviet Union (FSU), conventional oil depletion is evaluated on the basis of estimated mean reserves back-dated to the year of discovery. Both USA and FSU reported reserves need to be corrected to the mean value to be useful in forecasting (Kamalu, 2010).

Modeling future petroleum resources production or depletion is straight forward in countries not constrained by political policy, and that is a situation that does not exist so far within OPEC and FSU. And several methods can be identified on modeling future production or depletion. The creaming curve plotting cumulative means discoveries versus the cumulative number of new field. Yet another method is to plot the percentage of annual to cumulative production (or depletion) versus cumulative production (or depletion), and extrapolate the observed trend. Moreover, the future production (or depletion) can also be modeled with one or more bell-shaped

curves. The bell-shaped curve which was originated by K.M. Hubbert, a geophysicist, in 1956 to predict that U.S oil production eventually predicted that the US oil would peak in about 1970 and decline thereafter. He was rightly proved accurate because in 1971 the production of the 48 US lower states peaked and has remained on the down ward trend ever since. In other regions where severe exploration and production (E and P) episodes prevail, one has to use several such curves, each one related to its discovery pattern (Laherrere, 2002). Such a pertinent record for Nigeria is non-existent inspite of the strategic role of oil to the economy of the nation.

Oil production (or depletion) in the world outside OPEC and the FSU can be modeled with such a bell-shaped curve, despite the recent surge of deepwater production (Laherrere, 2002). And modeling the PSU petroleum is to recognize the overproduction of the late 1980s and the under-production during the 1990s that follow the collapse of the Soviet Union. What was under-produced in the 90s or under-developed in the Soviet era is what is now creating a new cycle. The regions mainly contributing will be East Siberia and Sakhalin as well as recent discoveries in the Caspian which together gave a peak of about 10Mb/d in 2010. According to Laherrere (2006b), modeling OPEC production (or depletion) is even more questionable, but it could peak in 2020 at between 40 and 45 Mb/d.

Combining the three crude models for OPEC, the FSU and the rest of the world gives a world ultimate recovery of 2200Gb with a peak at about 80Mb/d between 2010 and 2020. Non-conventional oil and other liquids, with an ultimate of about 800Gb, could peak in 2050 at around 20Mb/d. The combined model for all liquids gives a peak of 90Mb/d around 2015 (Laherrere, 2002).

As of 2009, world oil proven reserve as reported by the oil and gas journal was estimated at 1,342 billion barrels about 1% higher than the estimate for 2008. This proved reserve of crude oil according to international energy outlook (2009) are the estimated quantities that geological and

engineering data indicate can be recovered in future years from known reservoirs assuming existing technology and current economic and operating conditions.

However oil reserve include both crude reserves and other reserves with several reserve classification systems such as measured reserves, indicated reserves, probable reserves, inferred reserves and possible reserves (EIA, 2007). Apparently, other reserves are generally less well known and therefore less precisely quantifiable than proved reserves, and their eventual recovery is less assured.

According to oil and gas journal (OGJ), Nigeria had 36.2 billion barrels of proven oil reserve as of January 2007, while government plans to expand its proven reserves to 40 billion barrels by 2010. The majority of reserves are found along the country's Niger Delta in Southern Nigeria Guinea and Bight of Bonny.

Currently, Nigeria is the largest producer in Africa, the eleventh largest producer of crude oil in the world and has a potential of increasing oil production capacity to four million bbl/day by 2010. It is the eight largest exporter of crude oil, and a former major exporter to U.S.A, but currently to China.

The high rate of depletion of oil reserve worldwide is a cause for worry, as the Nigerian Economy is heavily dependent on the oil sector which according to the world bank, account for 95% export earnings and about 85 percent of government revenue.

Osahon (2014) said that Nigerian oil reserve had dropped from 40bn to 35bn barrels. This was attributed to reduction in oil production in the country. He said that some oil well in the Niger Delta had stopped production because they had attained "Maturity". He also said that vandalism and other unwholesome acts in the oil distribution process in the region accounted for the drop in production of crude oil.

He went on to say that; oil reserves are dropping and our output is dropping too what we are supposed to do to correct this, is to continue to explore and explore and explore for more oil. He

said that they started with 2D Seismic now they are at 3D Seismic and that 1,300 exploration wells have been drilled so far. Osahon (2014) said that we need to do more in this regard so as to have more reserves, continuing by saying that Nigeria has reached the plateau of production in the Niger Delta and we are already going down. He said that other things to do to shore-up the nation's reserves were, seismic data coverage and drilling of exploration wells, enhanced recovery methods, utilization of non-saddled reservoir and bitumen exploration.

According to Dune (2013), Nigeria and other OPEC countries' proven oil reserve figures from 1980-2010 are presented in table 2.2 shown below. Reserves figures of OPEC countries as of 2012 for Nigeria proven oil reserves are estimated at 37 billion barrels while that of natural gas is 5.246 trillion m³.

Table 2.1: OPEC share of world crude oil reserves as at 2012

OPEC country	Figure (billions of bbl)	% of total
Venezuela	297.7	24.8
Saudi Arabia	265.9	22.1
Iraq	157.3	11.7
Kuwait	101.5	8.5
UAE	97.8	8.1
Libya	48.5	4.0
Nigeria	37.1	3.1
Qatar	25.2	2.1
Algeria	12.2	1.0
Angola	9.1	0.8
Ecuador	8.2	0.7
Total =	1200.8	100

Source: (OPEC, 2013)

Two most influential scientific journals in the world, Nature and Science, have at some time also predicted the looming danger of dwindling oil reserves. A 1998 article in Science was titled “The next oil crisis looms large – and perhaps close” discussed by Kerr (1998); while a 1999 Nature article was subtitled “[A] permanent decline in global oil production rate is virtually certain to begin within 20 years” by Hatfield (1997). But contrary studies have adduced that the rate at which oil reserves are being utilized is less, if not equal to, than the rate at which it is being replenished as discussed in Sayne *et al.* (2015).

In cases where the theory of dwindling oil reserves has been categorically downplayed, Hubbert’s model of energy production has remained a source for reemphasis. The prediction by Hubbert in 1956 that oil production from the 48 contiguous United States would peak between 1965 and 1970 was accurate. Hubbert based his estimate on a mathematical model that assumes that the production of oil resource follows a bell-shaped curve—one that rises rapidly to a peak and declines just as quickly.

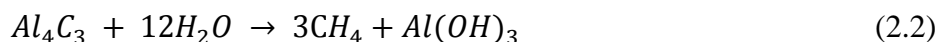
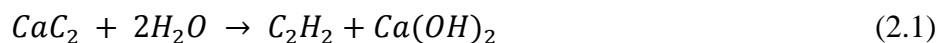
Nigeria comes as a case under study because of its current grandstanding in diversifying in emerging renewable energy - another energy form with similar abundant endowment and other viable but neglected energy sources coupled with the dire economic challenge inherent in its revenue generation. This approach is in contrast with the recent Hirsch report (Hirsch, 2005) which states “The peaking of world oil production presents the U.S. and the world with an unprecedented risk management problem. As peaking is approached, liquid fuel prices and price volatility will increase dramatically, and, without timely mitigation, the economic, social, and political costs will be unprecedented. Viable mitigation options exist on both the supply and demand sides, but to have substantial impact, they must be initiated more than a decade in advance of peaking.”

However certain limitations suffice in the study such as the preliminary difficulty in an unreliable database, exhibiting a wide range of estimates from different sources, with many

displaying brief internal inconsistencies. The definition of the various terms in wide use is another difficulty. In particular, the term reserve has been biased commercially, financially, and politically. Again, the estimates derived in this study cannot be passed off 100% because of unpredictable tendencies for oil reserves to grow by the application of current or emerging technology on the production of available resource. The rate of decline can vary from field to field, and this affects calculations on the size of the reserves. A further factor is the expected size of future demand for oil.

2.1.1.2 Origin and formation of crude oil

Scientists till now are entangled with the problem of explaining successfully the origin and formation of huge hydrocarbon deposits and notably a second theory has yet to be evolved. The ancient scientists were caught up with the idea of explaining reasons for such deposits. Their explanation was based upon the inorganic reactions, mainly on the activity of acetylene series. Some carbides produce hydrocarbons when reacted with water such as:



Assuming the availability of such carbides in earth's crust, they arrived at this axiom; in fact the deposits of such magnitudes could never be balanced with these ideas.

The cosmic hypothesis of Fujii & Yoshii (2011) depicts that the hydrocarbon vapors were already in cosmic clouds. Favorable conditions leading to precipitation of these clouds, rained hydrocarbons which were either absorbed or entrapped in earth's crust. A doctrine may possibly be explained if ever man comes across such giant deposits in a distant planet, where there happened to be no life at all.

Truly speaking, hydrocarbon vapors are present in certain planets. Oyedepo (2012) cited the presence of hydrocarbons in the atmosphere of Venus. These hydrocarbons must be distinguished from the early deposits, as purely derived from inorganic materials. Different and conflicting theories appeared in large number to explain the formation of petroleum deposits. The inorganic basis of petroleum formation had to be given up in favor of organic theory due to the following observations and facts as enumerated by Kamalu (2010):

- (a) The homologous series present in petroleum are found only in organic matters.
- (b) Nitrogen compounds in petroleum, especially plant derived porphyrins, comprise a very small amount but their significance is immense in the formation of petroleum from life source.
- (c) Optical activity, a pre-requisite of natural organics, is exhibited by petroleum.
- (d) Petroleum formation is a result of low temperature activity only.
- (e) Petroleum is always associated with sedimentary rocks, (even recent formations too) and not with igneous rocks.
- (f) Small quantities of petroleum (hydrocarbons) in recent sediments suggest, that the formation of petroleum is normal, continuous and does not require any server physic-chemical conditions.
- (g) Most organisms, like diatoms are found in petroleum.

Petroleum and other hydrocarbons were for long known to mankind since the beginning of civilization. The word petroleum derives from the Latin words *petra*, meaning rock, and *oleum*, denoting oil, which when combined literally means rock-oil. And this term was first used by Georgius Agricola, a German mineralogist. The ancient Greek word *naphtha* was often used to describe any petroleum or pitchlike substance. In older texts *naphtha* was often used as a synonym for petroleum, but this has now been phased out in English language. However, some languages, such as Russian or Arabic, still use variants of *naphtha* as the word for petroleum.

Often petroleum was treated as a curiosity without any serious use to society, although it was widely used in some parts of the world prior to industrialization. The eternal fires of Kirkuk, consisting of burning oil seepages, and the use of oil and natural bitumen in ancient Middle East has been studied by Beydoun (1997). The Roman Empire has been known to use coal for household heating in Britannia (Dearne & Branigan, 1995). The dreaded Greek fire, an early incendiary weapon used by the Byzantine Empire, is believed to have been partly based on petroleum or naphtha (Partington & Hall, 1999). Speight (2011) covers the history of petroleum utilization and terminology in greater detail. In many places all over the world where oil seepages occurred, local inhabitants drilled or dug surface wells and shafts to increase flow rate for collection purposes. These were the first hint of what would become the great rush for petroleum of modern times. In these early days, how petroleum was formed was not known and the knowledge of geology was far from sufficient to propose reasonable theories. However, scholars and philosophers nevertheless tried to explain the genesis of petroleum. The ancient Greeks were familiar with the existence of petroleum and some of their natural philosophers discussed its origin. Aristotle and other Greek philosophers proposed a theory based on the idea that that everything was made up from four different elements; earth, air, water and fire. Since petroleum seeped out of the ground and was very flammable, it was thought to be connected to fire and earth. Consequently, Aristotle thought that ores, minerals and hydrocarbons were the result of exhalations from the deep of the earth (Walters, 2006). His followers suggested that the foul smell that was typical for many forms of petroleum indicated that it was related to sulphur in some way.

Medieval development was chiefly driven by Arabic science. Several important developments in chemistry and alchemy were done by Arabian and Persian scholars. In the 8th century, an Arabian polymath, Abu Musa Jabir ibn Hayyan al Azdi, developed the foundation of chemistry and paved the way for many subsequent alchemists and scientists (Derewenda, 2008). Abu Bakr

Muhammad ibn Zakariya Razi was the first to distil petroleum and produce kerosene around 9th century. These experiments later inspired many European alchemists and scholars as manuscripts and books reached the medieval Europe.

Alembics, retorts and primitive chemical analysis were used by many alchemists to uncover the secrets of nature and provide better understanding of the world. However, few references to petroleum and its origin can be found in the literature from this time. Roger Bacon even made remarks about the inability of Aristotle and other Greek natural philosophers to adequately explain the origin of petroleum in his treatise *Opus Tertium*. It was not until the Renaissance that the first modern theories about the origin of hydrocarbons were developed. In 1546, Georgius Agricola in his manuscript *De Natura eorum quae Effluunt ex Terra* expanded on Aristotle's ideas about exhalations from deep underground and proposed that bitumen is a condensate of sulphur. Andreas Libavius in his chemistry textbook *Alchemia* of 1597 proposed that bitumen was formed from resins of ancient trees (Walters, 2006). These texts may be seen as the origin of the modern theories about oil formation. The Russian universal genius Lomonosov proposed and demonstrated in his book *On the Strata of the Earth* from 1763 (Peters, Walters, & Moldowan, 2005) the idea that petroleum and natural bitumen originates from transformation of coal and plant remnants due to subsurface heat and pressure. Already by 18th century, fossil evidence had indicated that coal and peat was related and that both originate from preserved vegetation remains. Remains of leaves and plant parts can actually be seen in certain coal and peat samples. Today, it is commonly accepted that coal has a biogenic origin and the modern coal geology is well documented, for instance, in Thomas (2002).

Whereas the origin of coal can be considered a settled question already in 18th century, the origin of petroleum remained an open debate for a long time. More detailed hypotheses and different versions of the biogenic origin of petroleum were created during the 19th century.

Several studies claimed that oil originated directly from organic remains or was created in a distillation process.

Meanwhile, the first seeds of the abiotic oil formation theory were planted in Europe. A French chemist, Marcellin Berthelot, described how hydrocarbon compounds could be created from the acid dissolution of steel (Walters, 2006). In Russia, the famous scientist Mendeleev proposed that petroleum was created in the depths of the earth from chemical reactions between water and iron carbides in the hot upper mantle. This theory was largely ignored and its supporters dwindled under the mounting evidence of biogenic petroleum creation at this time.

The beginning of the 20th century marks the development of modern petroleum geology. Early European studies of organic-rich rocks supported the biogenic origin of oil. Similar studies by the US Geological Survey (USGS) showed that Californian oil originated from organic-rich shale.

Iledare (2007) showed a connection between thermal properties in the ground and petroleum, when he produced hydrocarbons by heating organic matter. According to him, the carbon-ratio theory has been introduced in the early 1900s, which implied that petroleum resources occurrences are limited by the thermal history of a region. So, in essence, studies started to connect petroleum generation to some other naturally occurring distillation process of organic-rich sediments. Also, the birth of more advanced chemical analysis led to many other new discoveries that shed additional light on the origin of petroleum.

The connection between chlorophyll in living organisms and porphyrin pigments was established in 1934, a category of nitrogen compounds which originates from mainly chlorophylls, in fossil resources like petroleum, shale and coal. And it has also been shown that oils retain fractions that are optically active, just like biological matter. Also, it was shown that oil contains many chemical fossils and biomarkers, besides porphyrins, which could be traced to biological predecessors. Some founders of modern geochemistry however expanded the analysis with

geochemical fossils, allowing further comparison of structurally similar organic compounds in sediments and oil with their precursors in living organisms. Durand (1980) described a more comprehensive overview of the development of petroleum chemistry. The introduction of mass spectroscopy allowed new types of analyses that had the capacity to identify differences in isotopic composition.

It was found that stable carbon isotope compositions in oil were in line with biogenic origin as living organisms favour certain carbon isotopes than others. Tang *et al.* (2000) applied the carbon isotope studies to both petroleum and sedimentary organic materials and confirmed their biogenic origin. They presented more detailed modeling data for stable carbon isotopes. Moreover, geological field studies provided useful knowledge and valuable information relevant to petroleum genesis. Hydrocarbons were found to be abundant in ancient marine shales and limestones, and were uniformly absent in recent muds from a variety of environments. However, studies on rock samples from a wide variety of ages, lithologies, environments and depositions, revealed that the absolute concentration of hydrocarbons were greater in ancient rocks than in much younger sediments. The findings showcase the important role of kerogen in petroleum generation.

Hunt *et al.* (2002) opined that hydrocarbon yield from source rocks increase with both time and temperature. These studies gave the understanding of the geologic zone of intense oil generation, now known as the oil window (Hunt *et al.*, 2002).

Ahsan *et al.* (1997) reviewed how microorganisms could alter petroleum and cause biodegradation, resulting in heavy oil. They pointed out the importance of this process and related it to the discovered quantities of heavy oil prior to 1980. Ahsan *et al.* (1997) provide additional information on biodegradation. Theories and models on degradation and alteration could now explain the formation of all forms of petroleum within the biogenic framework.

Kenney *et al.* (2002) found that starting in the 1950s, several researcher's publications have already proposed a modernized version of Mendeleev's theory, relying on thermodynamic equilibrium for chemical reactions that basically allows spontaneous formation of methane at a high temperature and pressure, comparable to those of upper mantle region. The authors favouring the abiotic theory claim that the hydrogen-carbon system generates hydrocarbons under pressures found in the mantle of the earth and at temperatures consistent with that environment (Kenney *et al.*, 2002). There exists experimental proof that, in some conditions of high pressure and temperature (e.g. under a diamond anvil) it is possible to combine carbon and hydrogen to generate hydrocarbons (Kenney *et al.*, 2002).

Because many of the Russian-Ukrainian works were not translated and spread among the western scientists, perhaps the most well-known promotion of recent abiotic theory is likely the work of Gold (1999). J.F. Kenney, who served as the drilling manager of Gold's Siljan Ring project, also made significant efforts to spread the abiotic theory of petroleum formation. Kutcherov and several others amongst the Russian scientists have co-authored many papers with Kenney in their attempt to revitalize the Russian-Ukrainian theory and spread it within the scientific domain. The Russian-Ukrainian theory, Gold's ideas and some others was reviewed in a much greater detail by Glasby (2006).

The proponents of the abiotic theory also often claimed that hydrocarbons cannot be produced anywhere near the surface due to chemical constraints imposed by the thermodynamic second law. However, the claim ignores the fact that all life is in thermodynamic disequilibrium with its environment (Walters, 2006). And in some cases, it was clearly claimed that carbohydrates could be used as precursors being "typical biotic reagents" (Kenney *et al.*, 2002), even though others stated that neither proteins nor carbohydrates have been thought to play any role in petroleum formation for the last 40 years. Gold (1985) was influenced by Hoyle's idea and developed his own model. He stated that mantle methane is continuously injected into the crust of weak areas

such as the lithosphere's plate boundaries, ancient suture zones, and meteorite-impact sites. Under conditions of slow upward migration and cooling, some of this methane is assumed to polymerize and react in Fischer-Tropsch-analogous reactions resulting in longer hydrocarbon chains and crude oil of higher molecular weight.

Potter *et al.* (2004) also gave an overview of the Fischer-Tropsch synthesis related to abiotic petroleum, and Wang *et al.* (2008) provides an overview of abiogenic natural gas. More so, Gold (1999) later modified his hypothesis and he suggested that both coal and crude oil originate from mantle gas flows that feed bacteria living at extreme depths.

Nevertheless, Gold and others who support the abiotic theory viewed the drillings as a scientific success. As at 2009, permission for new drillings in the Siljan area was given to a company in collaboration with the researcher V.G. Kutcherov to test their improved version of the abiotic oil formation theory and where commercial amounts of abiotic petroleum could be located.

Studies of rocks from several places of the world shows that mantle hydrocarbons can be found widely, though the content of such hydrocarbons in mantle derived rocks are extremely low (Gold, 1992).

Hulston *et al.* (2001) reached a similar conclusion from an investigation of the Taranaki Basin in New Zealand. Globally significant amounts of abiotic oil in the crust can be ruled out. Other studies have managed to discover or produce trace amounts of abiotic oil.

From the available review of the situation it is seen that it is possible to create abiotic oil in some laboratory conditions of high pressure and temperatures and that minor amount of abiotic hydrocarbons may be created in the mantle. However, commercially interesting accumulations have never been found (Walters, 2006), nor has any oil ever been reported along major faults in continental shield areas where sedimentary rocks are not present (Peters *et al.*, 2005). Jenden *et*

al. (1993) concluded that the abiotic content of commercial natural gas is less than 200 ppm and that little confidence should be placed on the resource potential of abiotic natural gas.

Durand (2003) highlights how proponents of abiotic petroleum were generally chemists with very little geological knowledge. One example is how the Russian-Ukrainian school saw evidence of their theory in the fact that some oil reservoirs exist in non-sedimentary rocks such as granite, metamorphic or porous volcanic rocks. However, all the studies conducted by geologists demonstrated that these rocks served as reservoirs for oil expelled from a sedimentary source rock not far away through common migration or re-migration mechanisms. Somehow those results were never integrated into the abiotic theory. Durand (2003) also points to the media as a factor behind the apparent popularity of abiotic petroleum theories in public debates, despite the underlying shortcomings and lack of holistic treatment.

Kutcherov (2008) claims that there are a number of geological data, such as the giant and super giant petroleum deposits, can't be satisfactorily explained in the established biogenic petroleum theory. He reached the conclusion that only about 6% of the Saudi in-place reserves could have originated from a migration from the petroleum source rock. However, most of the studies were based on very old techniques for estimating the amount of oil generated by a source rock. Again the introduction of Rock-Eval methodology and new studies, such as Laherrere (1999) arrived at significantly higher estimates. The source rocks in the petroleum system would have generated more than 70 times the known in-place reserves of Saudi-Arabia, with very large amounts dispersed in the sediments as compared to the recoverable resources. In other words, no abiotic oil is needed to explain the in-place discoveries in Saudi-Arabia if only recent and updated geological studies are acknowledged. And indeed it is surprising to see Kutcherov (2008) refer to an old paper from 1982, meanwhile there are many more recent research papers.

Another argument presented by Kutcherov (2008) and some other proponents of the abiotic petroleum origin is the occurrence of petroleum deposits at great depths. Quite a number of oil

discoveries have been made at depths between 5000 and 10 000m, but the vast majority so far has been located less deep. The world inventory of oil fields, excluding USA and Canada, is about 14,000 fields with total reserves of 2000 billion barrels, but only less than 500 of them are located deeper than 5000m. It is therefore clear from the empirical discovery data that less oil has been found in deep horizons. This is as expected from the studies on temperature and its importance for oil occurrence.

Isaksen & Treffinger (2004) reviewed that temperature is the most sensitive parameter in hydrocarbon generation. About 90% of the world's oil and gas are found between the 60⁰C and 120⁰C isotherms, i.e. the golden zone, a clear trends show decreasing reserves with increasing reservoir temperature (Statoil, 2005). The maximum temperature where oil can be found is also an important topic. The geochemical approach of the 1980s were based on vitrinite reflectance and found that around 200⁰C was the maximum temperature for oil to be preserved and not transformed into gas. Isaksen & Treffinger (2004) and Hao *et al.* (2008) all showed how oil was cracked into natural gas at temperatures greater than 200⁰C that typically occurred at depths greater than 5000m. There are also recent claims that oil can be stable up to temperatures of 240 - 260⁰C (Domine *et al.*, 2002), which can explain occurrence of oil at more extreme places than previously thought possible.

More so, scientific support can be found for both the biogenic and abiotic theory of petroleum formation. However, theoretical studies, laboratory verification and discoveries of trace amounts of abiotic oil do not necessarily imply the existence of commercially significant deposits. The observations and actual discoveries that are made by geological studies will have to be integrated in any sound theory of petroleum formation. We cannot chose to disregard certain seemingly details or fail to integrate new results in a theory.

Petroleum geology and exploration is an empirical and pragmatic field that somehow has evolved by trial and error. Geologists and oil companies have learned where to drill and where

not to, and hence, developed a theoretical model that works and corresponds to reality. The developed model is consistent with the biogenic theory of oil creation.

In the real world, the “truth of the borehole” speaks louder than any theory, since it provides actual production flows. Needless to say, the drilling done in line with the biogenic theory of oil formation has resulted in a vast amount of oil that has been of benefit to mankind since the beginning of the oil era. This has been verified by geochemical and geological studies that are accepted by a broad majority of researchers and experts within the scientific establishment and the petroleum industry.

Modern petroleum geology does not deny the existence of commercial amounts of abiotic oil. The abiotic theorists would surely win more credibility if big oil fields could be found supporting their theory. Glasby (2006) highlights how researchers have been unable to cite any commercial occurrence of abiotic petroleum, despite a long list of promising places such as the Siljan Ring, offshore Vietnam, Eugene island in the Gulf of Mexico or the Dnieper-Donets Basin. In fact, the petroleum resources of those regions can be explained entirely within the biogenic framework (Glasby, 2006).

2.1.1.3 Peak oil and oil formation

The theory of peak oil is backed by phenomenological evidence, including geology, fluid mechanics, reservoir physics, statistical physics, economics, and actual observations. It is different from some peak oil disbelievers, who argue for the hypothesis where natural laws can possibly be mended or broken once the monetary price is high enough or where sufficiently strong political reasons exist.

Claims about a non-geological production peak due to factors like lack of investments, lack of access or similar factors can and should only be seen as agreement with the peak oil theory, since peak oil is about production flows not being able to meet demand, irrespective of what the cause may be. Ordinarily, it can be discussed when in time the peak will arrive, but the important

concept is the inescapable arrival of a production peak sooner or later. Therefore having this understanding is crucial for planning and necessary to be able to tackle the challenges that nature imposes on future development. Before going into details about what peak oil theory actually claims, it is necessary to define certain parameters. The term “finite resource” is frequently used but few people seem to ponder what it actually means. When it comes to natural resources, we argue that production limits are determined by the extraction and creation rates. If extraction of a resource is faster than replenishment rate the resource will be “finite” in the sense that it will eventually be exhausted. For instance, whales and their blubber oil were a finite resource. The whales were on the verge of becoming extinct despite their “renewable” nature prior to the use of crude oil (Bardi & Yaxley, 2005). The case of whaling in 19th century is an excellent illustration of a complete Hubbert cycle, where a resource has been extracted at a much faster rate than it could be replaced.

Whale oil and whale bone production show a very clear “bell shaped” curve. The main species being exploited at the time (right whales) was reduced to nearly complete extinction, with an estimate of only 50 females left alive in the ocean. One could even claim that whales were saved by the discovery of oil and the switch to petroleum-derived products. In addition, forests can be “renewable” if the annual outtake is no more than the annual growth. There are examples of how overextraction of wood fuels have turned renewable forests into finite resources that resulted in eroded, deforested areas without any usable wood fuel production, effectively depleting the resource (Johansson, 2007). Peat and coal are also finite resources as they call for thousands or millions of years to generate layers of only a few millimeters, while extraction can cut layers of many meters per day. Uranium is also a finite resource as it is originally created from the ashes of supernovas, requiring many millions or billions of years to accumulate commercially interesting concentrations (World Nuclear Association, 2006). And according to the standard, biogenic oil formation theory, millions of years are required for source rocks to be transformed

into petroleum and then accumulate in reservoirs while the reservoirs are drained within a few years or decades in most cases.

Hubbert (1956) proposed a model for the forecasting of finite resource production curves into the future. His model assumes that production levels begin at zero, before the production has started, and also ends at zero, when the resource has been completely exhausted. Between, the production curve passes through one or more maxima. And the actual shape of the production curves may vary, but then they are ultimately limited by the recoverable amounts of the finite resource in question. Normally, resource recoverability is influenced by technology, accessibility, restrictions and economics, as well as geology. It is however impossible to extract more petroleum resources than what is geologically available. The constraints imposed by technology, economics and some other factors will in practical reality make far less than all the geologically available volumes that are available for production. Hubbert (1956) initially proposed a bell shaped curve for an idealized production behaviour, representing various stages of maturity, without giving any exact mathematical description for it. Afterwards, he used the simple logistic function due to its mathematical tractability as well as simplicity. However, Bardi (2005) subsequently showed that mineral production will always result in a generally bell shaped curve save for very special assumptions, but then the shape was not necessarily symmetric.

Peak oil theory does not in any case say that decline will necessarily be exponential or symmetric (just as logistic curves do). The peak oil theory is a broad family of techniques and curves for forecasting future production where the core is analysis of production flows, and whether supply can meet a given demand or not. Cambridge Energy Research Associates (CERA) and their discussion of an undulating plateau instead of a distinct peak remains a case of “peak oil theory,” even though somewhat more concealed behind veils of accessibility and economical terms.

Colin Campbell and the Association for the Study of Peak Oil & Gas (ASPO), used the following definition: “the term peak oil refers to the maximum rate of the production of oil in

any area under consideration, recognizing that it is a finite natural resource, subject to depletion.” However, it does not state anything about which specific oil formation theory that is used nor does it clearly imply that only geological constraints are regarded or that future production is supposed to follow the Hubbert curve (i.e. the derivative of the logistic function).

Finiteness in this context is the key factor and is a function of the extraction rate as compared to the generation rate. The formation of a commercially interesting oilfield requires petroleum generation, migration, accumulation and entrapment with processes that take much longer time compared to the extraction of the recoverable oil by human activities. In a broader context, resources that are being consumed faster than they are replenished will evidently be subject to depletion. Depletion and peaking can then be modeled in many ways but the fundamental and notable property is that there is some form of limitation on production imposed by nature.

Bardi (2009) have explained the peak phenomenon on the basis of a simple theory based on the well-known Lotka-Volterra biological model. The basis of the theory is that, initially, the extraction of an abundant and cheap resource leads to economic growth and to increasing investments in further extraction. Gradually, however, the cheap resources are depleted and extraction costs become higher because of the need to extract lower quality deposits. In time, investments cannot keep pace with these rising costs; the growth slows down and, eventually, production starts declining. Here, “costs” are to be understood in monetary terms as well as energy costs, which grow for physical reasons related to the lower concentration and or lower quality of the resource. In other words, what creates the bell curve for an energy resource like oil is the variation with time of the net energy of extraction, also known as Energy Return on Energy Investment (EROEI). In the case of oil, the EROEI effect is enhanced also by physical factors related to the fall in reservoir pressure and also with the fact that less and less oil-bearing reservoir is in touch with the wells as the oil is progressively extracted (Bardi, 2009). It shows the historical trends of oil production in the US 48 lower states fitted with the theoretical

LotkaeVolterra model. The agreement shows that simple concepts from biological systems, such as predator-prey-relations without reproduction of the prey, can explain peaking behaviour in natural resource extraction.

According to Bardi (2007), abiotic oil formation theory changes the practical finite nature of petroleum and the behaviour that leads to production peaks, and this can be seen in two possible cases: a “weak” and a “strong” abiotic petroleum theory.

The “weak” abiotic oil theory: oil is formed abiotically at rates not usually higher than those that petroleum geologists assume for oil formation according to the conventional biogenic theory.

The “strong” abiotic theory: oil is formed at a speed sufficient enough to replace the oil reservoirs as we deplete them, that is, at a rate someworth like 10,000 times faster than known in conventional petroleum geology. In some cases, this version of theory claims that there exist true “oceans of oil” deep within the earth.

The weak abiotic theory does not change the fact that we are extracting oil at a rate much faster than it is being generated. So for that reason, it does not change the fundamentals of peak oil theory, nor does it remove the principle of a production peak at some point. As a matter of fact, one can easily swap biogenic petroleum formation theory for weak abiotic oil genesis, yet deriving the same type of arguments for a coming peak since it is production flows that matter. Mankind is still using oil at a much faster pace than it is regenerated by nature, and sooner or later the oil reserves will be depleted and force the onset of decline and peak production, regardless of biogenic or weak abiotic oil formation theory.

Strong abiotic oil genesis is however a completely different topic, as it would imply that oil is formed naturally at least as fast as we are currently extracting it or may be faster. In the most convenient (from a human viewpoint) version of the theory, the wells that are being exploited today are all connected to ultra-deep reservoirs that slowly (or rapidly) refill them. In this version of the theory, it allows for a virtually endless plateau production at some kind of level of oil

consumption. However, at some point production will eventually match the formation rate and, consequently, limit oil utilization.

Therefore, the arrival of a maximum sustainable production level is still valid but would be a plateau rather than a peak. This said occurrence would likely postpone the “end of oil” far into the future causing it to occur only when futuristic energy sources ought to have been invented.

It then goes without saying, however, that even the most enthusiastic supporter of the abiotic oil theory is not able to show any evidence for the “strong” version of the theory, at least in the research papers published in scientific journals. It is another matter in the popular press, where wild claims have often been made. Such claims are more like the result of wishful thinking than any quantitative evaluation based on sound scientific methodology.

2.2 Theory of Petroleum Resources Depletion

2.2.1 Depletion of petroleum resources

Ite, *et al.* (2016) while reviewing the work of Bartlett (1999) stated that “we live in the most energy intensive civilization the world has ever known and yet the average American knows nothing about energy”. They observed that things might get more critical before the public would be ready to hear the truth and that the truth might even piss everyone off when told. The truth, he said, may be in the form: “There is not enough oil in the US for self-sufficiency” or “wind and solar cannot meet 100% of our energy needs”. In the same manner, it will be the realization of bad dreams by fantastic Nigerians who erroneously believe in the everlasting of petroleum reserves.

In a more recent study, Salako, Sholeye & Ayankoya (2012) posed some questions and answers on oil peaking:

- When will United States oil production reach its peak? 2010? 2040? It happened in 1971.
- When will the world-wide volume of new oil discovered annually peak? It happened in 1960,

- When will world oil production peak? Between 2005 and 2010, depending on world economic growth rates, what happens then?

We move from a demand market to a supply market; with world economies bidding against each other for finite oil sources. It has already started.

- When will we reach one half of current world oil production?

Could be as early as 2035, then we will have added another three billion people who want to consume oil to the planetary load. (Think on what one-half current production means for the world economy). Continuing, Delaney said the amount of oil we can take from the ground each year is about to reach a limit that cannot be increased by ingenuity or development. That limit will then decrease forever, with short plateaux and little upward bumps on the way down. A similar limit for natural gas will follow in a few years. The act of taking oil or gas from the ground is called production. Since the start of oil production in the nineteenth century, the world has produced close to half of its ultimately recoverable oil resource. At the half-way point, the world will achieve its production peak - more oil will be produced in a year near the half-way point than ever before or ever after.

According to Akuru & Okoro (2011), the foregoing statement follows from a regularity of oil production statistics first observed by Hubbert (1982). If we consider a large enough number of oil fields, their aggregate production rate will display a characteristic curve. The group of fields will reach a peak rate of aggregate production when half of the ultimately recoverable oil has been produced. After the halfway point, the physical properties of oil reservoirs and the variation of the dates of discovery of the fields will cause the aggregate rate of production to slow progressively until all recoverable oil has been produced. Hubbert used this regularity of oil field statistics to predict correctly that the “lower 48 United States would produce half of their

ultimately recoverable oil resource by 1972, and that production from the lower 48 States would, consequently, peak in 1972”.

Walsh (2000) forecasted oil production by comparing discovery records with production records, and by exploiting the principle that oil cannot be produced until it has been discovered. A figure taken from Laherrere (2001) on forecasting future production from past discovery predicts a startling rate of decline of supply of oil between 2010 and 2050. His analysis was based on the energy need for petroleum resources and their diverse use in transportation, household and industry (Oyedepo, 2012).

As oil begins to decrease forever, transportation and then, economies would dwindle. In 1999, 69% of all US petroleum consumed was for power and transportation. Natural gas which would have taken the place of oil for transportation is critically dependent upon the source of energy for industrial processes and as a feed stock for many synthetic materials. Natural gas provided 44% of all energy used for households in 1999, and 42% of all energy used for industry and approximately 10% of all electricity (Walsh, 2000).

What Walsh failed to do however was to constructively predict peaking and exhaustion period for the US petroleum reserve as at the time of his analysis.

Walsh (2000) also applied parabolic techniques to this problem but on a quite different basis: where-in data from geological assessment of the resource-base was applied, and which does not display them as with the Hubbert-type parabola. A staged technique was devised to distribute the as-yet unproduced oil in parabolic form. This method is forward-looking in that past production is deducted from the geological assessment of the ultimate recoverable resource to determine the quantity of as- yet unproduced oil. This latter value is itself the sum of the known established reserves and the estimate of remaining undiscovered resources. The production - time pattern of the past is irrelevant in this methodology, an important advantage when the world production is

to be projected, given the severe dislocation that took place at the time of the oil crisis (Greene *et al.*, 2004).

Hirsch (2005) stated that the dire effects of petroleum depletion comes as economic, social and political demands premeditate mitigation efforts supposedly initiated far ahead of the time before peaking occurs. He went on to say that energy deficit, scarcity of liquid fuels, high oil prices and certain economic hardship are just few of such adverse effects anticipated in Nigeria for instance. He however gave no insight regarding the possible determination of the depletion rate, viz-a-viz peaking and exhaustion periods.

The failure of mainstream economists to acknowledge what is about to happen can be explained ideologically. The damage the post-peak decline will cause, will be mitigated by government direction of markets. New or restored regulation of markets by governments is offensive to free market dogma. Since the free market fundamentalists are on the driver's seat and reflexively deny the reality of problems that need collective action, only an economic crash will unseat them (Delaney, 2002).

However, Ojotaroeno (2010) reported that when the peak is surpassed then we will progressively have less energy at progressively higher prices, a rule that contradicts the law of demand and supply.

Energy is different from other commodities because nothing can be moved, built, manufactured, planted, fertilized, harvested, or mined without the liquid transportation fuels that petroleum provides so well and for which no competitive replacement has been ever found. Hence, new energy sources require large investment of energy itself.

Table 2.1 shows the predicted peak years, predictors, and regional production of conventional oil for the entire world, USA and Non-OPEC. Amongst the calculative predictions in table 2.1, Hubbert's has come true in 1971. However, some controversies still surround the estimates while some economists doubt the ever peaking of oil production.

Table 2.2: Predicted peak years, predictors and regional production of conventional oil

Predictor	Regional Production	Predicted Peak Year	Source
Walsh H. John	World Production	2015 ≤ Peak ≤ 2020	(Walsh, 2000)
M. King Hubbert	U.S.A	1970 ≤ Peak ≤ 1972	(Hubbert, 1959)
Duncan, Youngquist	World Production	2007	(Duncan, 1999)
Campbell Laherrere	World Production	>2010	(Campbell, 2005) (Laherrere, 2001)
International Energy Agency	Non-Opec	2010	(EIA, 2007)

2.2.2 Petroleum peak and the Nigerian context

In Nigeria, we are not unaware of long queues of automobiles at filling stations in all cities sometimes ago. The problem has been variously attributed to the refining capability, technology to do it, attempting to be at par with world petroleum price and, not reserve depletion question. It is thought that Nigeria imports refined petroleum not because her reserve is depleting, but, it will soon clearly dawn on her that it is the main reason.

According to Akuru & Okoro (2011) in their work entitled “Prediction of Nigerian oil depletion”, they agreed that petroleum oil would certainly reach a maximum peak and then begin to decline. They were not certain if Nigerian Oil had peaked or was on a downward trend. However, their finding shows the imminence of Nigeria exhausting its proven reserves notwithstanding the fact that it has grown dramatically in the last decade. Also, they did not pay attention to the factors that could help in the determination of the peaking, depletion and exhaustion of Nigerian petroleum resources.

It is important to get governments to accept that radical change will happen before proposing detailed solutions. Our society, perhaps our race, is known for formulating problem statements without the solution statements. Progressive adaptation will come from the collective imagination and experience of thousands or millions of people responding to a clear threat, by

governments to stop this natural phenomenon. In fact, their inaction will make the coming disruption of the world's economy much worse than it needs to be.

But how predictable is the oil peak? As the oil is discovered, the production output from a conventional reservoir follows a cycle of development involving steadily rising profile to reach a stable plateau, after which there is a long, slow period of decline. At this point of decline, if prices are high enough, more expensive enhanced oil recovery techniques may be justifiably involved which extend the life of the reservoir, a measure which affect neither the timing nor the magnitude of the peak seriously (Deffeyes, 2007). Even the combined output of a cluster of reservoirs shows a tendency for their joint production to rise to a peak and then fall. A curve of a parabola represents this pattern. Hubbert (1956), Duncan (1999), Youngquist (1999) and Akuru & Okoro (2011) have all projected oil production by parabolas determined in different ways, but based upon a common approach. In these works, the past pattern of production over time is used to predict the future. The ultimate recoverable resource is determined from the area of the parabolic curve so defined.

Campbell (2005) was of the opinion that the would-be petroleum products prices during and after depletion crisis can be used to explain the then dynamics of depletion phenomena and scenario. With normal discount rates, the earliest prices will be affected by a peak in production of conventional oil occurring between 2015 and 2020. Before this time, there will be non-depletional influences affecting the peak. But if price tend to increase regularly earlier than this date, it may be an indication that the peak is closer than estimated here (Campbell, 2007).

There is also the political aspect of the oil crisis. In fact, according to Davey (2002), when her oil production peaked in 1971, America helped to set up Islamic dissidents all over the world just to make sure she played the then USSR out of the energy game, but these fanatics are, today, terrorists on America and Israel as USSR crumbled. Yes, America has been "liberating and

smearing democracy on” Afghans, Iraqis, Kuwaitis, Saudi Arabians, and, perhaps, Iranians and Libyans in her oil politics to augment what she has at home(Greene *et al.*, 2004) .

Most of the recent Nigerian new oil discoveries are not actually discoveries but political gimmick to enable her secure bigger OPEC production quota. The underlying truth is that Nigerian oil reserve is irreversibly fast depleting.

The world profligacy in fossil fuel consumption of the tone of 80 million barrels per day will decline with the declining rate of world supply and the economic and military dominance of world powers. After all, if there is no “energy” in the guns they would not shoot.

2.2.3 Technical considerations

According to a petroleum expert, (Avuni, 2013), in his paper entitled “Policy and activity in the Nigerian petroleum sector presented at a technical meeting of National Association of Petroleum Explorationists (NAPE), he stated that Nigeria has the lowest reserve and production ratio for crude oil among Organization of Petroleum Exporting Countries (OPEC). Avuni said that the current oil reserve estimates in Nigeria stand at about 35 billion barrels. Average yearly reserves addition in the last 10 years is about 800 million barrels, while average yearly withdrawal rate over the same period is about 840 million barrels. This seems to suggest that our oil is fast depleting. The situation, of course, gets worse as we strive to attain a target daily production rate of four million barrels. What Avuni did not make us understand is how we could validate his assertion based on predictions and with regards to his estimates.

Ojotaroeno (2010) while reviewing the work of Walsh (2000), the future for the fossil fuels, analyzed the question of the peak in the production of the world oil. He looked at the special attributes of oil, the current surplus in oil production capacity, why the timing of the peak in world oil production is important, the economic policy response of higher oil prices, how the peak in the world production of oil can be predicted and how soon the peak in conventional oil production will affect prices. He noted that after discovery, the output of oil from a conventional

reservoir follow a cycle of development involving steadily rising production to reach a stable plateau, after which there is a long, slow period of decline. If prices are high, more expensive enhanced recovery techniques may be justified which may extend the life of the reservoir but in general cases, do not affect either the timing or the magnitude of the peak very greatly. A number of such reservoirs taken together therefore will follow the same pattern. This pattern he said could be described mathematically by a parabolic curve. Again, he only ended with the mathematical nature of the phenomenal description.

As the work of Delaney (2002) was reviewed by Halvorsen & Tim (2005), they observed that the amount of oil that can be taken from the ground each year was about to reach a limit that could not be increased by ingenuity or determination. The limit will then decrease forever, with short plateaus and little upwards bumps on the way down. A similar pattern for natural gas, he predicted, will follow shortly after that of the oil. According to him, since the start of oil production in the nineteenth century, the world has produced close to half of its ultimate recoverable oil resources. At the half way point, the world will achieve its production peak and more oil will be produced in a year near the halfway point than ever before or ever after. The above statements are in line with the observations of Campbell (2005) and Ivanhoe (2007).

Campbell & Laherrere (2005) in their work with title: Synthesis-what it all amounts to, reviewed the formation and entrapment of oil, how much has been found and when, how much remains and producing what remains. They concluded by drawing up three possible scenarios of supply and demand and pointed out that they all envisage a radical increase in the price of oil when the swing producers exercise their control. This price constraint will lead to a plateau of production for a certain period of time before the resource constraints drive production down.

2.2.4 Petroleum depletion concerns

The U.S.A. Energy Information Administration (EIA, 2006), at the request of the Principal Deputy Assistant Secretary for Fossil Energy of the Department of Energy carried out a study on accelerated depletion: Assessing its impact on domestic oil and natural gas prices and production. The need for the study was based on the concern raised about the effects of depletion on future oil and natural gas supply. The study was performed using the national Energy Modeling System (NEMS) in which a series of production based on different assumptions about the effects of depletion on future production and prices were made. A total of twelve (12) cases were examined. They include reference case, accelerated depletion, accelerated depletion with high and low world oil prices, accelerated depletion with rapid and slow technology, accelerated depletion with high rocky mount access and improved productivity technology, accelerated depletion with high rocky mount access and rapid technology, accelerated depletion with high natural gas imports. Their conclusion was among others that the accelerated depletion assumption leads to a lower projection for domestic natural gas prices.

Similar studies on accelerated depletion carried out by Duncan (1999), Edoreh (2003) and Iledare (2007), reported on a new method for predicting petroleum production which they have used to predict the production from 1996 to 2040 for the top 41 oil-bearing nations. In the new method, called numeric empiric model, all forecasts are based on both quantitative and non-measurable knowledge. Separate models were tailored for each of the world's top 41 oil-producing nations. The results indicate that although their production patterns were quite different, they all increased, peaked and subsequently decreased. Summing the forecast for the forty-one (41) nations gave a world all-time peak at 28.5 billion barrels per year in 2005. This value latter reduced to 10.9 billion barrels per year in 2014, representing a 62% fall in 35 years.

Duncan & Youngquist (2007) also reported on their prediction of the world peak production. This appeared to be an improvement over Duncan (1999). Production data from forty-two (42)

countries, representing 98% of world oil production were used in the work rather than their reserve estimates. A judgmental factor was also applied based on the structure, stratigraphy, and thermal maturity of oil basins and volumes of sediments in potential oil basins yet to be fully explored. These factors were combined with production numerical data to arrive at 2007 as the time for world oil production peak of 30.6Gb a year. This value will rapidly drop to 24.6Gb a year in 2020 and to 11.5Gb per year in 2040. This represents a 62% decline in 33 years. The numerical data for this work were obtained from software (World Oil Forecasting Program) which consists of two distinct, stand-alone models for each nation; the numeric forecasting and the heuristic forecasting models.

Ojotaroeno (2010) highlighted the parabolic projection of World Conventional Oil Production based on year 2000 resources assessment of the US Geological survey, as reported by Walsh (2000). This technique was used to estimate the timing and magnitude of the peak conventional oil production for the whole world. Mean value of the assessments was employed in the parabolic calculation to derive two boundary cases due to uncertainty concerning “reserve addition” which was assumed to contribute to output only after the peak has passed (case 1) and on the other hand to be continuously available throughout the production period (case 2). Two extreme cases of the assessment value (case 3 and 4) were defined as oil production based on the smaller oil resources at 95% probability (case 3) and the larger resources at 5% probability (case 4). Results obtained showed that world conventional oil peaked at 29.38 gigabarrels (Gb) per year in 2017 for case 1 while for case 2, it was 31.1 Gb/year in 2026. For case 3, it was 28.53 Gb/year in 2012 while it came to 30.64 Gb/yr in 2024 for case 4.

Walsh (2000) used the parabolic projection to predict the decline in world oil reserve. He defined change in reserve over one-year as the sum of discoveries less the production for that year, while the total discoveries he put as the sum of two different sources: Those arising from the normal exploration process and those that contribute to the reserves addition specified in the year 2000

assessment of the US Geological survey. He interpreted the reserves addition in two different ways that represented opposing boundary conditions while changes in reserves were calculated year-by-year to 2050 for each case starting from values published in the literature for the end of 1999. He concluded among others that changes in published reserves were not good prediction of when the peak in world conventional production of oil may be expected.

In his work, Laherrere (2001) tried relating oil depletion to natural and universal laws. He observed that events and activities in the universe consists of cycle, which when plotted are parabolic in nature; thus there is a beginning, rise, peak, decline and end. According to him, oil production will follow the same parabolic profile; however, the issue would be when its production peak would occur. He concluded by cautioning against believing only in oil production rise and peak while ignoring the decline and eventual exhaustion of oil. Laherrere didn't go further into the determination of the decline and exhaustion periods.

According to him, it was stated at the World Energy Congress in Tokyo Japan that there were no concerns regarding the supply of petroleum for the next 25 years. This statement was evidently based on the simple calculation that the reserves were sufficient to sustain consumption of oil for 43 years, and gas for 66 years. As Campbell (1997) earlier pointed out, it is absurd to imagine that production can last for a given number of years and then stop over-night when all oil fields are subject to decline during the latter half of their lives. Ivanhoe (2007) concluded that many reported reserves have a political content, and that there is a finite limit to supply.

2.2.5 Production and depletion forecasting

The starting point for determining discovery trends is to input valid data with any reserve revisions backdated to the discovery date, or better still the development data (especially offshore) (Deffeyes, 2007). In addition, it is necessary to take into account the analysis of distribution (the parabolic fractal) and the pattern of past discovery (reserves and number of field) and drilling (Kleinbaum & Kupper, 2008).

Laherrere (2001) pointed out that production forecasting needs to consider the correlation of discovery cycles, the Hubbert curve, the assessment of the ultimate recovery, and the depletion pattern. There is no single or simple solution, and experience is called for to judge how much weight to be given to the several factors.

That said, the range of realistic options is not large in most cases. He observed that given the importance of oil to the modern economy, it is vital to secure a sound inventory of petroleum reserves and resources, free from manipulation for political ends.

Laherrere (2006a) writing on the strength and weaknesses of the Hubbert curve identified three cases in which Hubbert curve ideally can be applied:

- Firstly, where there is a large population of fields, such that the sum of a large number of asymmetrical distributions becomes symmetrical (normal) under the central limit theorem of statistics.
- Secondly, where exploration follows a natural pattern unimpeded by political events or significant economic factors, as for example when OPEC artificially cut production: Hubbert modeling should not be used for the swing producers (Persian Gulf).
- Thirdly, where a single geological domain having a natural distribution of field is considered, political boundaries should be avoided.

He observed that Hubbert's modeling gives fair results only when the past data series have not been disturbed by economic and political factors, and when the past data series have passed the peak and when there is only one single cycle for discovery (Laherrere, 1999).

Campbell (2007) is of the opinion that oil depletion as a subject is a difficult and sensitive one, and there are many vested interests in the industry and government doing their best to confuse it. Although, he noted that, there are no particular technical challenges in measuring the reserves of an oil field or in assessing the potential for new discovery, public data are notoriously unreliable and inconsistent permitting, in turn, the full spectrum of unsubstantiated opinion. Further he

noted that, whereas a scientist would describe this oil depletion evolution as progress based on a growing knowledge of nature, the flat-earth fraternity, a conservative ideological group, will claim it as evidence that a resource-based approach to forecasting production is fundamentally flawed, insisting that minerals are inexhaustible and will never be depleted.

Kamalu (2010) developed simple physical models for Nigeria's crude oil and natural gas resources depletion. The models were based on actual Nigeria's crude oil and natural gas production data (sourced from the department of Petroleum Resources of the Ministry of Petroleum) during 1957 to 2007. He transformed the data into cumulative production data which he considered suitable for use in developing equations for predicting oil and gas production peak and exhaustion periods.

He formulated the resulting equations in line with sigmoidal, dumb-bell and free-fall profiles of natural resources depletion modeling techniques, and employed the MATLAB[®] software, implemented on a personal computer, in the determination of the model equation constants. From his work, he found out the following: 1) The peak and exhaustion periods to be within the range of years 2032 – 2080 and year 2928 respectively for crude oil; and within years 2099 – 2153 and year 3014 respectively for natural gas. 2) That non-associated gas production peak occurred between 67 to 73 years after that of crude oil, and its exhaustion occurred about 86 years after that of oil. According to him, Nigeria's crude oil is expected to peak in the year 2056; and natural gas in the year 2126, which is about 70years time gap. 3) That no one model can be used effectively to predict peaking and exhaustion times for both crude oil and natural gas. He said that a particular model predicts best for a particular petroleum component. 4) Also, that all the sigmoidal models do not agree with the Hubbert type model. He emphasized that in the Hubbert sigmoidal models the resulting production peaks fuse or merge into a single peak while, in the non-Hubbert sigmoidal models the resulting production peaks form a range of values. He highlighted that what most researchers reported in literature relied on this finding to criticize

Hubbert's method as inadequate because it considered petroleum resources reserves as geological rather than technical and economic. 5) According to him, the values of the petroleum resources depletion models' constants are specific to characteristic Nigerian petroleum resources (crude oil and natural gas) production data.

2.2.6 Resource cycle

Resource means wealth, supply of goods, raw materials, etc. which a person, country, etc. has or can use (Keyes, 2006). Natural resources, therefore, are a country's mineral wealth, potential world power, the productivity of the soil, etc. Depletion is the act of using up or emptying until little or none remains (Stulz-Karim & Economides, 2008). The gradual but sure emptying or using up of a nation's mineral wealth, in this case, oil, is referred to as natural resource depletion. A more general classification of resources as given by Meyer (1978) is outlined below (Stulz-Karim & Economides, 2008):

1. Resource is a concentration of naturally occurring solids or liquid petroleum or petroleum-like material, or natural gas, in or on the earth's crust in such a form that the economic extraction is currently or potentially feasible.
2. Discovered resources are those whose location, quality, and quantity are known from drilling and geologic evidence, supported by engineering measurements.
3. Undiscovered resources are resources surmised to exist on the basis of broad geologic knowledge and theory.
4. Reserve is that portion of the identified resources from which a usable mineral and energy commodity can be economically extracted at the time of estimation. Such commodities include, but are not necessarily restricted to, asphalt, without regard to mode of occurrence.'
5. Hypothetical resources are undiscovered material that may reasonably be expected to exist in a known producing basin under known geologic conditions.

6. Speculative resources are undiscovered materials that reasonably may be expected to exist in presently nonproductive basins.
7. Discovered sub-economic resources are known resources not economically producible on the date of estimation, i.e those too small or remote, at depths too great, or in water depth too great to be economic, or for which known producing technologies are not presently economical.
8. Undiscovered sub-economic resources are the material in hypothetical and speculative deposits which, if found, would not be economical to produce based upon existing technology.

All natural resources depletion phenomenon obey the law of diminishing returns because there is no replenishment of consumed natural resources and even if there is replenishment, its rate is very infinitesimal compared with its depletion rate. The growth models are applied in many fields: biology, forestry, zoology and ecology. Growth occurs in organisms, plants trees and bushes, animals and human beings (Stulz-Karim & Economides, 2008). The population growth law fits in here very well.

In chemistry and chemical engineering, growth occurs as a result of chemical reactions. In economics and political science, growth of organizations, supplies of food and material, and nations occur (Livernois & Uhler, 2007).

According to Laherrere (2006b), the universe is made up of cycles. Everything that is born will die: stars, days, species, humans, and civilizations. A stone thrown into the air follows a parabolic trajectory. If its velocity is more than 11km/s it will leave the gravitational field of the earth but will become part of the solar system returning back on an elliptic orbit. Even the sun and our planet have a life ending in death. Biological growth starts with an exponential multiplication as each cell divides, but because of constraints (mainly space and family planning for human beings) growth declines and population reaches a maximum and declines to death. Ultimately nothing is linear, even the production of a finite natural resources (Hubbert, 1982).

As Duncan & Youngquist (2007) puts it, because gas and oil are exhaustible resources, the discovery history of these fuels in any particular area must be characterized by a beginning, a period of increase, a period of decline, and ultimately, an end, but the transition between the period of increase and the period of decline is of utmost interest. Hubbert (1956), in his famous paper of 1956, predicted that U.S oil production (from the lower 48 states) would peak in the 1970s at the top of a bell-curve, the area of which represented the total endowment of oil, which he estimated at 200billion barrels. He was vilified at the time as being a pessimist, but was amply vindicated when the country's production indeed did peak in 1971 (Sorrell *et al.*, 2009).

The Hubbert curve is a derivative of the logistic curve which was introduced in 1845 by the Belgian mathematician, Verhulst, as a law of population growth (Heinberg, 2003), as shown in equation (2.1):

$$CP = y = \frac{U}{(1 - e^{-b(t-tm)})} \quad (2.3)$$

The equation of Hubbert curve for annual production, as shown in equation (2.2):

$$P = \frac{2Pm}{(1 + \text{Cosh}(t - tm))} \quad (2.4)$$

Where,

CP = volume

U = ultimate value

P = annual production

Pm = peak production

t = time

tm = peak time

However, there are variants such as the Gauss curve, the Cauchy curve, the sine wave and even the parabola (Geverjohn, 2005).

According to Adelana *et al.* (2011), resource taken out of the environment eventually goes back to the environment via series of processes almost undefined in nature. According to them,

resources availability is dynamic and depends on the state of technology at that time, and this overrides the general assertion of growth limit based on known (estimated) physical limits hence depletion is a forecasting problem where probabilistic statements are used (Elliott, 2005).

2.3 Empirical Literature

The natural oil supply is fixed because petroleum is naturally formed far too slowly to be replaced at the rate at which it is being extracted.

Going from Hubbert Peak theory, we saw that the short and medium term oil production decline occurs in a predictable manner based on geological circumstances, governmental policies and engineering practices. The shape of the decline curve varies depending upon whether one considers a well, afield or a set of fields.

We noted that an individual oil well will usually produce at its maximum rate at the start of its life; the production rate eventually declining to a point at which it no longer produces profitable amounts. Here the shape of the decline curve depends on the oil reservoir in question and the reservoir-drive mechanisms. Wells in water-drive and gas-cap drive reservoir will often produce at a near constant rate until the encroaching water or expanding gas cap reaches the well to cause a sudden decline in oil production.

Most oil is found in a small number of very large oil field, and according to Hubbert peak theory, production normally starts off slowly, rising faster, then, slowing down and flattening until it reaches a peak after which declines. In the later stage, production is said to enter a period of exponential decline in which the decline becomes less and less steep. However, oil depletion may never actually reach zero, but eventually becomes very low.

There are factors which can modify this curve, and they include,

- (a) Inadequate demand for oil, which reduces steepness of the curve, pushing its peak into the future.

- (b) Sharp price increases when the production peak is reached, as production fails to meet demand.
- (c) Development of new drilling technology or marketing of unconventional oil can reduce the steepness of the decline as more oil is produced than what was initially anticipated.

Generally, the trend for most natural resources depletion is first increasing return, followed by a constant trend of resource value and finally diminishing return. But if there is no commensurate replenishment of the produced (consumed) resources, the diminishing return will continue until the resource is finally exhausted (flatten). Law of diminishing return is hence applicable to natural resource depletion with a general model given by the equation below;

$$P = C_1t - C_2t^2 \quad (2.5)$$

Where, P is the annual resource production and t is the time in years.

Though much works have been carried out in the study of petroleum from its origin, formation, exploration and production, utilization and economics, more still need to be done, especially in the area of petroleum reserves peaking and depletions. The peaking and depletion are usually forecasted from a predictive modeling of some reservoirs.

Several researchers had tried to predict the world reserves, at different times. The outcome is that the majority of these researches confirm that petroleum resources is not limitless, hence the study of its reserve, peaking and depletion cannot be overemphasized.

Among other researchers, Hubbert (1956), a geophysicist work stands out on this area of petroleum peaking and depletion as he was able to use the bell- shape curve originated by him to predict that the lower 48 US States oil will peak around 1970 and thereafter decline. He was remarkably correct as the lower 48 US States oil actually peaked in 1971 and declined thereafter. Hubbert's research works in this area of petroleum resources predictions remains outstanding in the sense that it has stood a test of time in its validity of use and application. Although it

shortcomings abound, which could be attributed to the non-generality of its scope of application. His model assumes that production levels will normally begin at zero, before the production has started, and then equally ends at zero when the resource has been exhausted, and in between the production curve passes through one or more maxima. A major limitation could be said to be that there is no found literature that talked about him validating his model.

By exploiting the principle that oil cannot be produced until it has been discovered, Walsh (2000) have forecasted the US oil production by comparing available discovery and production records. He based his analysis on the energy need for petroleum resources viz-a-viz their diverse use for various purposes. However, at the time of his analysis, he did not do an articulated prediction for the peaking and exhaustion period for the US petroleum reserve.

Laherrere (2001), a key researcher in this field related oil depletion to other natural and universal laws, stating that events and activities in the universe consists of cycle, which when described on a graphical plot are parabolic in nature, with a beginning, rise, peak, decline and end stages. He was able to establish that oil production follow same parabolic profile. He eventually cautioned against ignoring oil decline and exhaustion, with emphasis only in the production rise and peak.

Campbell (2007) noted that there are no particular technical challenges in measuring the reserves of an oil field or in assessing the potential for new discovery, stating that public data are notoriously unreliable and inconsistent permitting, in turn, the full spectrum of unsubstantiated opinion.

Like Hubbert (1956) and Laherrere (2001), Duncan & Youngquist (2007) agree that because gas and oil are exhaustible resources, the discovery history of these fuels in any particular area must be characterized by a beginning, a period of increase, a period of decline, and ultimately, an end, but that of the transition between the period of increase and the period of decline was not properly substantiated.

In his own input, Bardi (2009) outlined the peak phenomenon on the basis of a simple theory based on Lotka-Volterra biological model. The basis is that, initially, the extraction of an abundant and cheap natural resource will lead to economic growth and to increasing investments in further extraction, but gradually the cheap resources are depleted and then extraction costs become higher as the need to extract lower quality deposits rises.

Akuru & Okoro (2011) have also said that petroleum oil would certainly reach a maximum peak and then begin to decline. Although they were not certain if Nigerian oil had peaked or was on a downward trend, their finding basically showed the imminence of Nigeria exhausting its proven reserves. However, they did not consider the factors that could aid Nigerian petroleum peaking, depletion and exhaustion determination. Again, their analysis was not based on a composite reservoir.

Looking at the high rate of depletion of oil reserve worldwide, and as the Nigerian economy is heavily dependent on the oil sector which accounts for over 90 percent of her export earnings and about 85 percent of government revenue, Osahon (2014) put it in the light that Nigerian oil reserve keeps dropping, which he attributed to the reduction in the country's oil production. He stated that some oil wells in the Niger Delta had stopped production because they had attained "Maturity".

2.4 Analytical tools and concepts for the research

2.4.1 Models and Modeling

Kamalu (2010) explained that a model is a miniature representation of something, a pattern of something to be made, an example for imitation, emulation, a description or analogy used to help visualize something that may or may not be directly observed, a system of postulates, data and inferences presented as a mathematical description of an entirety or state of affairs. According to him, while modeling is the concise description of the total variation in one quantity by partitioning it into a deterministic component given by a mathematical function of one or more

other quantities plus a random component that follows a particular probability distribution. It is therefore the mathematical representation of physical systems, he opined.

Basically, models are useful in estimation, prediction, calibration and optimization. Estimation involves the determination of the value of the regression function that is associated with a specific combination of prediction variable values which are the parameters. Prediction determines future values of the response variables, yielding the variables (dependent and independent) using the parameter values. Calibration on another hand allows for the quantitative conversion of measurements made on one of the scale, while optimization addresses the substitution of an output value for the response variable and solving for the associated predictor variable values (Marker, 1996).

According to Marker (1996), the existence of numerous models is dependent on their various applications. These include sequence and series models, limit and continuity (i.e. finite and infinite), linear and nonlinear models, static and dynamic models, growth and decay models, cycles and oscillation models, deterministic and probabilistic stochastic models.

In the words of Rehr & Friedrich (2006), for the purpose of building a model, the basic steps include data collection, model development, model fitting and model validation. Usually, models are developed following some basic steps which provide universal framework. And although details may vary from one method to another, an understanding of the common steps combined with underlying assumptions needed for the analysis is essential. In the model development stage, plots of the data process knowledge and assumptions about the process are used to determine the form of the model to be fitted to data. By use of the developed model, an appropriate model fitting method is used to estimate the unknown parameters in the model. Then the model is assessed to find out if the assumptions are valid or otherwise. The model can now be used to provide answers to scientific or engineering questions that prompted it.

Generally, model development is based on fundamental physical and chemical laws (Kamalu, 2010). Models are often characterized by assumptions which are indispensable, but this is provided that they are made with sound judgment and within phenomenal context, since a model containing all parameters will be very complex. Again, mathematical consistency must be followed for models to be solvable. Model verification is also essential in order to ensure their workability and usability.

These are implicit assumptions in form of statements, based on properties inherent to the process modeling methods (Ivanhoe, 1995). Any particular model has its own specific assumptions but in general, typical assumptions for process modeling include the following:

- The process is physical, chemical or statistical in nature
- The means of the random error are zero
- The random errors have a constant standard deviation and follow a normal distribution
- The data are randomly sampled from the process
- The exemplary variables are observed without error.

Developed models serve as prediction tools in engineering application. To be able to determine future values of the dependent variables, fitted models with estimated parameters are used to compute uncertainties by plugging the values of the prediction or independent variables. This gives the idea of the future of the process in question with the associated assumption(s) in place (Ivanhoe, 1995).

Kleinbaum & Kupper (2008) opined that: achieving an optimization process with models involves substituting an output value for the response variable and subsequently solving for the associated predictor variable values. In such an effort, the model serves as a link between input and output. Models developed for the purpose of optimization usually require a cause and effect relationship between the predictor and the response variable. For example, quadratic models are

usually used for the calculus determination of maximum and minimum for the applicable optimization decision.

To verify a model to ascertain if it fits a data is a crucial exercise for effective modeling. Model validation is probably the most important step in the model building sequence (Laherrere, 2002). More of the times, the validation of a model is seen to consist of nothing more than quoting the statistics from the fit which measures the fraction of total variability in the response that is accounted for by the model. However, a high value does not guarantee that the model fit the data well and the use of such model will rather not generate answers to the underlying engineering questions, if critically considered (Hubbert, 1982).

Crude oil forecasting is an area that has attracted much attention in recent times, mainly because of the increasing dependence on hydrocarbon as the primary source of world energy. Different model types have been applied in the forecasting of crude oil. They include numeric and heuristic forecasting models (Laherrere, 2001).

The numeric forecasting model (N-Model) is quantitative using hydrocarbon fuel production data and mathematics on a translated coordinate system to produce an intermediate “helper” forecast for each nation. This so-called “guide” forecast (G-forecast) is a purely mathematical prediction of future production. In some cases, the G-forecast can provide useful information about the shape of future oil production curve by providing a lower boundary on the estimated ultimate recovery (EUR) and the probable shape of the future production curve. However, in other circumstances, it is not useful as in the situation of the OPEC production quota-limited countries. The N-model produces the G-forecast, the best forecast we are able to make based solely on historic production data, and mathematics. The G-forecast is just one of many items of information that may be used in the second model portion of the world oil forecasting program (Laherrere, 2001).

By definition, “heuristic” denotes a method of solving a problem for which no algorithm exists. It involves trial and error, as in iteration. In this context, heuristic knowledge indicates soft, qualitative or judgmental knowledge (Laherrere, 2001). Although judgmental is lacking in the numeric model, it is crucial for oil forecasting in the heuristic model (H-model). The H-model provides the user with a powerful interface for oil forecasting chief of which is a three-curve graph for each nation with years on the X-axis, and production on the y-axis.

2.5 Knowledge Gap to be filled

- i. In all of these contributions, no particular description has been given to the Nigerian situation regarding the depletion, peaking and exhaustion of her petroleum resources by considering a cumulative of all her existing reservoirs as a single composite, which is a more reliable basis for such analysis. The theory of composite reservoir has been considered in this work, and this is to mirror the entire oil production well as one entity.
- ii. Nigerian energy researchers have not shown concern to apply the composite reservoir concept in their analysis of the Nigerian petroleum resources production, discovery and reserve. Data obtained from DPR and analyzed in this work can be used by other Nigerian researchers for extended predictive works.
- iii. Again, following the global recession on world petroleum reserves, the aforementioned scholars attempted to alert the world or their respective nations of origin about the danger that looms in the future fossil energy supplies as this form of energy depletes exhaustibly. Non was specific to Nigeria.
- iv. While attempting to forecast different countries and continents reserve limits, peaking and exhaustion periods, strangely and unfortunately, the records of such information for Nigeria are very limited, and as good as nonexistent. This is a matter of serious concern for a country that depends so much on this natural resource for her income earnings.

- v. No particular description or prediction has been focused on peaking and exhaustion with specific dates and volumes for Nigeria. This work is set out to achieve this.
- vi. No strong caution has been sounded to the Nigerian government based on detailed research as this, for her to take precautionary steps to shore up her economy.
- vii. There is no known developed model for the Nigerian petroleum depletion, peaking and exhaustion predictions, based on the composite reservoir concept; which is the thrust of this work.
- viii. No sound and judgmental assumptions were defined in the reviewed literatures, as provided by this work.

CHAPTER THREE

MATERIALS AND METHODS

3.1 Materials

The Nigerian petroleum production data from 1957-2014 were collected from Department of Petroleum Resources (DPR), Ministry of Petroleum and Minerals at No 7 Kofo Abayomi Street, Victoria Island Lagos, Nigeria. It is a one stop shop where annual and cumulative production data from all oil companies operating within the country and within this time range are stored/documented (accounted for). See Appendix B: Oil, Gas and Condensate Reserve 1958 – 2014 (DPR, 2016). Other materials used include a computer laptop and MATLAB software, and a scientific calculator.

3.2 Method

3.2.1 General flowchart for the methodology

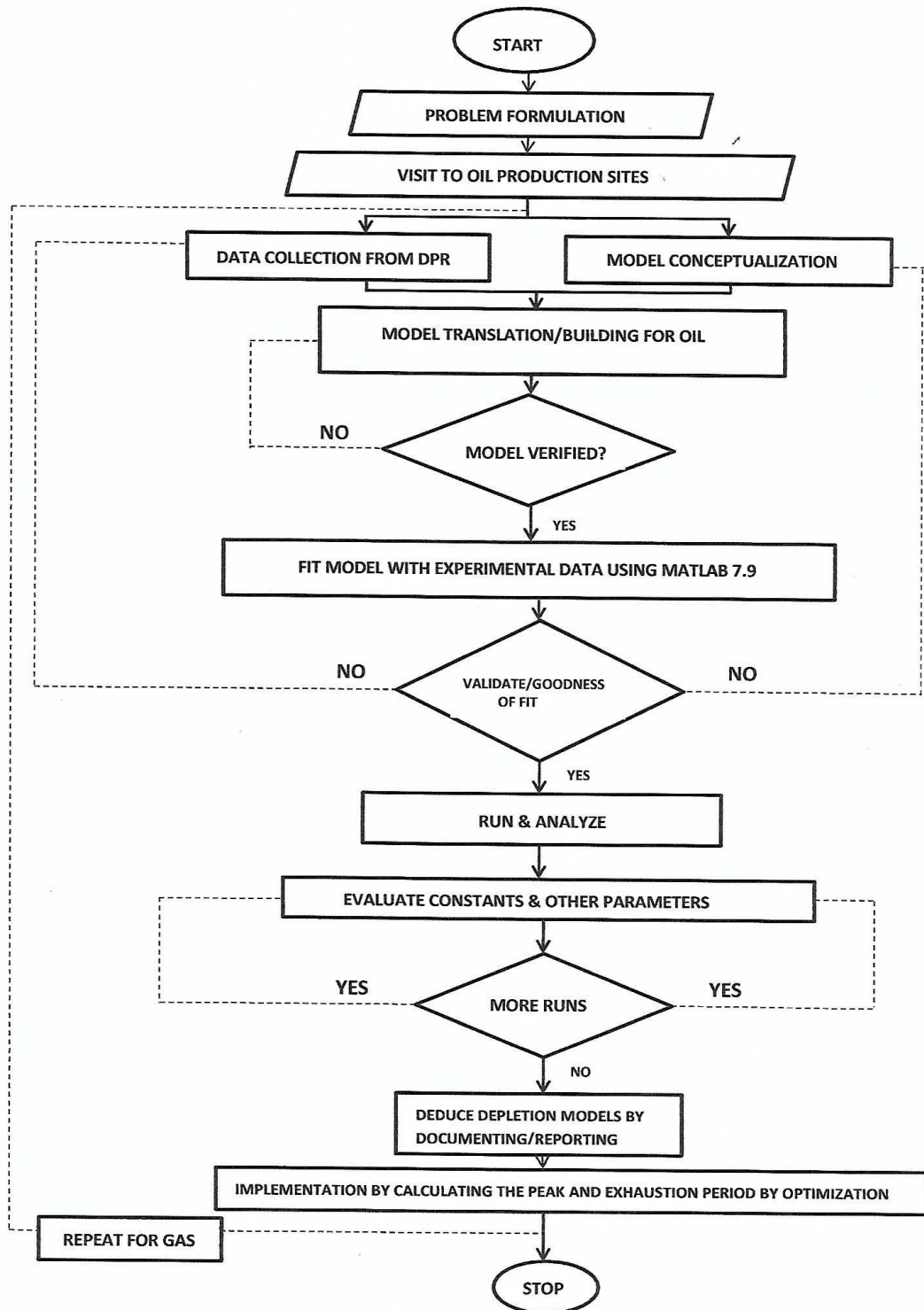


Figure 3.1: General flowchart for the methodology/work plan for modeling composite reservoir depletion profiles for Nigerian petroleum reserves.

Figure 3.1 shows the general flowchart of method for modeling of petroleum resources depletion using a composite underground reservoir. It comprises of pertinent decisions and/or actions to be undertaken to develop the predictive models. The primary task is the collection of the required petroleum production data following visits to different fields to observe production and the conceptualization and development of the models. The models were fitted with experimental data, and this was validated and the goodness of the various model fit was studied, while all the constants were evaluated, and the generated graphs were used to predict the peaking date and volume, and the exhaustion year and volume for both petroleum oil and gas.

Petroleum depletion models were developed using Hubbert concept (Hubbert, 1956), and scatter diagrams for the time history of oil and gas production in Nigeria from 1957-2014. These were plotted using MATLAB 7.9. The models were validated by superimposing them on the scatter diagram profiles to determine their goodness of fit which are declared by the R squared (R^2) of the MATLAB computer package. Other statistical goodness of fit were also declared by the MATLAB 7.9 package on the models. The curve-fitting of the models on S-shaped sigmoidal scatter diagrams were continued until the best R^2 was established and tested with sensitivity or trend test analysis, as shown in Appendix G.

3.2.2 Computational technique

Using the data obtained from Nigerian Petroleum Resources Department (DPR), the cumulative production profiles from 1957 to 2014 of Nigerian petroleum production were established and projected.

With the cumulative production profiles established, postulations and formulations of mathematical production models were then carried out. Curve fittings of the trial models into the profiles by computer were also done.

In the optimization of the model, the analysis GUI of the MATLAB curve-fitting toolbox was applied. The numerical value of the first differential of the model were obtained and the profile of which analytical solution of the first derivative of the model were superimposed using the MATLAB curve-fitting toolbox, to obtain the peak or the trough of the annual rate. If this does not happen i.e not getting a peak in the second plot (optimization), the model is not predictive, just like when the cumulative plot does not give a sigmoidal profile. In other words, for a model to make accurate predictions, the cumulative plot must taper out to give a sigmoidal profile of known ultimate value, and in the optimization plot it must give a dumb bell with a peak.

Optimization of the models, after replacement of the constants with their computed numerical values, by way of maxima and minima points determination were carried out using the software to ascertain the peak period by aggregation or otherwise. Of course, the calculation of exhaustion period is simply done by equating the relation $\{P = f(t)\}$ to zero, that is, assuming zero quantity of petroleum remains, and calculating the time period in the relation by iteration.

Deduction of depletion model and profile was done by subtracting production from total reserve at every interval (year), to get a new profile when replotted. The calculation for the peak and exhaustion periods of the depletion relation $\{D = f(t)\}$ is similar to that of production.

Appendices C and D illustrates the data in tabular form to provide clear picture of the variables that have been studied. The data obtained from the above process tables were further processed using the statistical regression tools. Plotting of the raw data for oil and gas against time period were then carried out and the profiles of these plots noted. In this study, the dependent variable is Nigerian petroleum (oil and gas), while the independent variable is the time period in years. In these resultant nonlinear models, the dependent variable is denoted by P as a function of independent variable t i.e $P = f(t)$.

The non-linear least squares technique was applied on the nonlinear model equations. From the computations, the numerous arbitrary constants in the models were evaluated and put back into the model equations.

Plots of these models obtained were also made to find which models gave almost identical curves as the curves of the plots made from raw data for oil and gas obtained from the Department of Petroleum Resources (DPR). These nonlinear operating equations which have Nigerian petroleum resources production and depletion as function of time were the forecasting/predicting tools so desired for Nigerian petroleum industry. The areas under these curves were calculated by the software and compared with those of the Nigerian total reserve data.

Like the Hubbert technique, optimization technique was invoked upon the established operating equations, since they govern Nigerian petroleum production, to obtain the peak time period and quantity, and, follow the trend of depletion to exhaustion.

3.3 Petroleum Resources Depletion Model Development

3.3.1 Assumptions and mathematical models using transfer functions

Assumptions in the model development

- a. One composite oil well representing all oil wells drilled by all the oil companies in Nigeria.
 - i. Drilling in Nigeria is carried out by different oil companies with different production data and different oil wells at different places in Nigeria. So we must use all the data as coming from one oil well in Nigeria, that oil well must be a composite well.
 - ii. This composite oil well allows for holistic view of all wells assuming that they have same and similar characteristic.

- iii. This assumption will help to narrow the model formulation to fewer components and for the evaluation of constants there-in instead of generating multiple and unending models if each of the wells were considered separately.
- b. The data collected from DPR is the sum total of all the production, discovery and reserves from all the oil companies in Nigeria.
 - i. These data (production, discovery and reserve) were obtained from different oil companies independently from their own oil exploration and drilling activities for different parts of the country.
 - ii. The major oil companies and smaller ones usually send or transmit all production, discovery and reserve data either from the oil well, or field to DPR, which in turn stores it as a Nigerian Petroleum Data for oil, gas and condensate. The records are updated quarterly, biannually and yearly.
 - iii. The DPR is saddled with the responsibility of collating all data of production, discovery and reserve. In fact it is a one stop shop for all information on petroleum data.
- c. The composite oil well has one mean input and one mean output.
 - i. Since all the drillings done by the major oil drilling companies are done on the Nigerian wells, these drillings can be taken as coming from one composite reservoir.
 - ii. Although in a reservoir there are several input but for the modeling to work well instead of having so many models that will not have a tangible solution, we choose one input.
 - iii. Because we want to study the modalities or flow regime of the input to composite reservoir, we will go for one input that will give us the characteristic disturbances or any other phenomenon in Nigeria crude flow in the ground.

- iv. The one mean input can also help us because the known input functions in control (step input, ramp input, etc) have their representative meaning in physical life. Different input have different interpretation of flow regime.
- d. The material balance around the well will result in first order ODE.
 - i. From literature, a macroscopic material balance around a well (composite or single) have been identified to be of first order ODE (Campbell (2007); Delaney (2002); Deffeyes (2007); Duncan (1999)).
 - ii. From process control concepts, most flow in a tank results in first order differential equations; even if it results in second order, it will still degenerate into first order.
- e. The density of the crude entering the reserve, in the reserve and out from the reserve is the same.
 - i. This is so because density is a function of temperature. If there is no appreciable change in the temperature, then the density virtually remains the same; even in literature, the temperature of oil in the ground and that drilled out are virtually the same with a difference of 1-2⁰C.
- f. At the beginning (t = 0), there is initial positive volume of reserve, V₀.
 - i. The rock decides how much oil to let go. That means all the oil in the ground cannot be produced.
 - ii. There must be some quantity of oil discovered before drilling can take place. If there is no volume of oil in the ground or well, the drilling/production cannot commence.
- g. The nation's crude will finish when the rate of change of the reserve is zero (Hubbert concept, ie $\frac{dQ_R}{dt} = 0$); $\frac{dQ_D}{dt} = \frac{dQ_P}{dt}$
- h. $q_0 = \frac{V}{R}$

where,

q_o = quantity of oil flowing out

V = volume of oil in reservoir

R = restriction of volume out

- i. The restriction of flow from tank is expressed as afore-written, where R is constant restriction or valve constant.

3.3.2 Initial and boundary conditions for the model development

Initial condition

$$V_{(t)} = V_O, \text{ at } t = 0 ; V_O > 0$$

Boundary conditions

Data collection: 1957 to 2014

Modeling:

$$\text{Time} = 0 < t < \infty$$

$$\text{Volume} = V_{t=\infty} < V_{(t)} < V_O ; V_O > 0$$

3.3.3 Mathematical modeling by transfer function

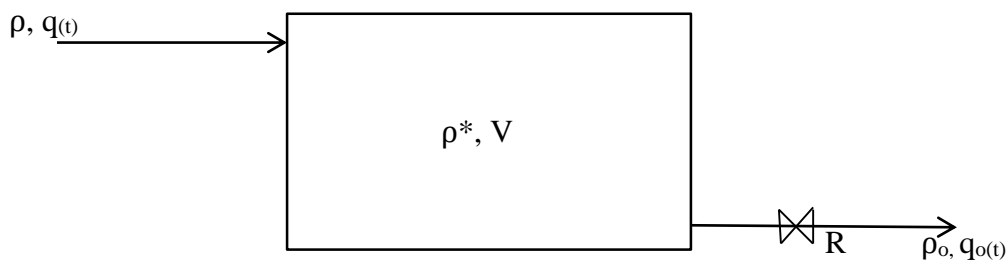


Figure 3.2: Typical petroleum composite reservoir.

Following the concept of mass conservation, the net mass flow is equal to the rate of accumulation of mass in the reservoir, given as:

Mass flow in – mass flow out = rate of accumulation of mass in the reservoir

Mathematically,

$$\rho q - \rho_o q_o = \frac{d(\rho^* V)}{dt} \tag{3.1a}$$

At constant density (ρ), assumption (e), equation (3.1a) becomes;

$$q - q_0 = \frac{d(V)}{dt} \quad (3.1b)$$

But $q_0 = \frac{V}{R}$, from assumption (h), hence equation (3.1b) becomes

$$q - \frac{V}{R} = \frac{dV}{dt} \quad (3.2)$$

Solving equation (3.2) by Laplace transformation;

Taking Laplace transform of eqn (3.2); $Q(s) - \frac{V(s)}{R} = sV(s) - V_0$

Making $V(s)$ subject of the formula, we have eqn (3.3)

$$V(s) = \frac{V_0}{[s + \frac{1}{R}]} + \frac{Q(s)}{[s + \frac{1}{R}]} \quad (3.3)$$

Note: 1) Equation (3.3) is the generalized equation for the model development.

2) $V_0 \neq 0$, assumption (f). If it is zero, then there will be no drilling.

3) The conventional approach would have been to obtain the mathematical models based on deviation variables from steady state, but because this work is clearly based on the formation and drilling of oil at all times, deviation variables were not put into consideration in the derivation of the models.

4) The varied input function could be interpreted as the rates of discovery of oil wells, which eventually add to the composite from which production of oil/gas resources are carried out.

3.3.3.1 Varying the input function $Q(s)$ in the generalized equation (3.3)

(i) For model 1

*If the input function is $Q(s) = \frac{1}{s}$, then eqn (3.3) becomes

$$V(s) = \frac{V_0}{[s + \frac{1}{R}]} + \frac{1}{s[s + \frac{1}{R}]} \quad (3.4)$$

Simplifying by taking partial fraction of the above, we have

$$V_{(S)} \equiv \frac{SV_0 + 1}{S[S + \frac{1}{R}]} \equiv \frac{A}{S} + \frac{B}{[S + \frac{1}{R}]} \equiv \frac{A[S + \frac{1}{R}] + BS}{S[S + \frac{1}{R}]}$$

$$\text{if } S = 0: 1 = \frac{A}{R}, A = R$$

$$\text{if } S = -\frac{1}{R}: 1 - \frac{V_0}{R} = -\frac{B}{R}, B = V_0 - R$$

$$\text{Therefore, } V_{(S)} = \frac{R}{S} + \frac{V_0 - R}{[S + \frac{1}{R}]} \quad (3.5)$$

Taking the inverse Laplace transformation of eqn (3.5) and simplifying gives equation (3.6);

$$\boxed{V_{(t)} = R + (V_0 - R)e^{-\frac{1}{R}t}} \quad (3.6)$$

Equation (3.6) is the model (Model 1) established for the cumulative production of oil/gas, for

the step input function, $Q_{(S)} = \frac{1}{S}$.

Differentiating equation (3.6) and simplifying gives equation (3.7);

$$P_{(t)} = \frac{dV}{dt} = 0 - \frac{1}{R}(V_0 - R)e^{-\frac{1}{R}t} = \left(-\frac{V_0}{R} + 1\right)e^{-\frac{1}{R}t}$$

$$\boxed{P_{(t)} = \left(1 - \frac{V_0}{R}\right)e^{-\frac{1}{R}t}} \quad (3.7)$$

Equation (3.7) is the annual production (for model 1) established for the oil/gas, for the unit step

input function, $Q_{(S)} = \frac{1}{S}$.

Differentiating $P_{(t)}$ in eqn (3.7), and optimizing;

$$\text{At peak, } \frac{d^2V}{dt^2} = \frac{dP}{dt} = 0 = -\frac{1}{R}\left(-\frac{V_0}{R} + 1\right)e^{-\frac{1}{R}t}$$

$$\text{Hence, } V_0 = R \quad (3.8)$$

Equation (3.8) do not contain the peak time factor and hence cannot be used in the determination of the peaking periods for the petroleum resources.

(ii) For model 2

*If the input function is $Q(s) = \frac{1}{s^2}$, then the generalized equation (3.3) becomes equation (3.9);

$$V(s) = \frac{V_o}{[s + \frac{1}{R}]} + \frac{1}{s^2[s + \frac{1}{R}]} \quad (3.9)$$

Simplifying and taking partial fraction of equation (3.9)

$$V(s) \equiv \frac{s^2 V_o + 1}{s^2[s + \frac{1}{R}]} \equiv \frac{A}{s} + \frac{B}{s^2} + \frac{C}{[s + \frac{1}{R}]} \equiv \frac{As[s + \frac{1}{R}] + B[s + \frac{1}{R}] + Cs^2}{s^2[s + \frac{1}{R}]}$$

$$\text{if } S = 0: 1 = \frac{B}{R}, B = R$$

$$\text{if } S = -\frac{1}{R}: \frac{V_o}{R^2} - 1 = \frac{C}{R^2}, C = R^2 + V_o$$

$$\text{Highest power of } S: V_o = A + C, A = V_o - C = V_o - (R^2 + V_o), A = -R^2$$

$$\text{Therefore, } V(s) = -\frac{R^2}{s} + \frac{R}{s^2} + \frac{R^2 + V_o}{[s + \frac{1}{R}]} \quad (3.10)$$

Taking the inverse Laplace transformation of eqn (3.10) and simplifying gives equation (3.11);

$$V(t) = -R^2 + Rt + (R^2 + V_o)e^{-\frac{1}{R}t}$$

$$\boxed{V(t) = (t - R)R + (R^2 + V_o)e^{-\frac{1}{R}t}} \quad (3.11)$$

Equation (3.11) is the model (model 2) established for the cumulative production of oil/gas, for

the unit ramp input function, $Q(s) = \frac{1}{s^2}$.

Differentiating eqn (3.11) and simplifying gives eqn (3.12);

$$P(t) = \frac{dV}{dt} = R - \frac{R^2 + V_o}{R} e^{-\frac{1}{R}t}$$

$$P(t) = R - \frac{R^2 + V_o}{R} e^{-\frac{1}{R}t} \quad (3.12)$$

Equation (3.12) is the annual production (for model 2) established for the oil/gas, for the unit

ramp input function, $Q(s) = \frac{1}{s^2}$.

Differentiating $P(t)$ in eqn (3.12), and optimizing;

$$\text{At peak, } \frac{d^2V}{dt^2} = \frac{dP}{dt} = 0 = \frac{R^2 + V_0}{R^2} e^{-\frac{1}{R}t}$$

$$\text{Hence, } V_0 = -R^2 \quad (3.13)$$

Again, equation (3.13) do not contain the peak time factor and hence cannot be used in the determination of the peaking periods for the petroleum resources.

(iii) For model 3

*If the input function is $Q(s) = \frac{1}{[s+\frac{1}{R}]}$, then the generalized equation (3.3) becomes

$$V(s) = \frac{V_0}{[s+\frac{1}{R}]} + \frac{1}{[s+\frac{1}{R}]^2} \quad (3.14)$$

Simplifying and taking partial fraction of equation (3.14)

$$\frac{V_0[s+\frac{1}{R}] + 1}{[s+\frac{1}{R}]^2} \equiv \frac{A}{[s+\frac{1}{R}]} + \frac{B}{[s+\frac{1}{R}]^2} \equiv \frac{A[s+\frac{1}{R}] + B}{[s+\frac{1}{R}]^2}$$

Comparing equation above;

$$B = 1$$

$$A = V_0$$

$$\text{Therefore, } V(s) = \frac{V_0}{[s+\frac{1}{R}]} + \frac{1}{[s+\frac{1}{R}]^2} \quad (3.15)$$

Taking the inverse Laplace transformation of eqn (3.15) and simplifying gives;

$$V(t) = V_0 e^{-\frac{1}{R}t} + t e^{-\frac{1}{R}t} = (V_0 + t) e^{-\frac{1}{R}t} \quad (3.16)$$

Equation (3.16) is the model (model 3) established for the cumulative production of oil/gas, for the unit exponential decay input function with zero ultimate value, $Q(s) = \frac{1}{[s+\frac{1}{R}]}$.

Differentiating eqn (3.16) and simplifying gives eqn (3.17);

$$P(t) = \frac{dV}{dt} = e^{-\frac{1}{R}t} - \frac{1}{R}(V_0 + t)e^{-\frac{1}{R}t}$$

$$P(t) = e^{-\frac{1}{R}t} - \frac{1}{R}(V_0 + t)e^{-\frac{1}{R}t} \quad (3.17)$$

Equation (3.17) is the annual production (for model 3) established for the oil/gas, for the unit exponential decay input function with zero ultimate value, $Q(s) = \frac{1}{[S + \frac{1}{R}]}$.

Differentiating $P(t)$ in eqn (3.17), and optimizing;

$$\text{At peak, } \frac{d^2V}{dt^2} = \frac{dP}{dt} = 0 = -\frac{1}{R}e^{-\frac{1}{R}t} - \frac{1}{R}e^{-\frac{1}{R}t} + \frac{1}{R^2}(V_0 + t)e^{-\frac{1}{R}t}$$

$$\text{Hence, } V_0 + t = 0 \quad \text{or} \quad t_{pk} = -V_0 \quad (3.18)$$

(iv) For model 4

*If the input function is $Q(s) = \frac{1}{S[S + \frac{1}{R}]}$, then the generalized equation (3.3) becomes

$$V(s) = \frac{V_0}{[S + \frac{1}{R}]} + \frac{1}{S[S + \frac{1}{R}]^2} \quad (3.19)$$

Simplifying and taking partial fraction of equation (3.19)

$$\frac{V_0S[S + \frac{1}{R}] + 1}{S[S + \frac{1}{R}]^2} \equiv \frac{A}{S} + \frac{B}{[S + \frac{1}{R}]} + \frac{C}{[S + \frac{1}{R}]^2} \equiv \frac{A[S + \frac{1}{R}]^2 + BS[S + \frac{1}{R}] + CS}{S[S + \frac{1}{R}]^2}$$

$$\text{if } S = 0: 1 = \frac{A}{R^2}, A = R^2$$

$$\text{if } S = -\frac{1}{R}: 1 = \frac{C}{R^2}, C = -R$$

$$\text{Highest power of } S: V_0 = A + B, B = V_0 - A = V_0 - R^2$$

$$\text{Therefore, } V(s) = \frac{R^2}{S} + \frac{V_0 - R^2}{[S + \frac{1}{R}]} - \frac{R}{[S + \frac{1}{R}]^2} \quad (3.20)$$

Taking the inverse Laplace transformation of eqn (3.20) and simplifying gives;

$$\begin{aligned} V(t) &= R^2 + (V_0 - R^2)e^{-\frac{1}{R}t} - Rte^{-\frac{1}{R}t} \\ &= R^2 - R^2e^{-\frac{1}{R}t} + V_0e^{-\frac{1}{R}t} - Rte^{-\frac{1}{R}t} \\ &= R^2(1 - e^{-\frac{1}{R}t}) + (V_0 - Rt)e^{-\frac{1}{R}t} \end{aligned}$$

Multiplying through by a lifter factor F, the model becomes;

$$V_{(t)} = F(R^2(1 - e^{-\frac{1}{R}t}) + (V_0 - Rt) e^{-\frac{1}{R}t}) \quad (3.21)$$

Equation (3.21) is the model (model 4) established for the cumulative production of oil/gas, for the unit exponential decay input function with a non-zero ultimate value, $Q(s) = \frac{1}{s[s + \frac{1}{R}]}$.

On differentiation with respect to time, eqn (3.21) which is a cumulative production-time history yields an annual production-history, eqn (3.22)

$$P_{(t)} = \frac{dV}{dt} = F(R^2(0 + \frac{1}{R}e^{-\frac{1}{R}t}) + (0 - R)e^{-\frac{1}{R}t} - \frac{1}{R}(V_0 - Rt) e^{-\frac{1}{R}t})$$

$$P_{(t)} = \frac{dV}{dt} = F((Re^{-\frac{1}{R}t}) + (-R - \frac{V_0}{R} + t) e^{-\frac{1}{R}t})$$

$$P_{(t)} = \frac{dV}{dt} = F(t - \frac{V_0}{R})e^{-\frac{1}{R}t}$$

$$P_{(t)} = F(t - \frac{V_0}{R})e^{-\frac{1}{R}t} \quad (3.22)$$

Equation (3.22) is the annual production (for model 4) established for the oil/gas, for the unit exponential decay input function with a non-zero ultimate value, $Q(s) = \frac{1}{s[s + \frac{1}{R}]}$.

Differentiating $P_{(t)}$ in eqn (3.22), and optimizing;

$$\text{At peak, } \frac{d^2V}{dt^2} = \frac{dP}{dt} = 0 = F(-\frac{V_0}{R} + t)(-\frac{1}{R}e^{-\frac{1}{R}t}) + Fe^{-\frac{1}{R}t}$$

$$0 = [(-\frac{V_0}{R} + t)(-\frac{1}{R}) + 1]Fe^{-\frac{1}{R}t}$$

$$0 = [\frac{V_0}{R^2} - \frac{t}{R} + 1]Fe^{-\frac{1}{R}t}$$

$$\text{Hence, } Rt - R^2 - V_0 = 0 \quad (3.23)$$

$$\text{And, } t_{pk} = R + \frac{V_0}{R} \quad (3.24)$$

(v) For model 5

*If the input function is $Q(s) = \frac{1}{s[s + \frac{1}{R}]^2}$, then the generalized equation (3.3) is

$$V(s) = \frac{V_0}{[s + \frac{1}{R}]} + \frac{1}{s[s + \frac{1}{R}]^3} \quad (3.25)$$

Simplifying and taking inverse Laplace transformation of eqn (3.25) and simplifying gives

$$V_{(t)} = F(R^3(1 - e^{-\frac{1}{R}t}) + (V_0 - R^2t - \frac{Rt^2}{2}) e^{-\frac{1}{R}t}) \quad (3.26)$$

Equation (3.26) is the model (model 5) established for the cumulative production of oil/gas, for

the input function $Q(s) = \frac{1}{s[s + \frac{1}{R}]^2}$.

On differentiation with respect to time, eqn (3.26) which is a cumulative production-time history yields an annual production-time history, eqn (3.27).

$$P_{(t)} = F(\frac{t^2}{2} - \frac{V_0}{R}) e^{-\frac{1}{R}t} \quad (3.27)$$

Equation (3.27) is the annual production (for model 5) established for the oil/gas, for the input

function $Q(s) = \frac{1}{s[s + \frac{1}{R}]^2}$.

Differentiating $P_{(t)}$ in eqn (3.27), and optimizing;

$$\text{At peak, } \frac{d^2V}{dt^2} = \frac{dP}{dt} = 0; \quad Rt^2 - 2R^2t - 2V_0 = 0$$

$$\text{And, } t_{pk} = R \pm \frac{\sqrt{(R^4 + 2RV_0)}}{R} \quad (3.28)$$

The results obtained by further changing the transfer functions are as shown in table 3.1.

Table 3.1: An array of developed models

Model Number	Input Function	Cumulative Production/Discovery $V(t)$ (Q_P or Q_D)	Annual Production/Discovery $P(t)$ ($\frac{dQ_P}{dt}$ or $\frac{dQ_D}{dt}$)	Peak Time ($t_{pk} = t_{1/2u}$)
1.	$\frac{1}{s}$	$R + (R-V_0) e^{-t/R}$	$\left(\frac{V_0}{R} - 1\right) e^{-\frac{t}{R}}$	at t_{pk} : $V_0 = R$
2.	$\frac{1}{s^2}$	$(t-R) R + (V_0 + R^2) e^{-t/R}$	$R - \frac{1}{R} (V_0 + R^2) e^{-t/R}$	at t_{pk} : $V_0 = -R^2$
3.	$\frac{1}{s + \frac{1}{R}}$	$(V_0 + t) e^{-t/R}$	$e^{-t/R} - \frac{1}{R} (V_0 + t) e^{-t/R}$	$-V_0$
4.	$\frac{1}{s\left(s + \frac{1}{R}\right)}$	$R^2 (1 - e^{-t/R}) + (V_0 - Rt) e^{-t/R}$	$\left(\frac{V_0}{R} - t\right) e^{-t/R}$	$Rt - R^2 - V_0 = 0$
5.	$\frac{1}{s\left(s + \frac{1}{R}\right)^2}$	$R^3 (1 - e^{-t/R}) + (V_0 - R^2 t - R \frac{t^2}{2}) e^{-t/R}$	$\left(\frac{V_0}{R} - \frac{t^2}{2}\right) e^{-t/R}$	$Rt^2 - 2R^2 t - 2V_0 = 0$
6.	$\frac{1}{s\left(s + \frac{1}{R}\right)^3}$	$R^4 (1 - e^{-t/R}) + (V_0 - R^3 t + \frac{R^2 t^2}{2} - \frac{Rt^3}{6}) e^{-t/R}$	$\left(\frac{V_0}{R} - \frac{t^3}{6}\right) e^{-t/R}$	$Rt^3 - 3R^2 t^2 - 6V_0 = 0$
7.	$\frac{1}{s\left(s + \frac{1}{R}\right)^4}$	$R^5 (1 - e^{-t/R}) + (V_0 - R^4 t - R^3 \frac{R^3 t^2}{2} - \frac{R^2 t^3}{6} - \frac{Rt^4}{24}) e^{-t/R}$	$\left(\frac{V_0}{R} - \frac{t^4}{24}\right) e^{-t/R}$	$Rt^4 - 2R^2 t^3 - 24V_0 = 0$

8.	$\frac{1}{s\left(s + \frac{1}{R}\right)^n}$	$R^{n+1} (1-e^{-t/R}) + \left(V_0 - R^n t - R^{n-1} \frac{t^2}{2!} - \dots - R^{n-(n-2)} \right) \frac{t^{n-1}}{(n-1)!} - \frac{R^{(n-1)} t^n}{n!} \right) e^{-\frac{t}{R}}$	$\left(\frac{V_0}{R} - \frac{t^n}{n!} \right) e^{-t/R}$	$Rt^n - nRt^{2n-1} - (n!)V_0$
9.	$\frac{1}{s\left(s + \frac{1}{C}\right)}$	$RC + V_0 e^{-t/R} + \frac{RC}{R-C} (Ce^{-t/C} - Re^{-t/R})$	$\frac{RC}{R-C} (e^{-t/R} - e^{-t/C}) - \frac{V_0}{R} e^{-t/C}$	$\frac{RC}{R-C} \ln \left[\frac{RC - V_0(R-C)}{R^2} \right]$
10.	$\frac{1}{s^2\left(s + \frac{1}{R}\right)}$	$R^2 (t-2R) + [V_0 + R^2 (t+2R)]e^{-t/R}$	$R^2 (1-e^{-t/R}) - \left(\frac{V_0}{R} + Rt \right) e^{-\frac{t}{R}}$	$-\frac{V_0}{R^2}$

3.3.4 Hubbert concept

If we represent the cumulative production by the symbol Q_P , the cumulative proved discoveries by Q_D (producible discovery: not all discovery are producible), and the proved reserves by Q_R , then for each year,

$$Q_D = Q_P + Q_R \text{ or } Q_R = Q_D - Q_P \quad (3.29)$$

The relation between rates of change of these quantities with time is obtained by taking the derivative with respect to time of equation (3.29), giving

$$\frac{dQ_D}{dt} = \frac{dQ_P}{dt} + \frac{dQ_R}{dt} \text{ or } \frac{dQ_R}{dt} = \frac{dQ_D}{dt} - \frac{dQ_P}{dt} \quad (3.30)$$

in which;

- $\frac{dQ_D}{dt}$ is the rate of discovery,
- $\frac{dQ_P}{dt}$ is the rate of production, and
- $\frac{dQ_R}{dt}$ is the rate of increase of the proved reserves (Hubbert, 1956).

The manner in which the three quantities Q_D , Q_P , and Q_R vary with respect to time during the entire history of petroleum resources production from start to finish must be approximately as follows:

- 1) The cumulative production Q_P , when plotted as a function of time, will increase slowly during the early stages of petroleum resources production, increasing even more rapidly with time to about the halfway point, and then continue its ascent by rising slowly, and finally leveling off as the ultimate figure Q_∞ , as production ceases (Hubbert, 1956).
- 2) The curve of proved reserves Q_R will normally start at zero, rise gradually up until a maximum is reached at about the halfway point, and then gradually decline to zero.
- 3) As petroleum must be found before it can be produced, the curve of cumulative proved discoveries Q_D must closely replicate that of cumulative production, except that it must

plot ahead of the production curve by some time interval Δt , which on itself may vary during the cycle (Hubbert, 1956).

A plot of the family of the three curves Q_D , Q_P and Q_R is as shown in Figure 3.3 below, as they may be expected to appear in the case of cumulative production, of petroleum in Nigeria.

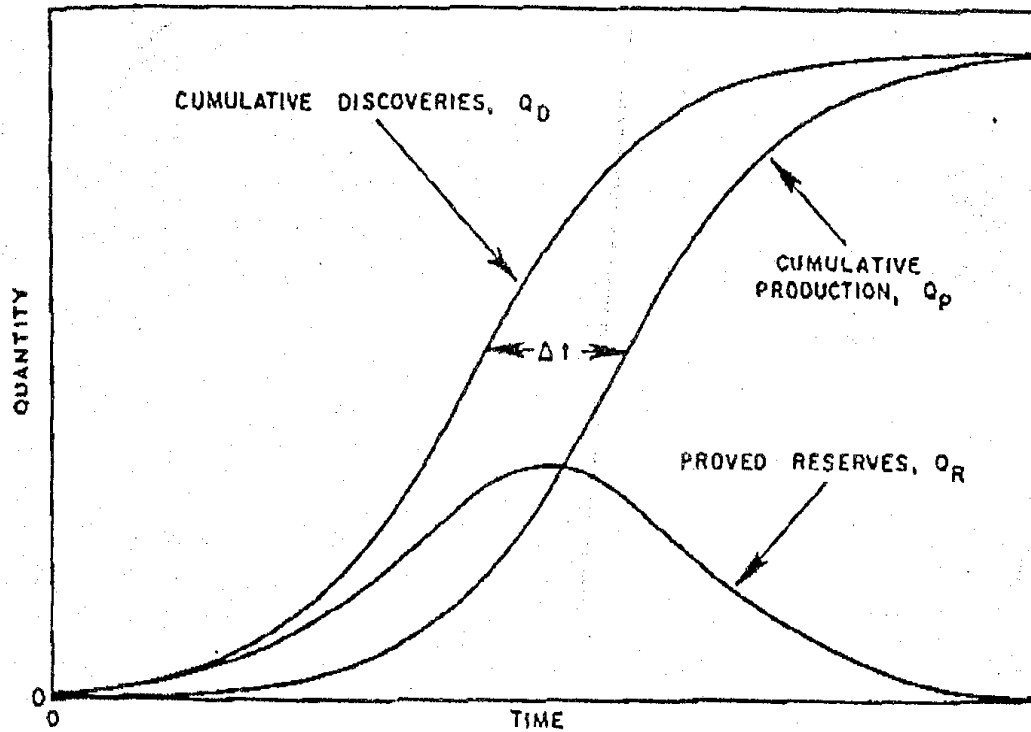


Figure 3.3: Typical cumulative discoveries, production, and proved reserves (Hubbert, 1956).

Because of the close similarity between the curve of cumulative proved discoveries and that of cumulative production, it follows therefore that the study of the discovery curve must give one a preview of what production will do at a time of approximately Δt in the future (Hubbert, 1956).

Taking the time derivatives of the three curves shown in Figure 3.3 gives us the rate of discovery, rate of production, and rate of increase of proved reserves, which are all plotted as functions of time in Figure 3.4. It is important to note that the rate of discovery will reach a peak at about mid-range, then thereafter, gradually decline to zero.

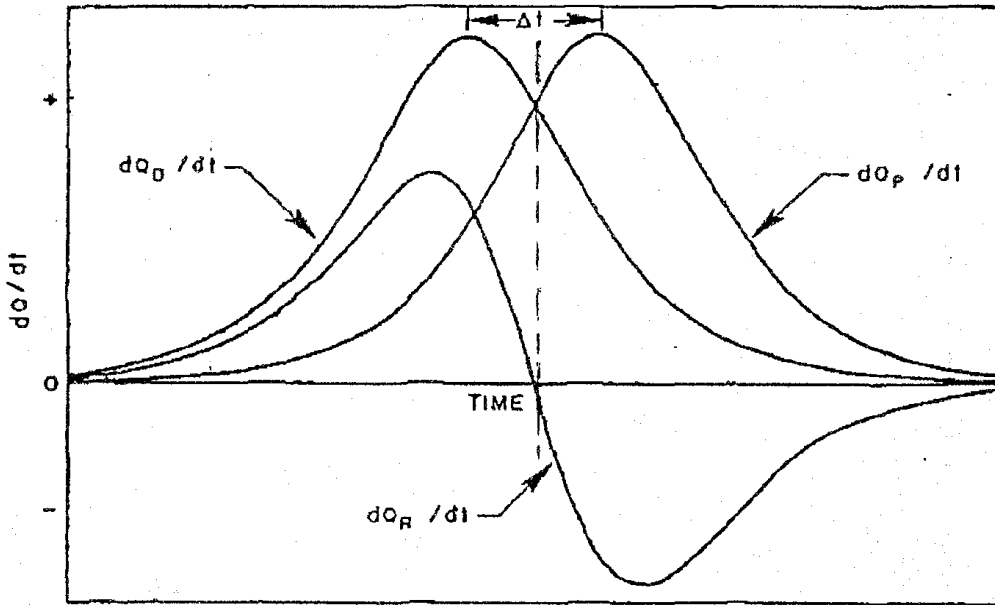


Figure 3.4: Rates of discovery, production and change of proved reserves (Hubbert, 1956).

The rate of production will reach a peak at a time about Δt after that of discovery, and then the increase of proved reserves will change from positive to negative about halfway between the discovery and production peaks. The reserves themselves, Q_R , will reach a maximum at the same time.

The relations between the three curves at this midpoint can be observed by noting that when reserves reach their maximum value, their derivative with time becomes zero.

$$\frac{dQ_R}{dt} = 0 \quad (3.31)$$

Which when inserted into equation (3.30) gives;

$$\frac{dQ_D}{dt} = \frac{dQ_P}{dt} \quad (3.32)$$

This tells us clearly that when reserves reach their maximum value, the curve of discovery rate and production rate will cross, with the production curve going up and that of discovery going down. This is shown in Figure 3.4.

$$\text{Then from equations (3.29) and (3.30), we see that; } y = Q \quad (3.33)$$

Which is a cumulative (sigmoidal) discovery or production profile with t as time (an independent variable in years) and y (or Q) is a dependent variable as petroleum volume in (mmB or bscf).

When equation (3.33) is differentiated with respect to time, we obtain; $\frac{dy}{dt} = \frac{dQ}{dt}$ (3.34)

The equation (3.32) which is an annual production or discovery model has a dumb-bell profile. Recall that although, $y \equiv Q_D \equiv Q_P$, i.e in shapes, but the numerical values of the characteristic constants of the model for production and discovery are not necessarily going to be the same, so that their coefficients of correlation (R^2) are not the same.

It is like solving two non-linear equations simultaneously or graphically, at their intersection, $\frac{dQ_P}{dt} = \frac{dQ_D}{dt}$ (the rates are equal at the intersection) and the time when this occurs is when the nation's petroleum reserve is finished (see figure 3.4).

3.3.4.1 Hubbert equation for discovery/production and peaking

The Hubbert equation for oil and gas resources discovery, production and peaking is as shown in equation (3.35);

$$Q_t = \frac{Q_{max}}{1 + ae^{-bt}} \quad (3.35)$$

Where, Q_t = cumulative discovery/production

Q_{max} = ultimate value

Differentiating eqn (3.35) for the production-time history, we have,

$$\frac{dQ}{dt} = P = \frac{1 + ae^{-bt} (0) - Q_{max}(-abe^{-bt})}{(1 + ae^{-bt})^2} = \frac{abQ_{max}e^{-bt}}{(1 + ae^{-bt})^2} \quad (3.36)$$

Where, P = annual production

And at the peak:

$$\frac{d^2Q}{dt^2} = 0 = \frac{(1 + ae^{-bt})^2 (-ab^2e^{-bt}) - (abQ_{max}e^{-bt})^2 (1 + ae^{-bt})(abQ_{max}e^{-bt})}{(1 + ae^{-bt})^4} \quad (3.37)$$

Simplifying equation (3.37), we have eqn (3.38)

$$-ab^2Q_{max}e^{-bt} - a^2b^2Q_{max}e^{-2bt} = -2a^2b^2Q_{max}^2e^{-2bt}$$

$$-ab^2Q_{max}e^{-bt} = a^2b^2Q_{max}(1 - 2Q_{max})e^{-2bt}$$

$$ab^2Q_{max}e^{-bt} = a^2b^2Q_{max}(2Q_{max} - 1)e^{-2bt}$$

$$1 = a(2Q_{max} - 1)e^{-bt}$$

Taking log of the above;

$$\ln 1 = 0 = \ln a(2Q_{max} - 1) - bt$$

$$- \ln a(2Q_{max} - 1) = -bt$$

$$\ln a(2Q_{max} - 1) = bt$$

$$t_{pk} = \frac{1}{b} \ln a(2Q_{max} - 1) \quad (3.38)$$

But from $Q_t = \frac{Q_{max}}{2}$, at peak, $Q_{max} = 1$ or $Q_t = \frac{1}{2}$

$$\text{Hence, } t_{pk} = \frac{1}{b} \ln a \quad (3.39)$$

Note: a and b are constants which will be declared by the analysis GUI of MATLAB curve-fitting toolbox.

3.4 Model Validation

3.4.1 The sigmoidal and dumbbell plots using MATLAB 7.9

The data obtained from Department of Petroleum Resources, Ministry of Petroleum and Minerals, Nigeria, were used in plotting scatter diagrams, and the models $y = Q_D = Q_P$ were superimposed to obtain sigmoidal profiles. Then, the analysis section of the MATLAB software was used to produce the differentiated data (dumb-bell) with time from the sigmoidal data. This differentiated data which are dumb-bell profiles were plotted and the differentiated models $\frac{dy}{dt} = \frac{dQ_P}{dt} = \frac{dQ_D}{dt}$ superimposed on them and the intersections of the discovery Q_D and production Q_P read off.

3.4.1.1 Cumulative plots

In cumulative plots, the result is always sigmoidal in profile. Hence, the data for discovery is many times bigger than that for production and so they are plotted separately. The combined plots will show the discovery profile pushing production profile down to almost align with zero (x-axis). The difference between their y-axis values is the cumulative reserve.

3.4.1.2 Dumb-bell plots (Rate plots)

Because of the difference between the apparent and real interception points, the rate of change plots (Dumb-bell) are done in duplicates: the 1st showing the apparent intersection, while the 2nd shows enlarged real intersection points. The real intersection point will show the point at which the reserve is exhausted so that the rate of discovery will be equal to rate of production which is a very important point for the petroleum industry of a country.

3.5 Model Resolution and Determination of Model Constants

3.5.1 Method of solution of model equations

A pair of models were developed from solving the first order ODE using Laplace transformation techniques. The ODE models developed were easily solved using Laplace transformation techniques because they are time dependent models. The first solution is a cumulative which is sigmoidal in shape. The second solution is a derivative of the first and it is an annual rate dumbbell.

The two models must plot and predict for them to be accepted. When the first cumulative model is superimposed on a cumulative scatter diagram, it must plot and taper out asymptotically giving us a sigmoidal profile with ultimate reserve.

The analysis GUI of MATLAB curve-fitting toolbox was applied to produce first derivative numerical values which should be a dumbbell in their scatter diagram profile. At the same time the first derivative of the cumulative model was obtained as an analytical solution and superimposed on the dumbbell profile. This analytical solution should predict the numerical

values with an accuracy near 100%, which is equivalent to R^2 values very close to 1, else the model is faulty.

3.5.2 Determination of model constants

The superimposition of the cumulative (sigmoidal) model on a cumulative data allow the MATLAB curve-fitting toolbox to declare the model constants and goodness of fit, if the model fits to the data up to 95% and above. The differentiated plot of the dumbbell using the declared data of the analysis section does not have a new constant, it merely uses the values of the constants of the cumulative model, hence the R-squared (R^2) must be 1.

3.6 Sensitivity or Trend Test Analysis

The sensitivity analysis of this work was performed on the chosen model 5, 6 & 7, as this was chosen over other mathematical models that were developed. Model five (5) was adjudged the best model because:

- 1) It plotted both the cumulative production and discovery data
- 2) It was able to predict because the curve tappers out asymptotically
- 3) It has a positive initial volume (V_0)
- 4) It has the best R^2 as declared by the MATLAB software

The modeling was carried out to predictively model the production peak, ultimate oil and gas discovery by using the cumulative of the raw data on oil and gas production and discovery gotten from DPR.

For a work of this magnitude to actually worth its value, then sensitivity or trend test analysis must be performed.

Sensitivity or trend test analysis is a statistical tool that is used to study how the uncertainty in the output of a mathematical model or system can be apportioned to different sources of uncertainty in its input. It actually works on the principle of changing the model or the variables

of the model and observing the behaviours. While the changes in the models are being carried out, we observe the following:

- i. The values of the objectives as per the strategy
- ii. Values of the decision variables

The interpretation of sensitivity analysis is that the higher the sensitivity figures, the more sensitive the output is to change in that input and vice-versa.

In this work, detailed mathematical models were developed from first principle and assumptions of certain conditions of a physical system were invoked. The traditional process in model formulation has already been carried out from material balance on a composite reservoir with approximate physical system and solving correctly the resulting equations there-in.

However, there is no way a model can be developed without assumptions which introduce a level of error. The effect of the error on any developed model determines its capability to represent the system. If the error is so much the model will fail to capture some or all key characteristics of the physical system and therefore must be discarded or modified. If the error is within the acceptable and practical limits, the developed model will duly represent the physical system and therefore can be used in further analytical interpretations.

Thus sensitivity can be calculated by finding the percentage change in the output and percentage change in the input and by dividing the percentage change of output over percentage change of input. i.e
$$\text{Sensitivity} = \frac{\% \text{ change of output}}{\% \text{ change of input}}$$

With regards to the cumulative production models developed in this work, the cumulative production is expected to rise from zero at zero time to the ultimate recovery and there after remaining constant throughout the time. In the course of this work, i.e the trend test analysis, it was found that the most crucial determinant of the acceptability of model 5 or any other model are dependent on the cumulative value of production for the first ten years. Cumulative

productions are expected to be non-negative throughout the time, and there should be no production at zero time.

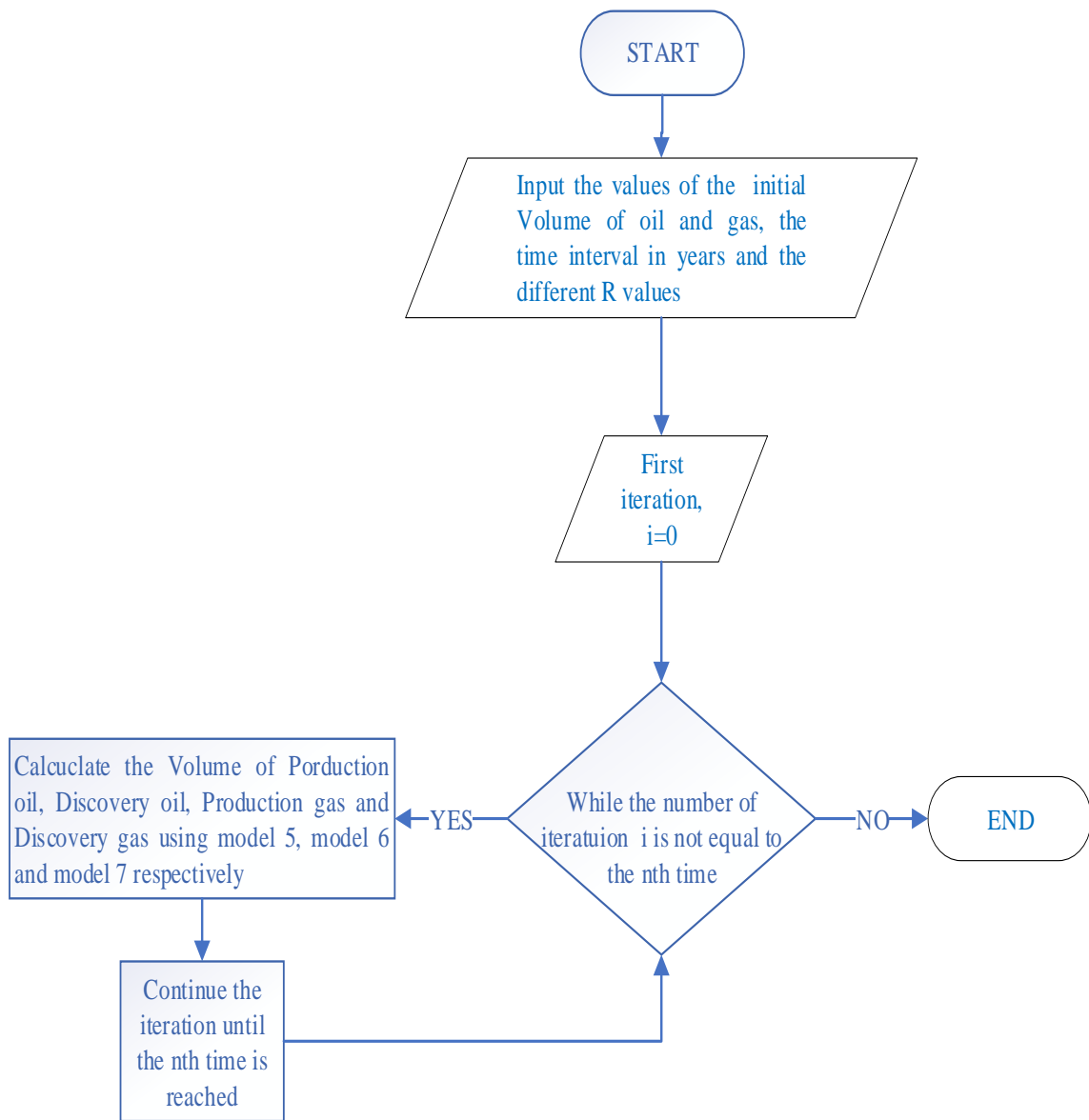


Figure 3.5: Sensitivity analysis flowchart

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Results

4.1.1 Curve fittings of cumulative and dumb-bell plots for model validation

The curve-fitting done with the models in Table 3.1 and data from Appendices B, C and D are shown in figures 4.1, 4.2, 4.3 and 4.9, in which the models (1, 2, 3 and 10 respectively) did not fit or predict the data accurately. Also shown are figures 4.4 – 4.8 (A, B, C) for oil, figures 4.4 – 4.8 (D, E, F) for gas, and figures 4.10 (A, B, C) for oil, figures 4.10 (D, E, F) for gas, in which cases the models (4, 5, 6, 7, 9 and 11) both fitted and predicted the data adequately. The difference between their respective y-axis values gave their cumulative reserves.

The rate of change of discovery and production with time are also shown in figures 4.4 – 4.6 (D_O) and (D_{OI}), or (D_G) and (D_{GI}). Also presented are figures 4.7 (D_{OI}) and (D_{GI}), figures 4.8 (D_{OI}) and (D_{GI}), and figures 4.10 (D_{OI}) and (D_{GI}). The D_{OI} 's represent graphs of real intersections for oil, while D_{GI} 's represent graphs of real intersections for gas. But D_O and D_G refer to graphs of apparent intersections for oil and gas respectively. Because of the difference between the apparent and real interception points, the rate of change plots (Dumb-bell) were done in duplicates: the 1st showing the apparent intersection, i.e Figures 4.4 – 4.6 (D_O and D_G), while the 2nd shows enlarged real intersection points, i.e Figures 4.4 – 4.6 (D_{OI} and D_{GI}), figures 4.7 (D_{OI} and D_{GI}), figures 4.8 (D_{OI} and D_{GI}), and Figure 4.10 (D_{OI} and D_{GI}) for the control model 11.

The result tables of models 4 – 7, and 9, with that of the control model 11 (Hubbert) are shown in Tables 4.1 – 4.6 as read out from their respective plots. Table 4.7 is a comprehensive result table of all the models, deductions, and inferences or remarks.

The results of the sensitivity or trend test analysis are shown in Figures 4.11 (A – D), figures 4.12 (A – D), and figures 4.13 (A – D), respectively for the good models 5, 6 and 7, for both oil and gas production and discovery.

4.1.1.1 Plots of cumulative data for oil and gas discovery/production for models 1-7, 9, 10, 11

The cumulative plots for oil and gas production and discovery for the developed models and control model (Hubbert) are presented in this section.

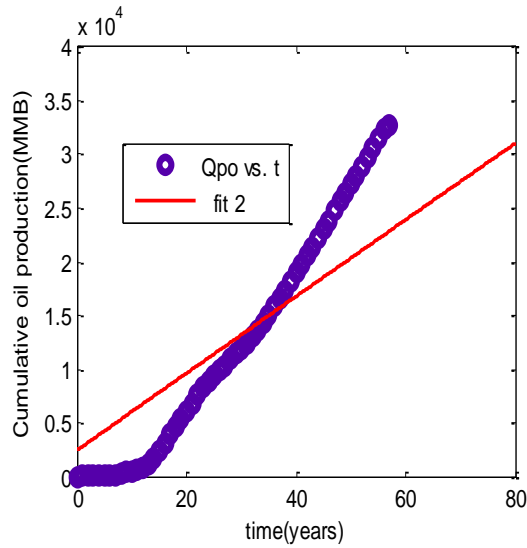


Fig 4.1: Cumulative oil production versus time for model 1 ($R^2 = 0.7969$). Model 1 does not fit, does not predict.

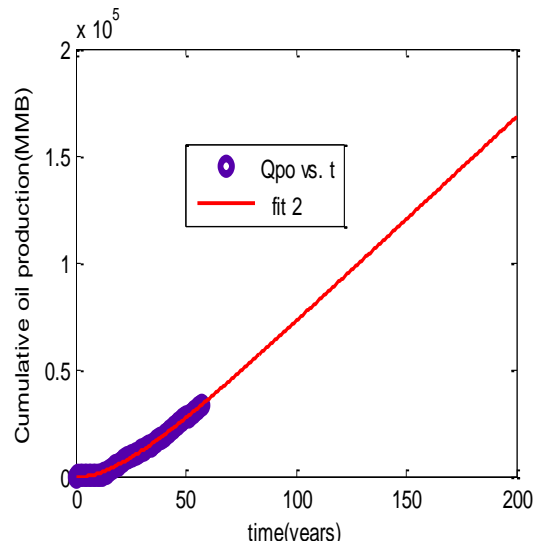


Fig 4.2: Cumulative oil production versus time for model 2 ($R^2 = 0.9978$). Model 2 fitted, but does not predict.

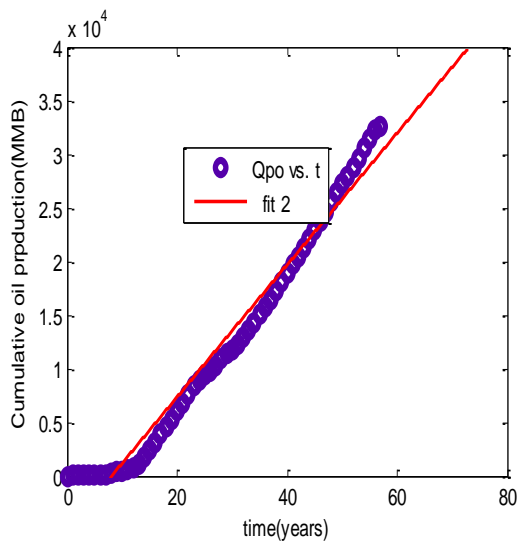


Fig 4.3: Cumulative oil production versus time for model 3 ($R^2 = 0.9708$). Model 3 does not fit, does not predict.

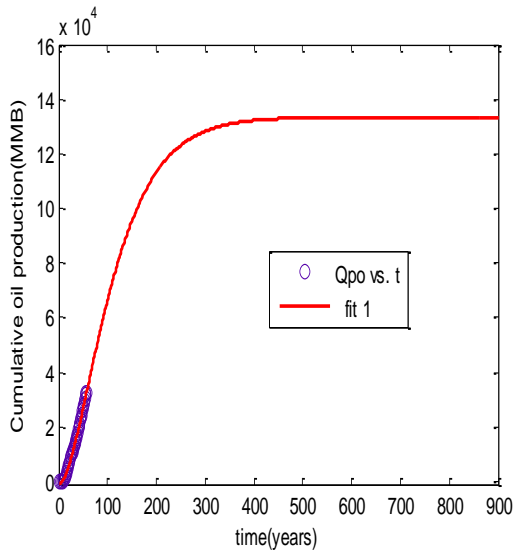


Fig 4.4A: Cumulative oil production versus time for model 4 ($R^2 = 0.9978$).

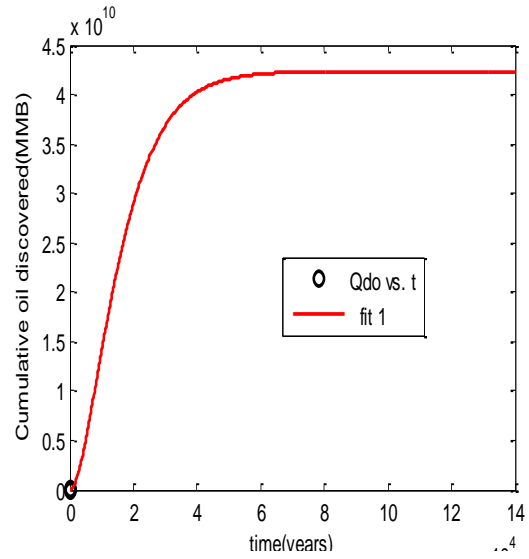


Fig 4.4B: Cumulative oil discovery versus time for model 4 ($R^2 = 0.9983$).

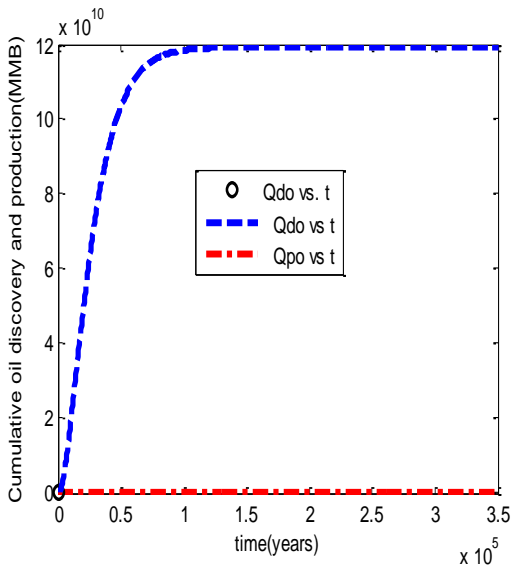


Fig 4.4C: Cumulative oil discovery and production versus time for model 4.

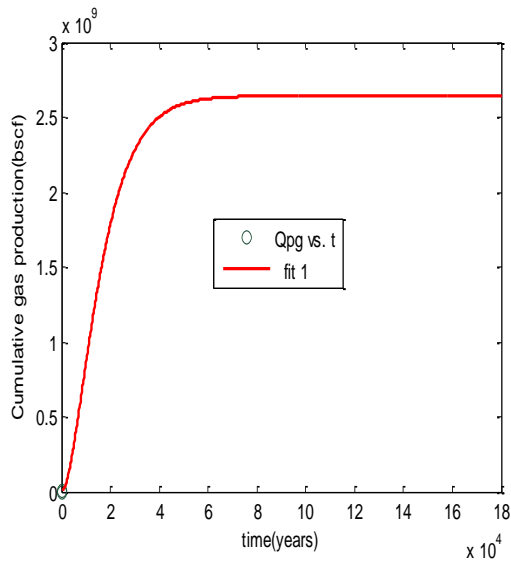


Fig 4.4D: Cumulative gas production versus time for model 4 ($R^2 = 0.9872$).

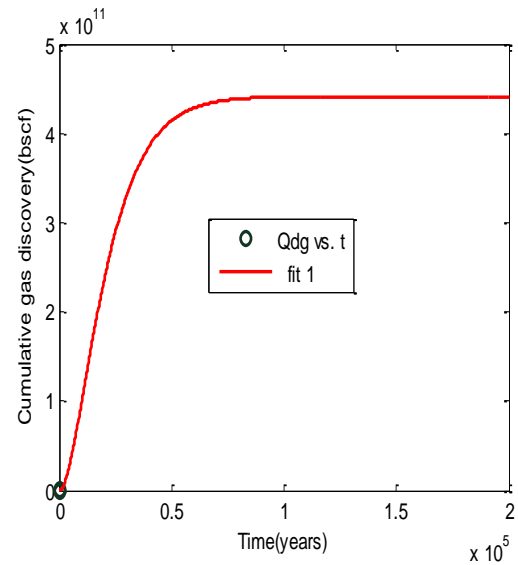


Fig 4.4E: Cumulative gas discovery versus time for model 4 ($R^2 = 0.9989$).

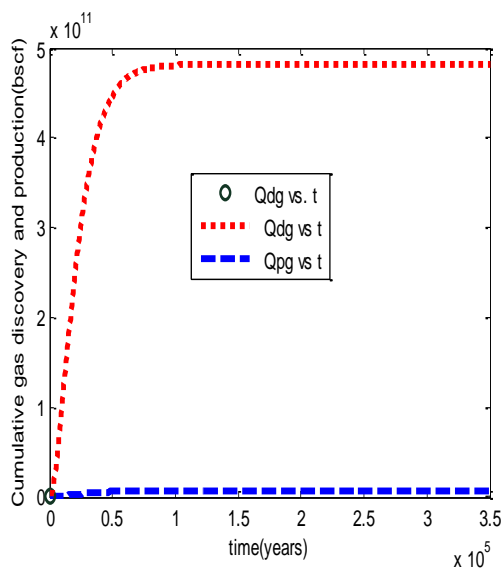


Fig 4.4F: Cumulative gas discovery and production versus time for model 4.

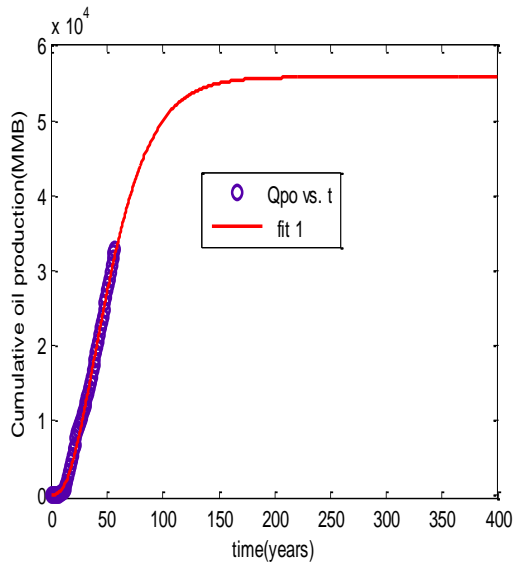


Fig 4.5A: Cumulative oil production versus time for model 5 ($R^2 = 0.9964$).

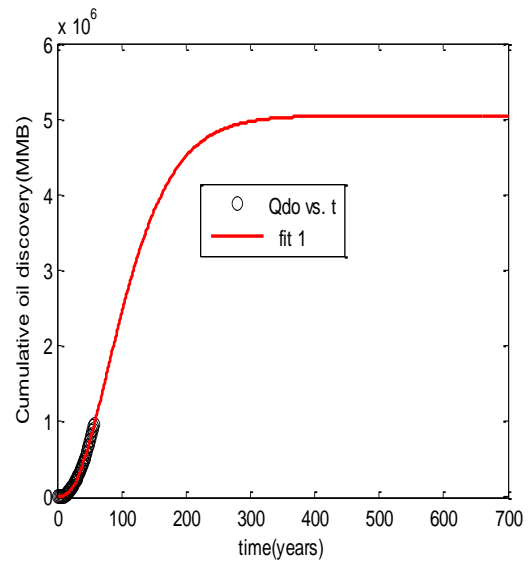


Fig 4.5B: Cumulative oil discovery versus time for model 5 ($R^2 = 0.9957$).

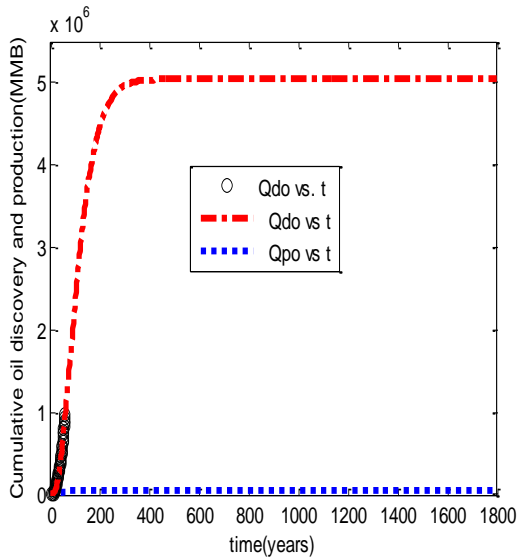


Fig 4.5C: Cumulative oil discovery and production versus time for model 5.

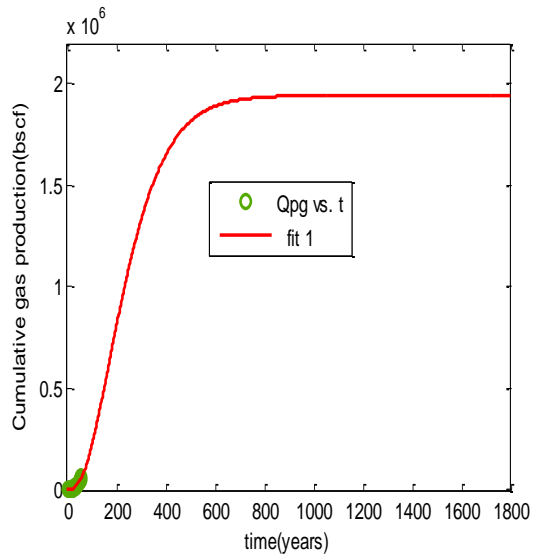


Fig 4.5D: Cumulative gas production versus time for model 5 ($R^2 = 0.9983$).

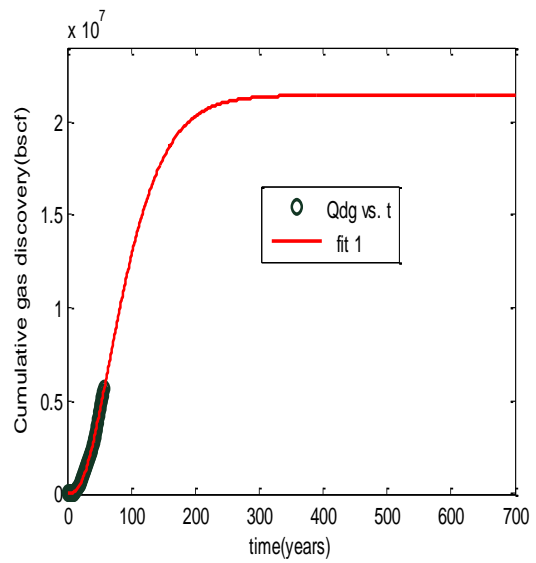


Fig 4.5E: Cumulative gas discovery versus time for model 5 ($R^2 = 0.9979$).

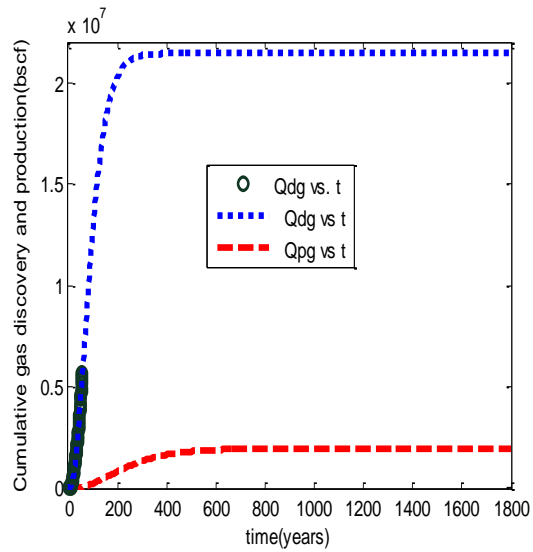


Fig 4.5F: Cumulative gas discovery and production versus time for model 5.

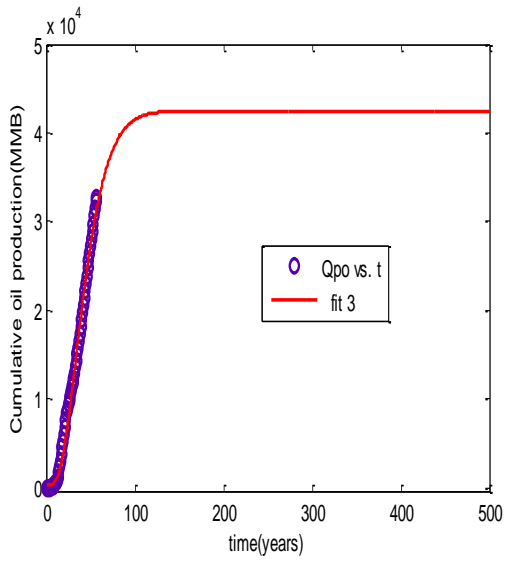


Fig 4.6A: Cumulative oil production versus time for model 6 ($R^2 = 0.9925$).

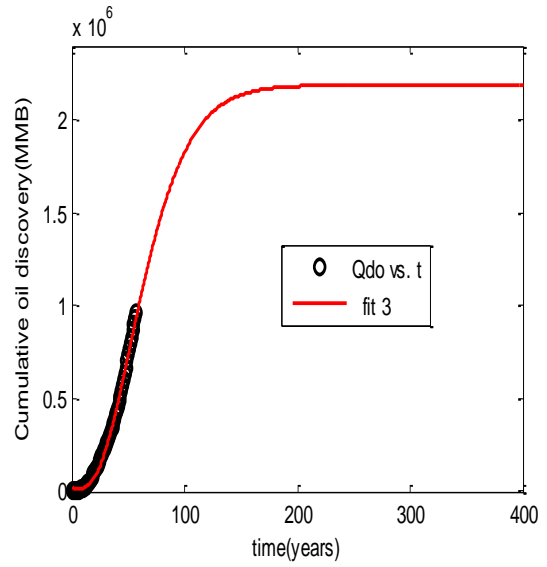


Fig 4.6B: Cumulative oil discovery versus time for model 6 ($R^2 = 0.9953$).

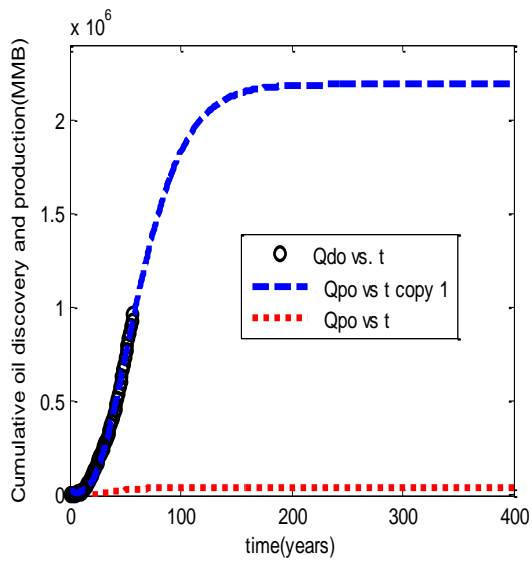


Fig 4.6C: Cumulative oil discovery and production versus time for model 6.

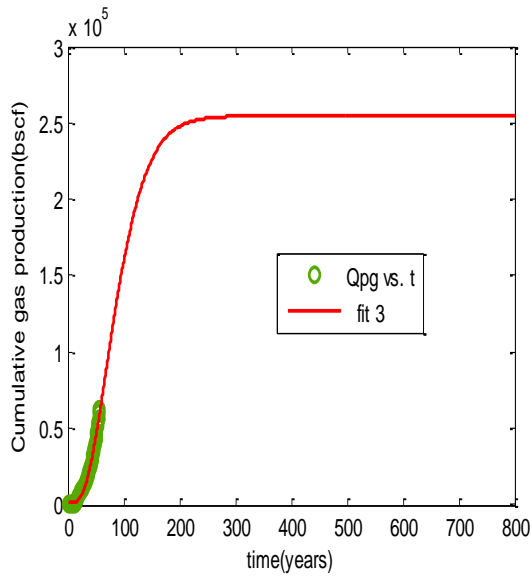


Fig 4.6D: Cumulative gas production versus time for model 6 ($R^2 = 0.9919$).

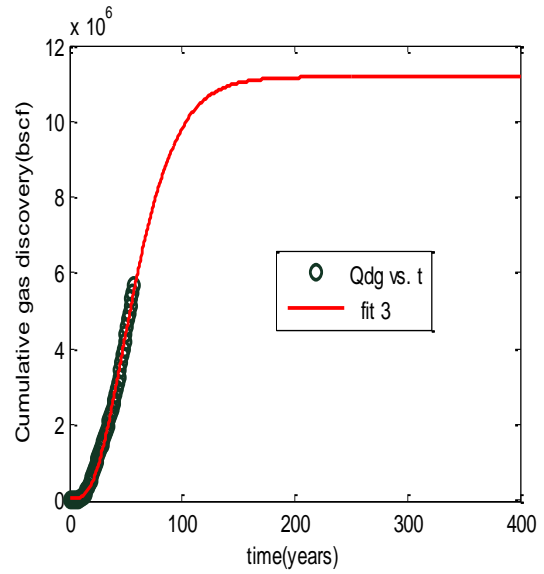


Fig 4.6E: Cumulative gas discovery versus time for model 6 ($R^2 = 0.9950$).

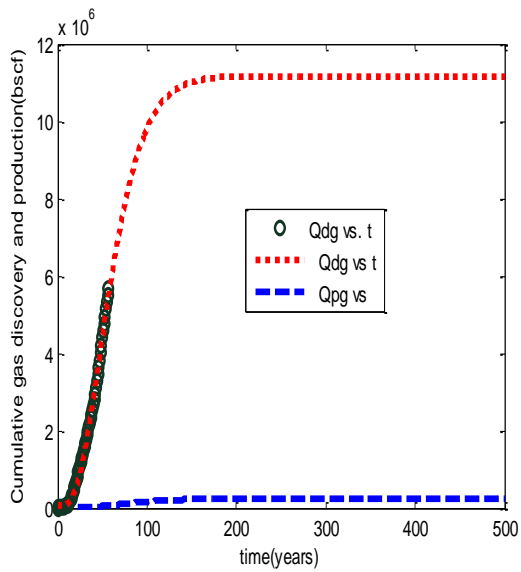


Fig 4.6C: Cumulative gas discovery and production versus time for model 6.

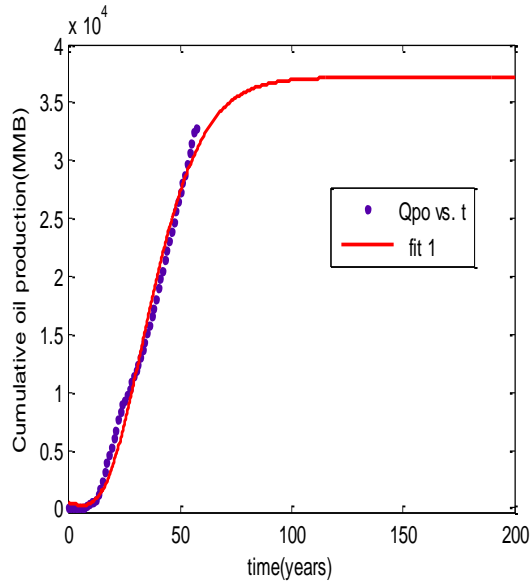


Fig 4.7A: Cumulative oil production versus time for model 7 ($R^2 = 0.9872$).

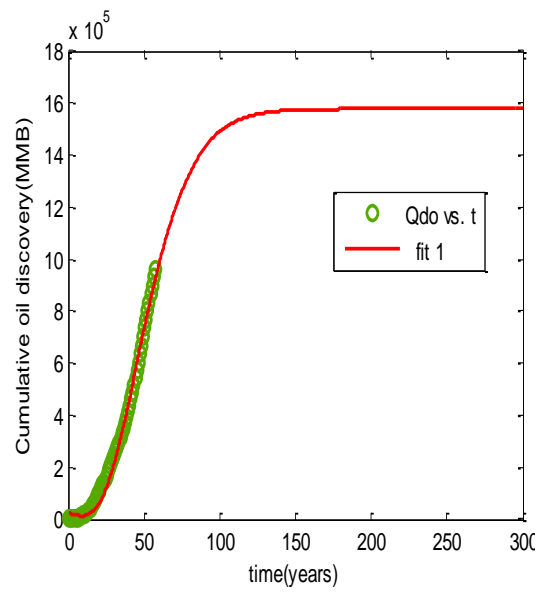


Fig 4.7B: Cumulative oil discovery versus time for model 7 ($R^2 = 0.9912$).

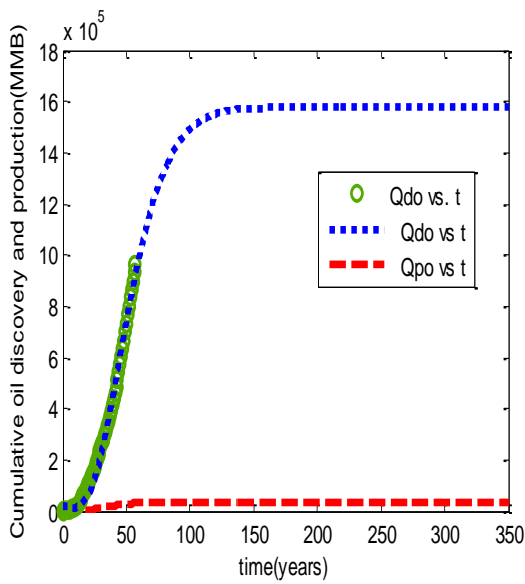


Fig 4.7C: Cumulative oil discovery and production versus time for model 7.

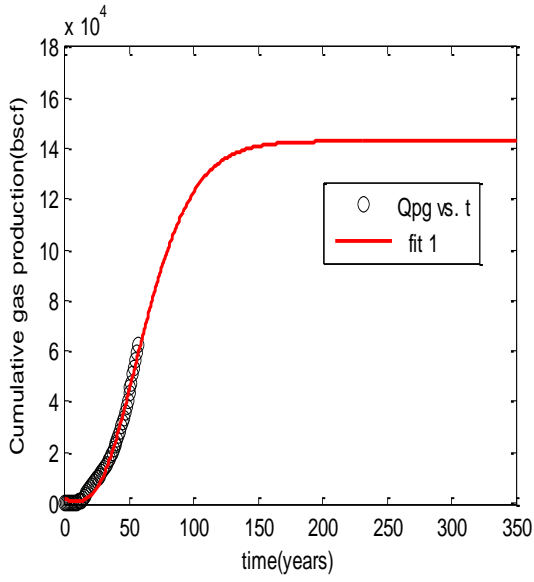


Fig 4.7D: Cumulative gas production versus time for model 7 ($R^2 = 0.9873$).

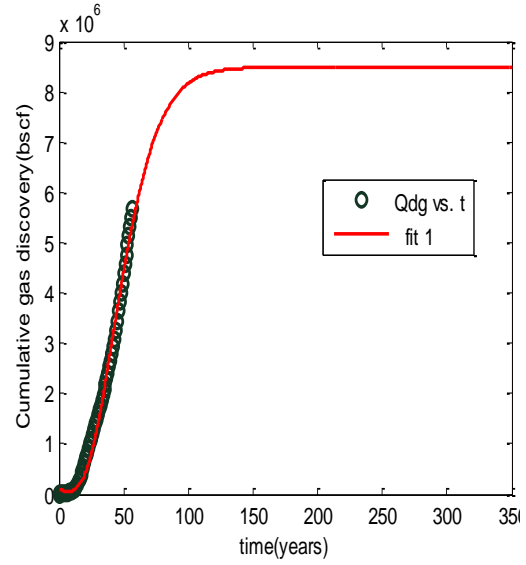


Fig 4.7E: Cumulative gas discovery versus time for model 7 ($R^2 = 0.9910$).

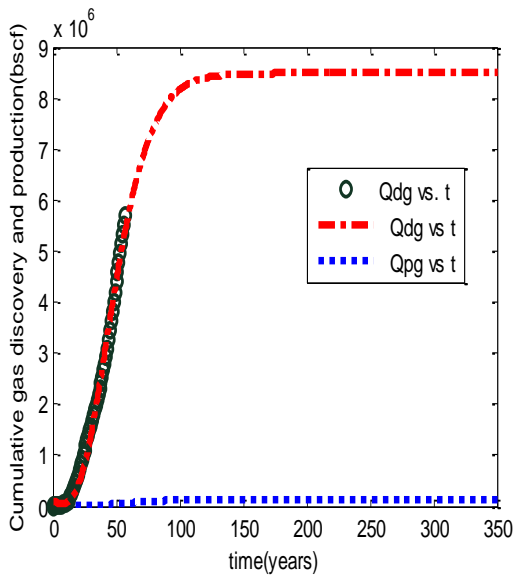


Fig 4.7F: Cumulative gas discovery and production versus time for model 7.

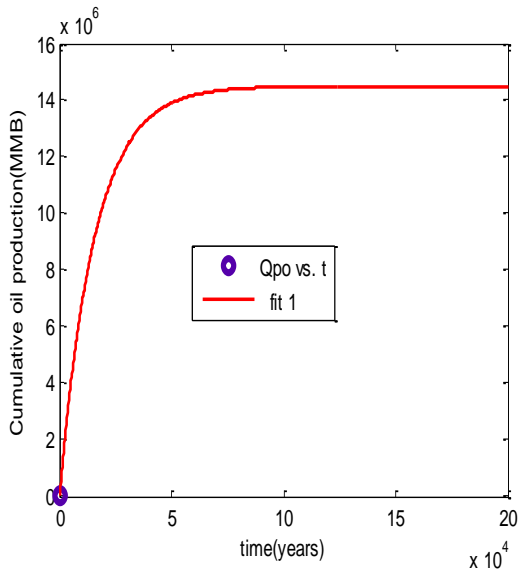


Fig 4.8A: Cumulative oil production versus time for model 9 ($R^2 = 0.9979$).

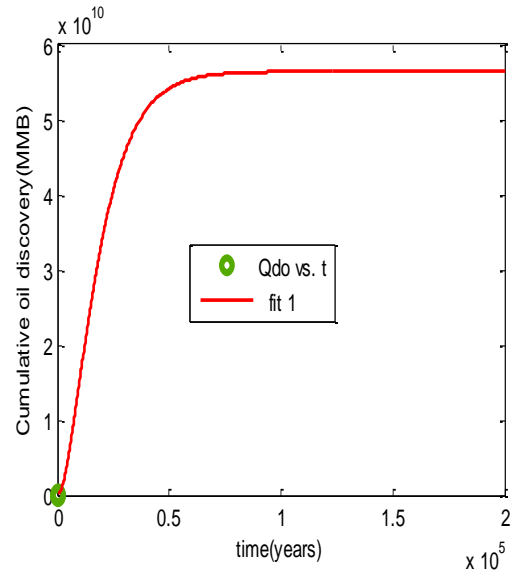


Fig 4.8B: Cumulative oil discovery versus time for model 9 ($R^2 = 0.9983$).

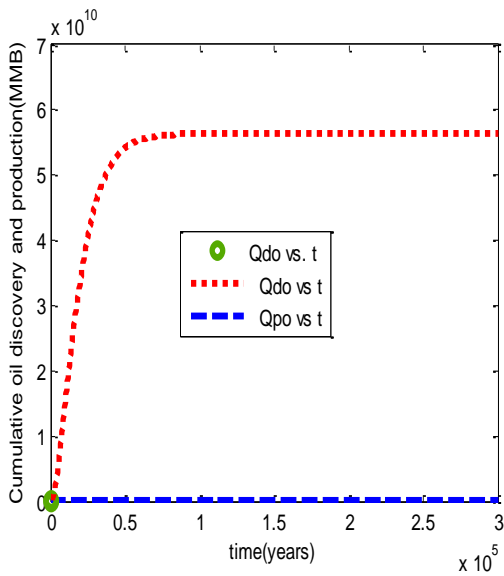


Fig 4.8C: Cumulative oil discovery and production versus time for model 9.

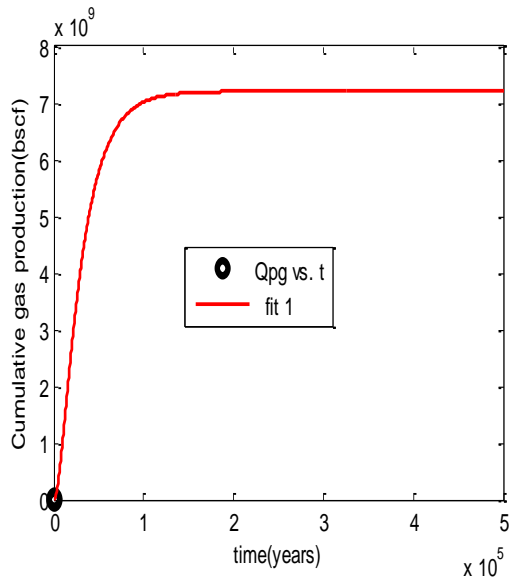


Fig 4.8D: Cumulative gas production versus time for model 9 ($R^2 = 0.9872$).

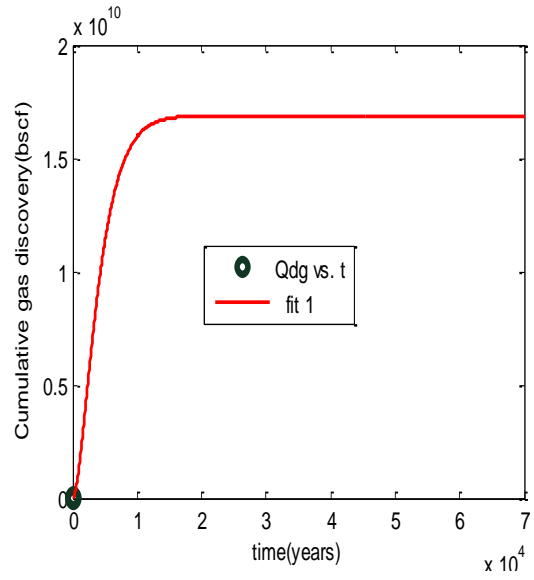


Fig 4.8E: Cumulative gas discovery versus time for model 9 ($R^2 = 0.9988$).

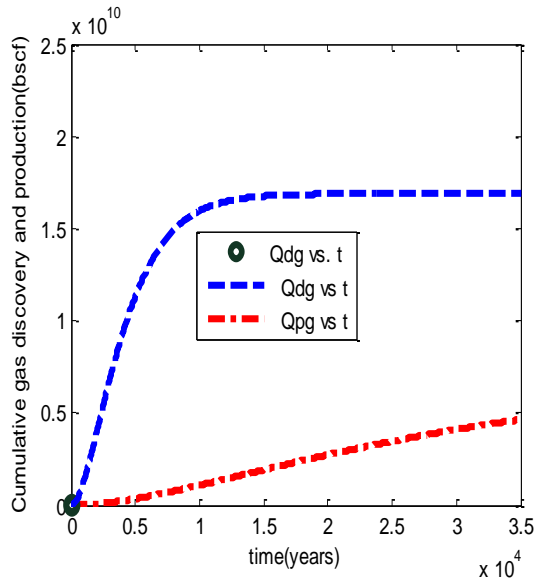


Fig 4.8F: Cumulative gas discovery and production versus time for model 9.

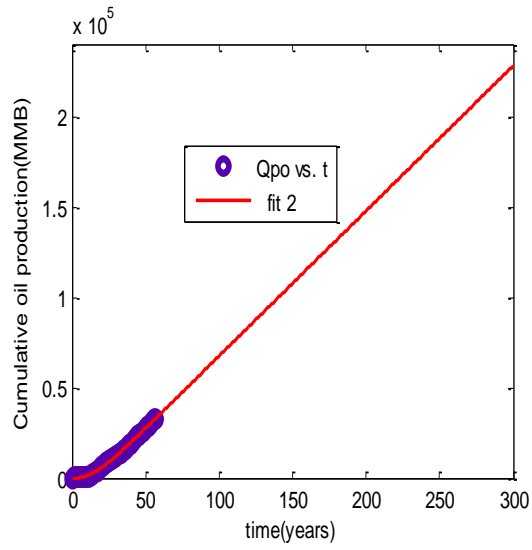


Fig 4.9: Cumulative oil production versus time for Model 10 ($R^2 = 0.9979$)
Model 10 fitted, but does not predict

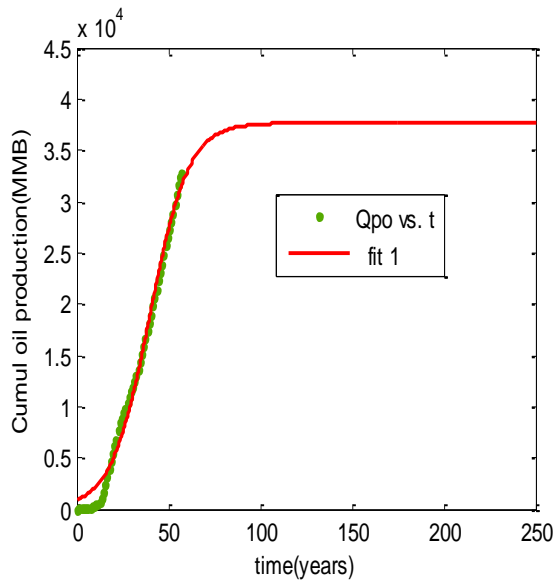


Fig 4.10A: Cumulative oil production versus time for control model 11, Hubbert ($R^2 = 0.9900$).

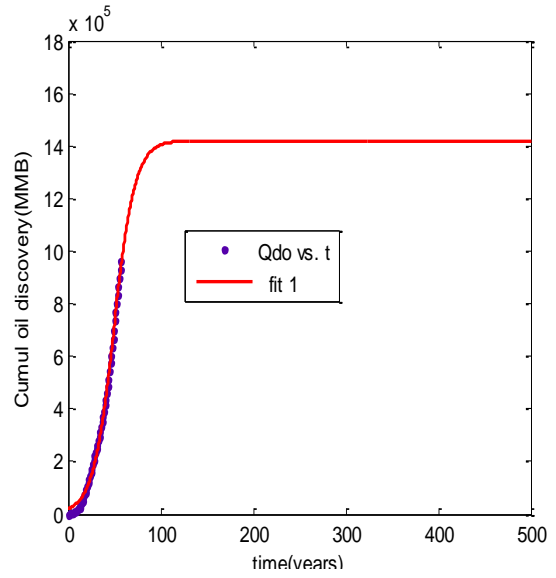


Fig 4.10B: Cumulative oil discovery versus time for control model 11, Hubbert ($R^2 = 0.9967$).

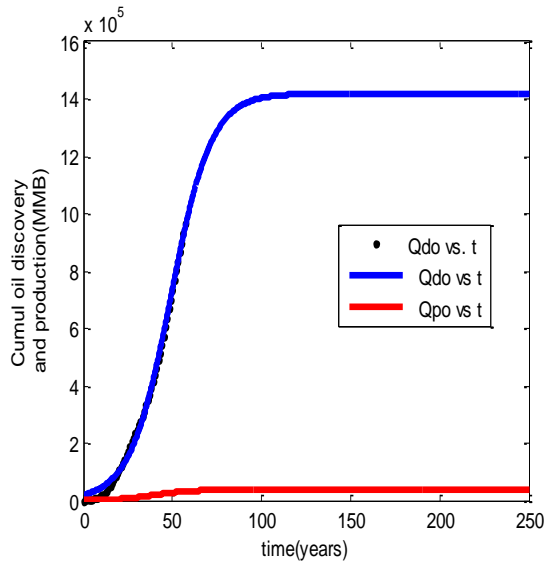


Fig 4.10C: Cumulative oil discovery and production versus time for control model 11, Hubbert.

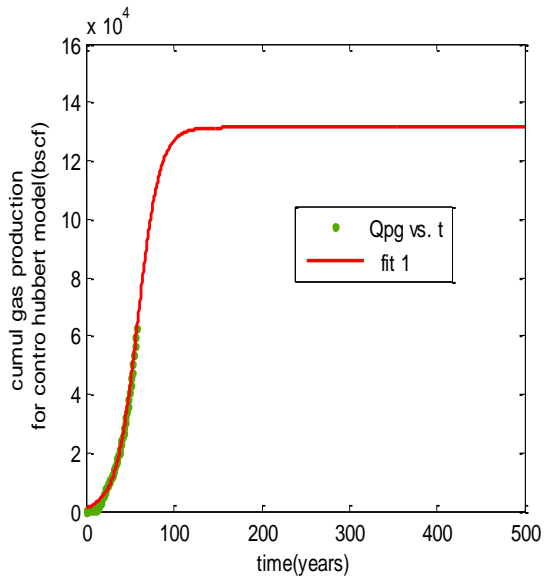


Fig 4.10D: Cumulative gas production versus time for control model 11, Hubbert ($R^2 = 0.9954$).

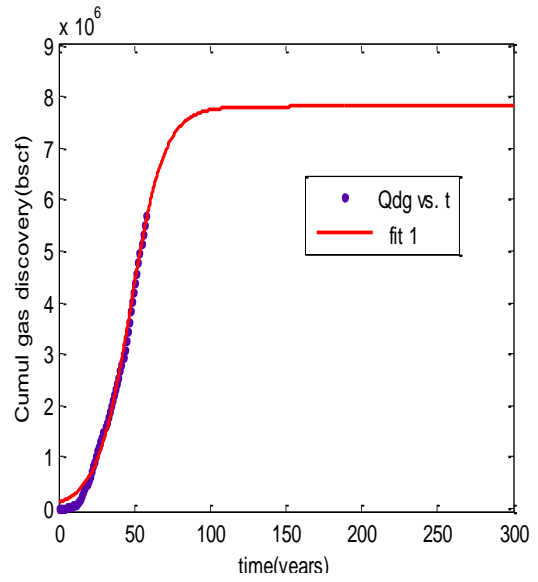


Fig4.10E: Cumulative gas discovery versus time for control model 11, Hubbert ($R^2 = 0.9944$).

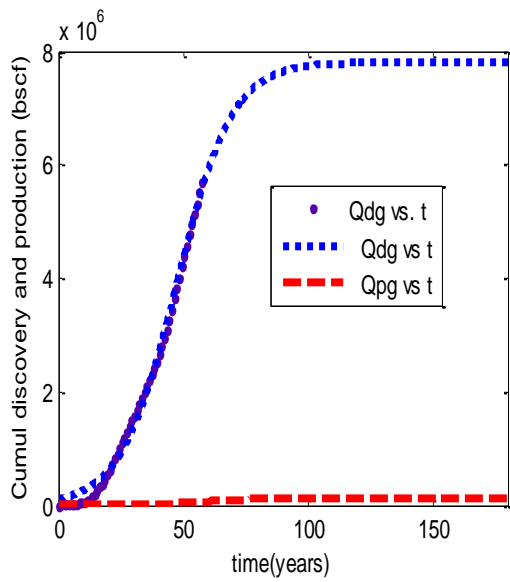


Fig 4.10F: Cumulative gas discovery and production versus time for control model 11, Hubbert.

4.1.1.2 Plots showing rate of change of oil and gas discovery/production (dumb-bell plots) for models 4-7, 9 and 11

The dumb-bell plots for the oil and gas production and discovery for the good models are presented in this section.

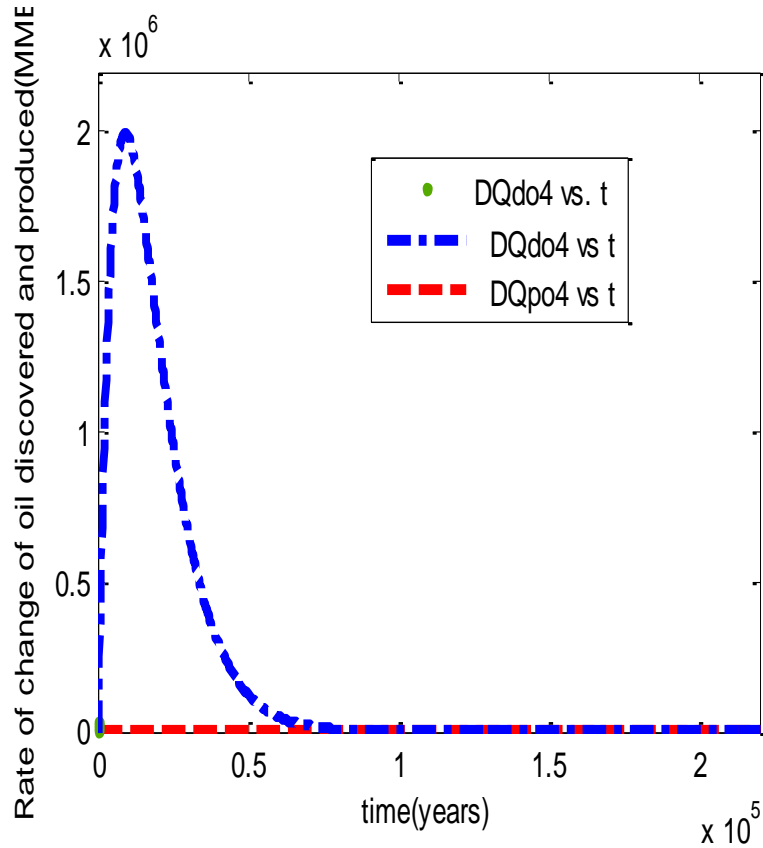


Fig 4.4(D₀): Rate of change of oil discovered and produced versus time for model 4 (apparent intersection).

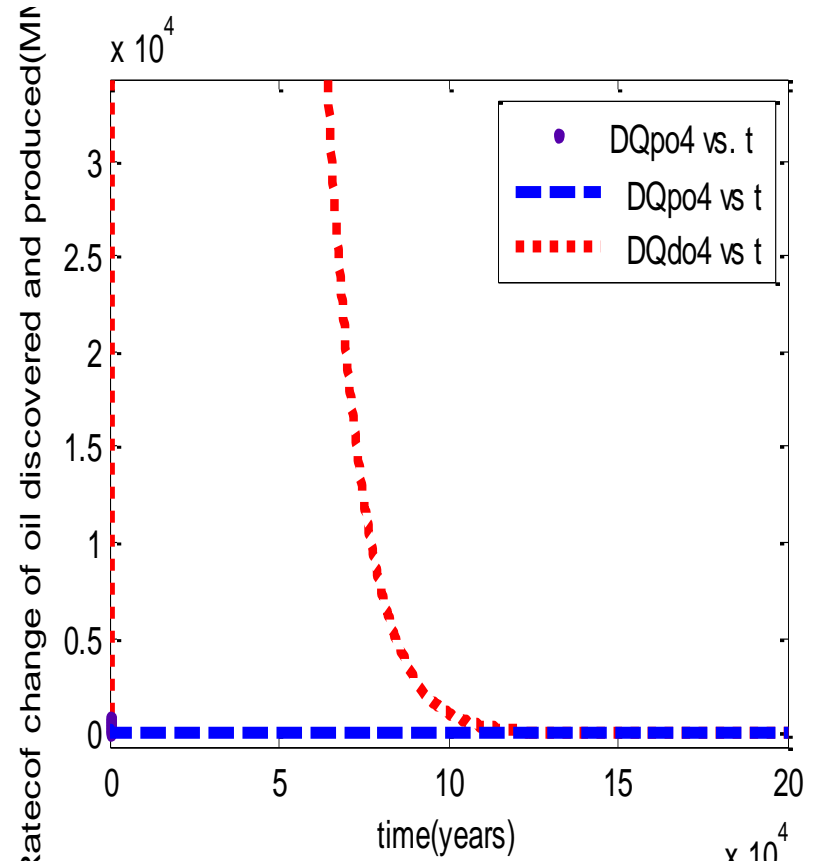


Fig 4.4(D_{0I}): Rate of change of oil discovered and produced versus time for model 4 (real intersection).

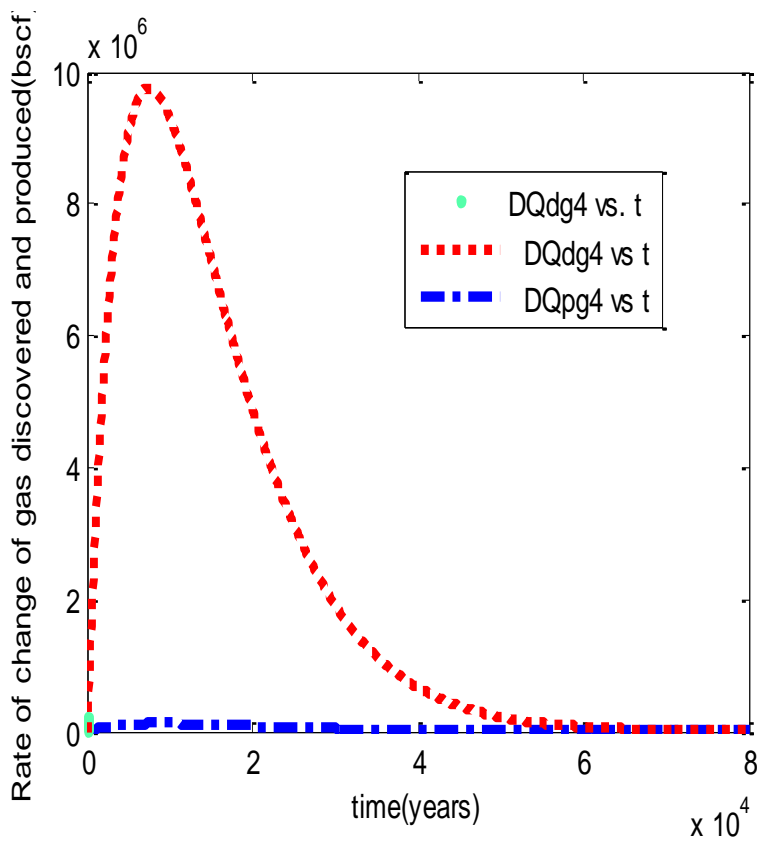


Fig 4.4(D_G): Rate of change of gas discovered and produced versus time and model 4 (apparent intersection).

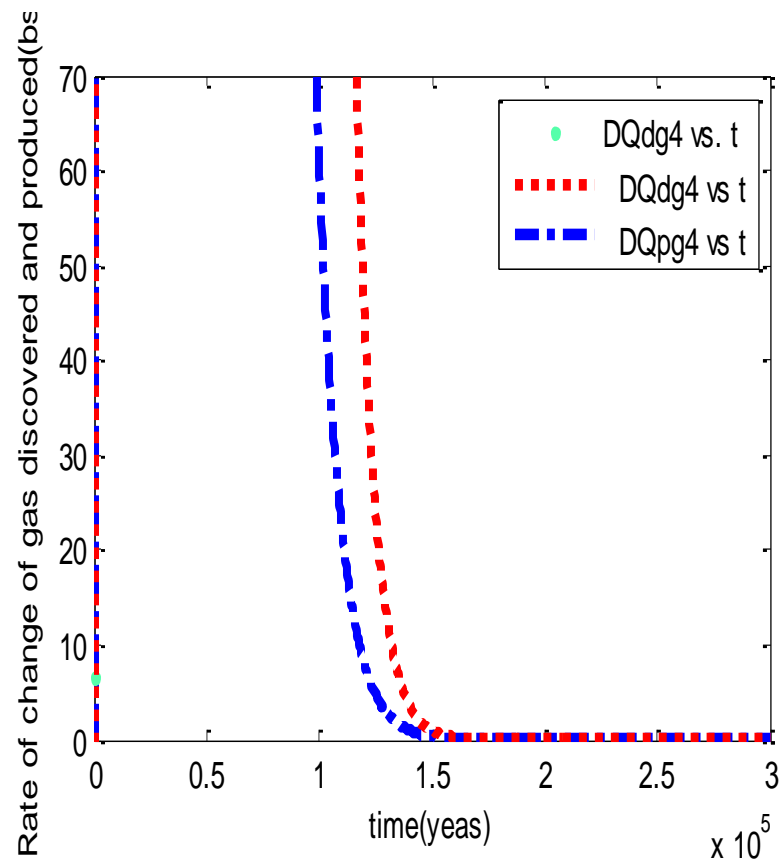


Fig 4.4(D_{GI}): Rate of change of gas discovered and produced versus time for model 4 (real intersection).

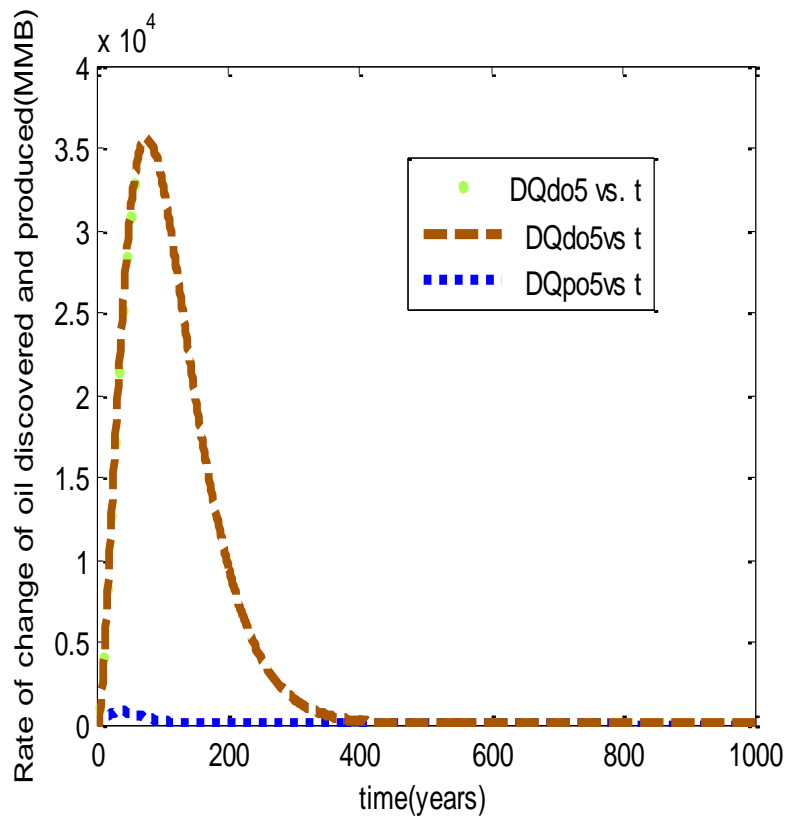


Fig 4.5(D₀): Rate of change of oil discovered and produced versus time for model 5 (apparent intersection).

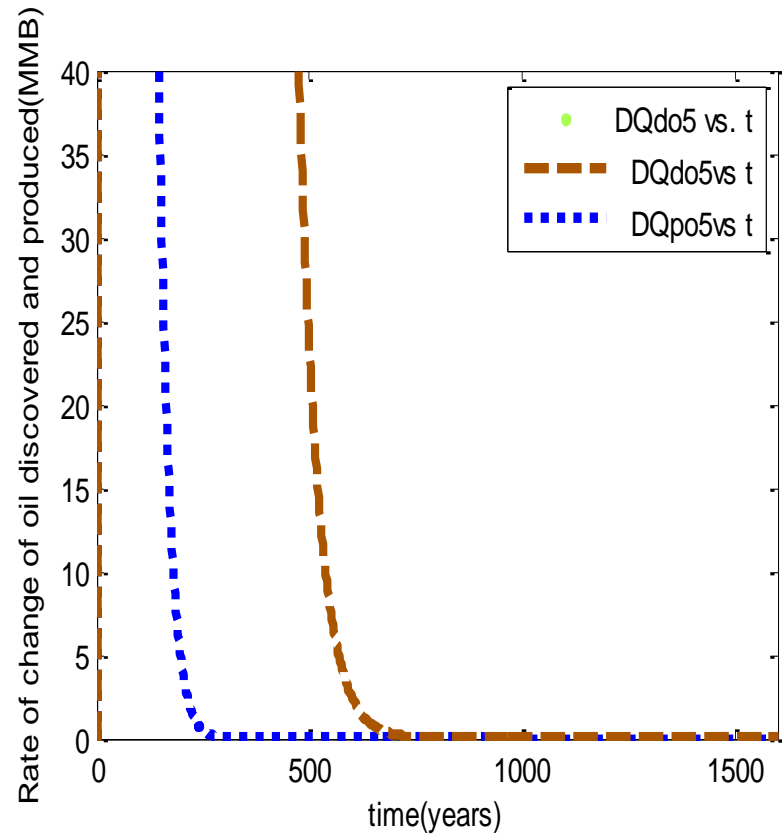


Fig 4.5(D₀₁): Rate of change of oil discovered and produced versus time for model 5 (real intersection).

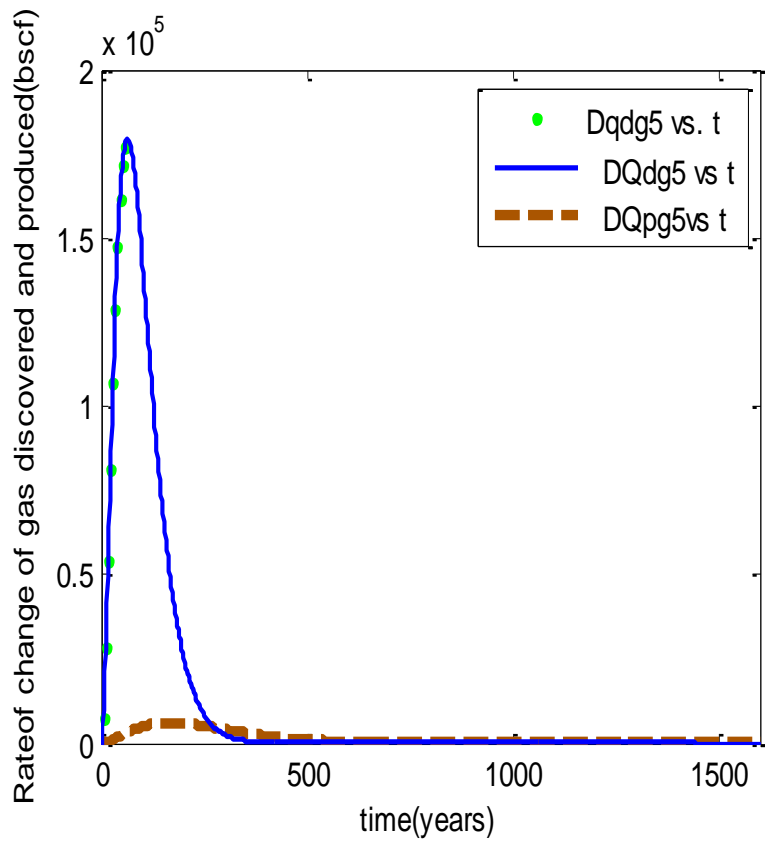


Fig 4.5(D_G): Rate of change of gas discovered and produced versus time for model 5 (apparent intersection).

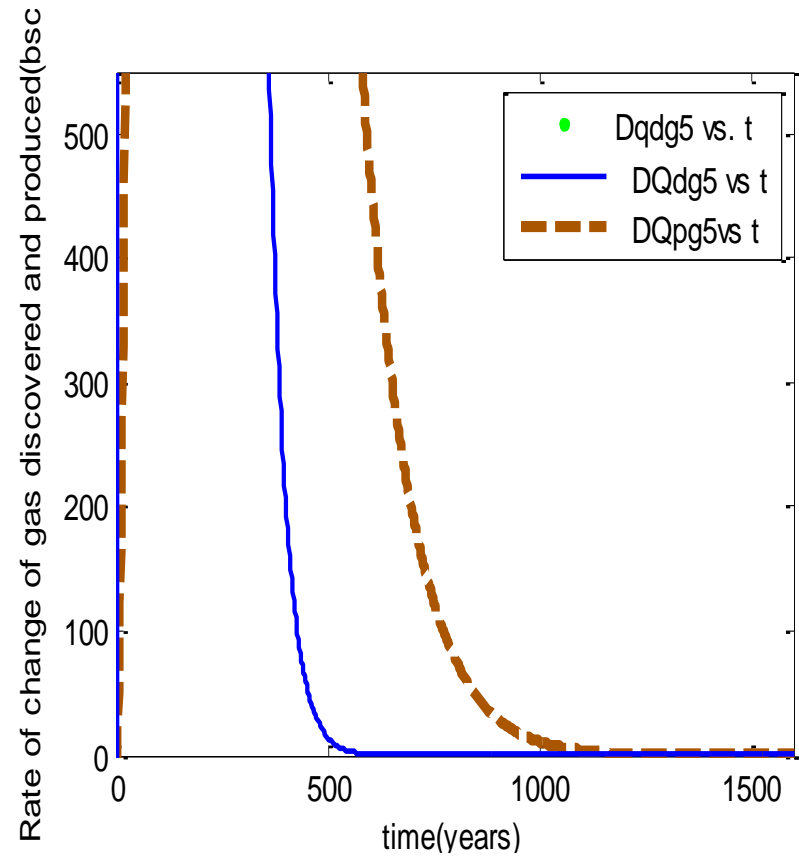


Fig 4.5(D_{GI}): Rate of change of gas discovered and produced versus time for model 5 (real intersection).

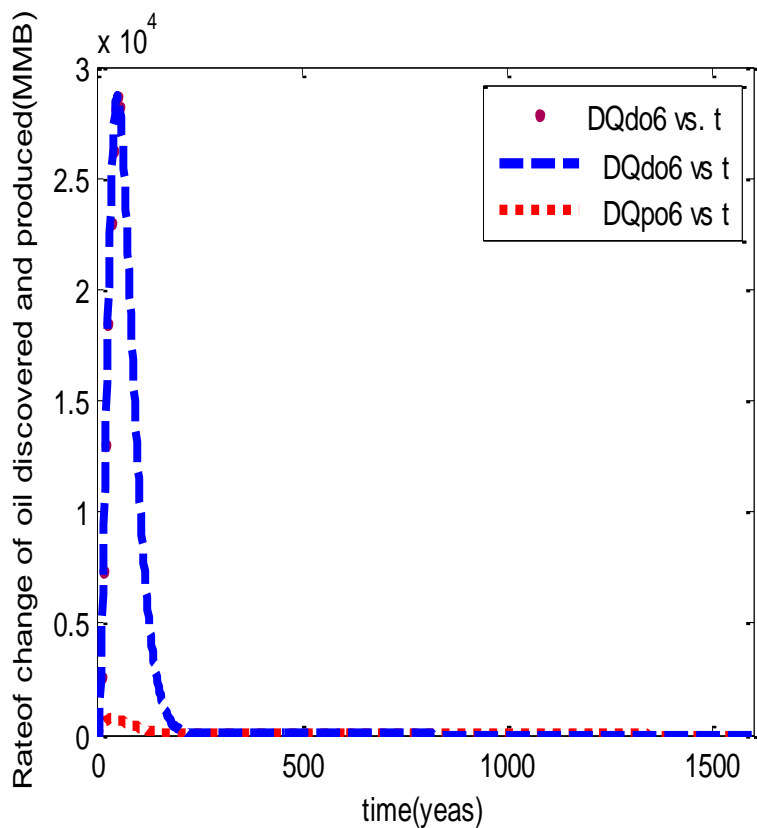


Fig 4.6(D_O): Rate of change of oil discovered and produced versus time for model 6 (apparent intersection).

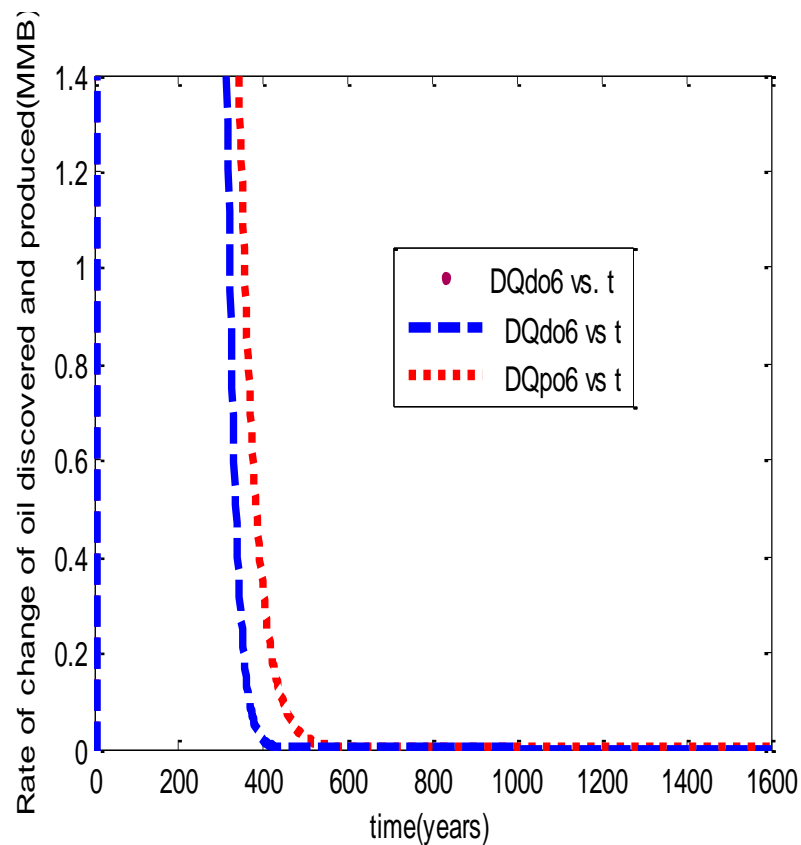


Fig 4.6(D_{OI}): Rate of change of oil discovered and produced versus time for model 6 (real intersection).

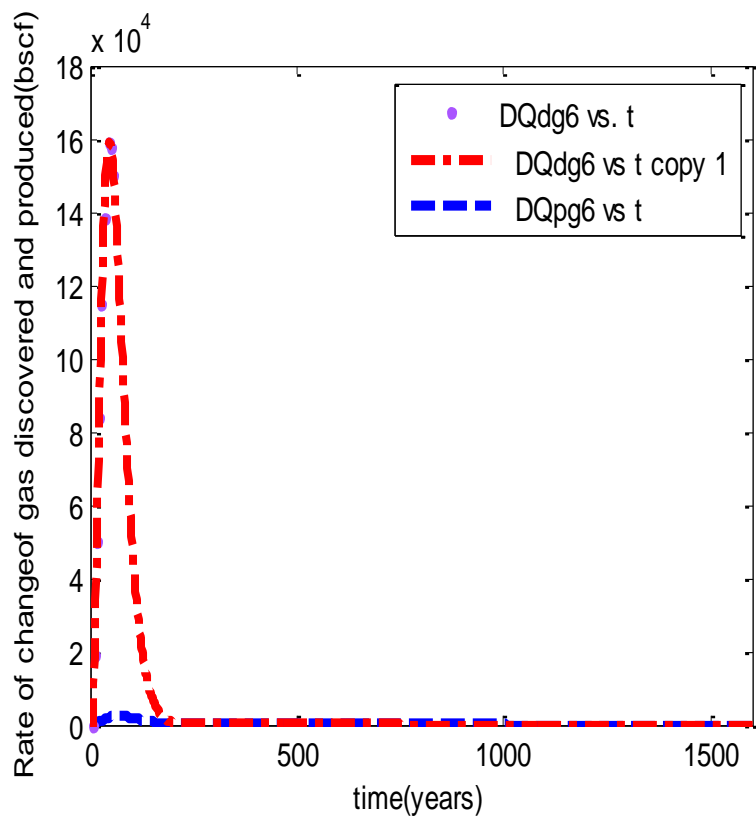


Fig 4.6(D_G): Rate of change of gas discovered and produced versus time for model 6 (apparent intersection).

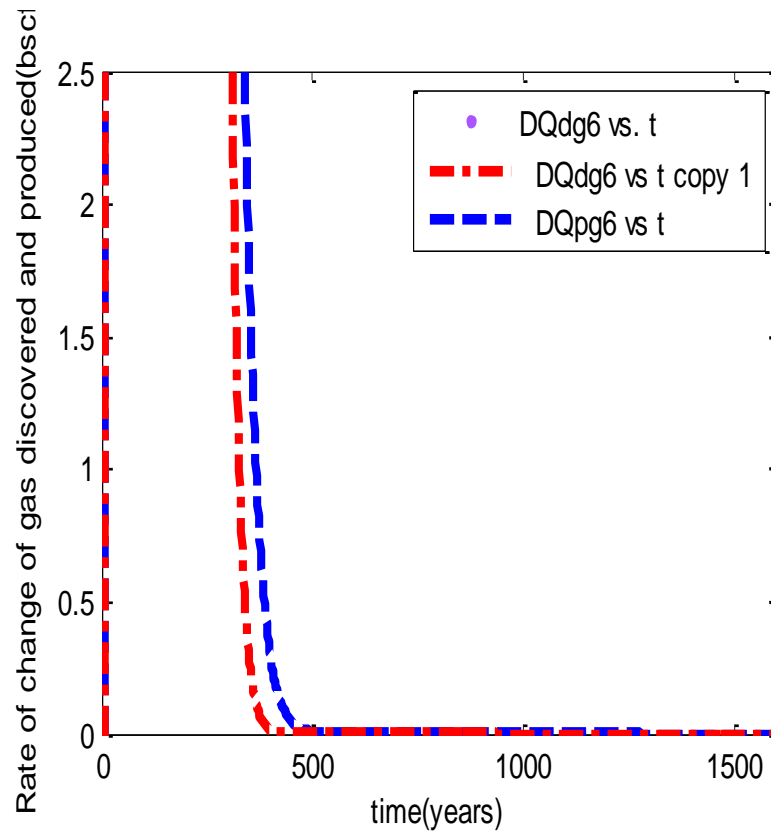


Fig 4.6(D_{GI}): Rate of change of gas discovered and produced versus time for model 6 (real intersection).

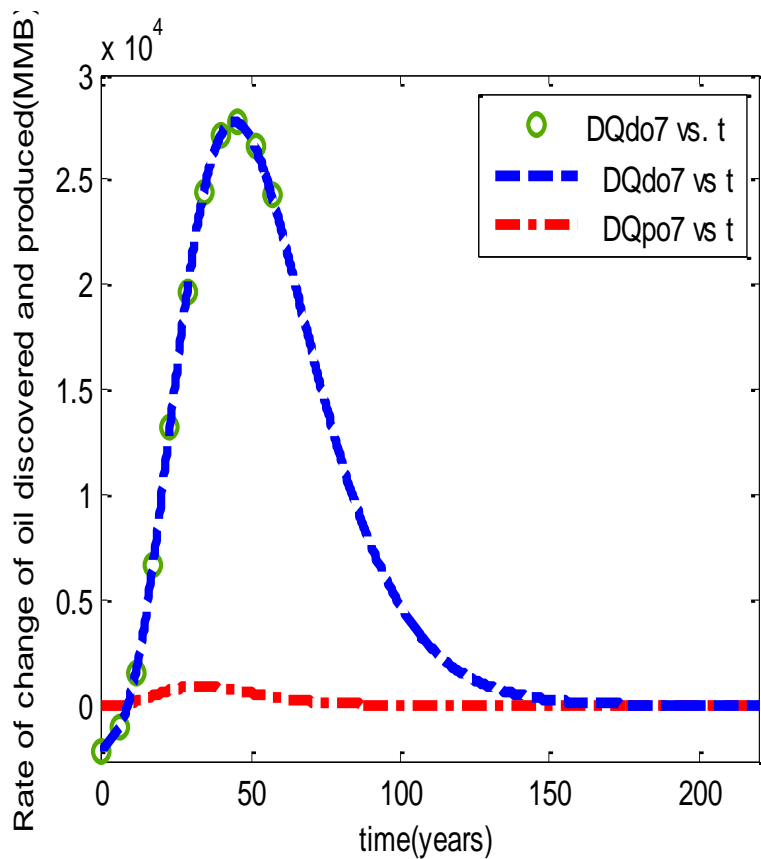


Fig 4.7(D_{O1}): Rate of change of oil discovered and produced versus time for model 7 (apparent intersection).

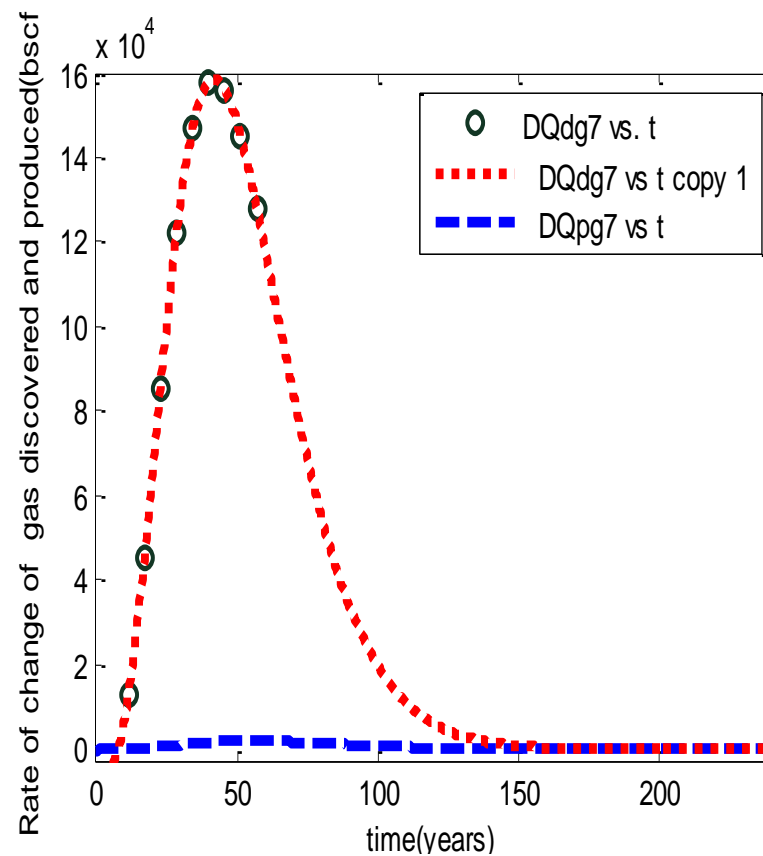


Fig 4.7(D_{G1}): Rate of change of gas discovered and produced versus time for model 7 (real intersection).

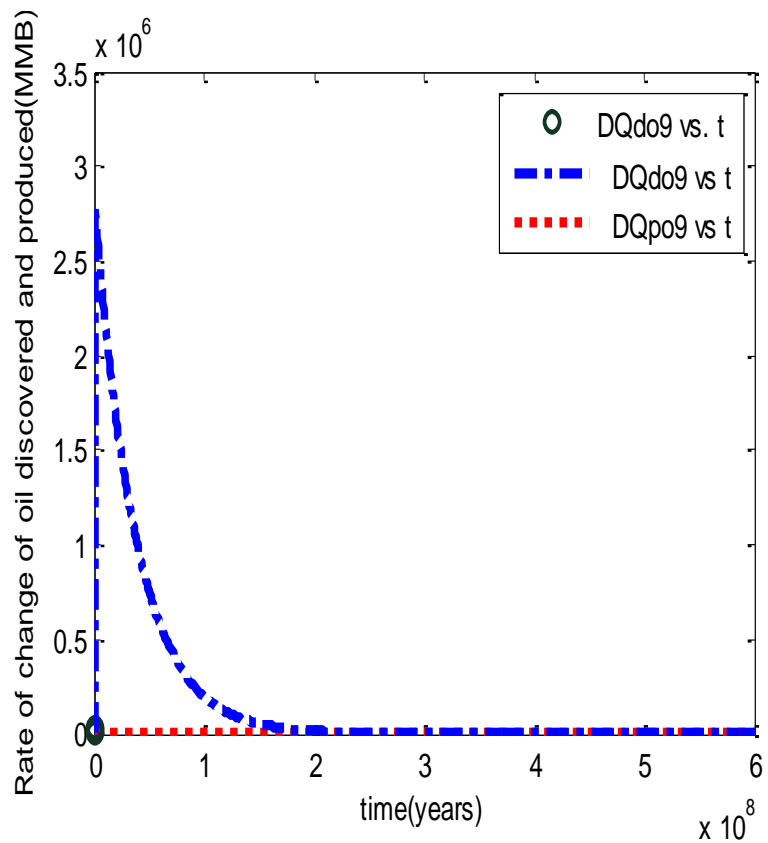


Fig 4.8(D_{OI}): Rate of change of oil discovered and produced versus time for model 9 (apparent intersection).

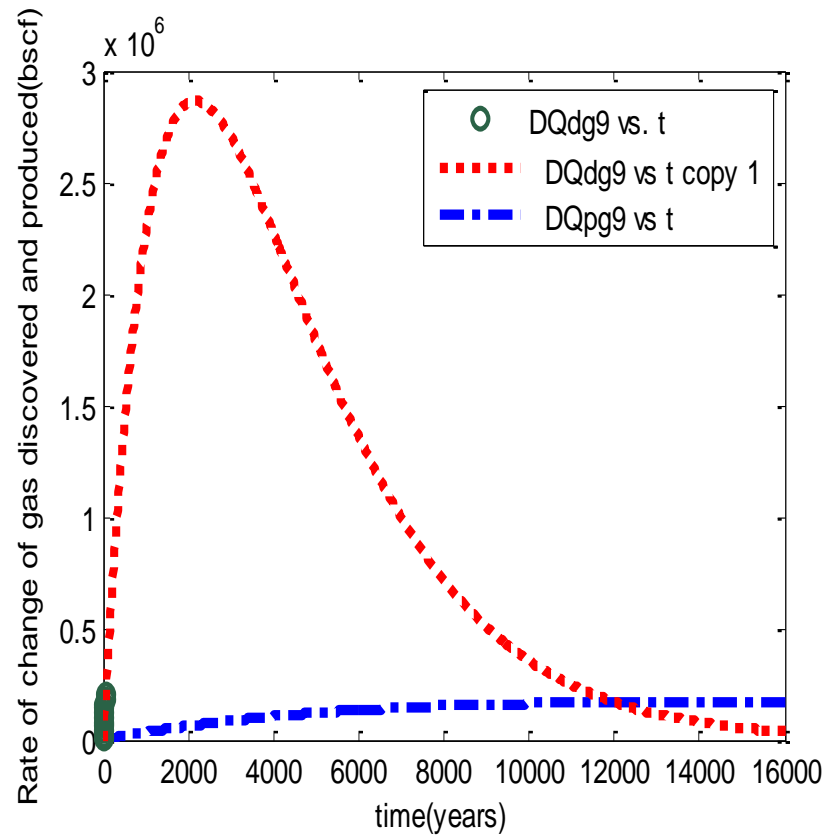


Fig 4.8(D_{GI}): Rate of change of gas discovered and produced versus time for model 9 (real intersection).

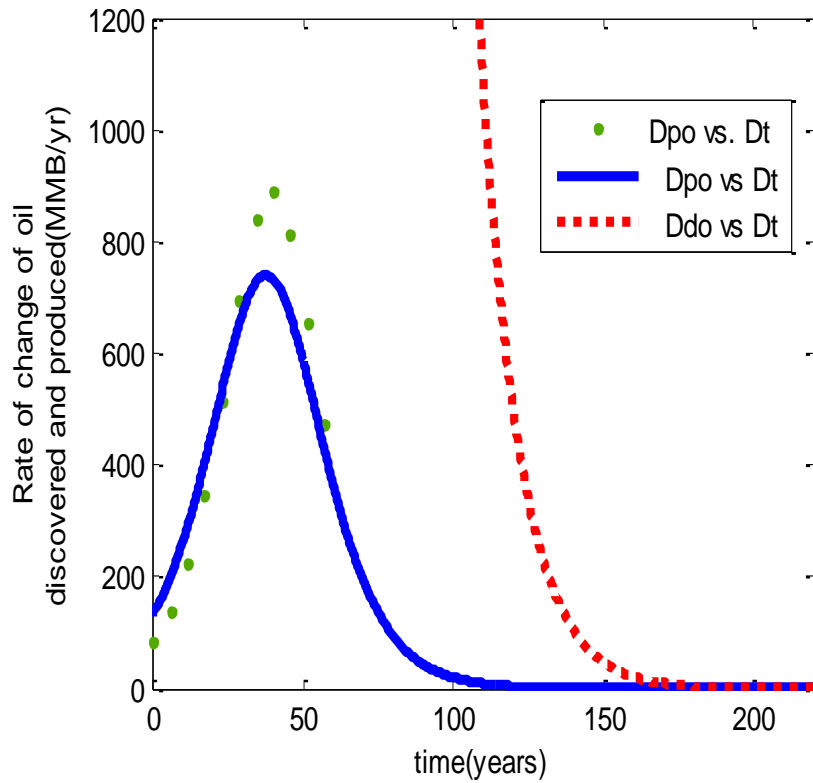


Fig 4.10(D_{OI}): Rate of change of oil discovered and produced versus time for control model 11 (Hubbert) (apparent intersection).

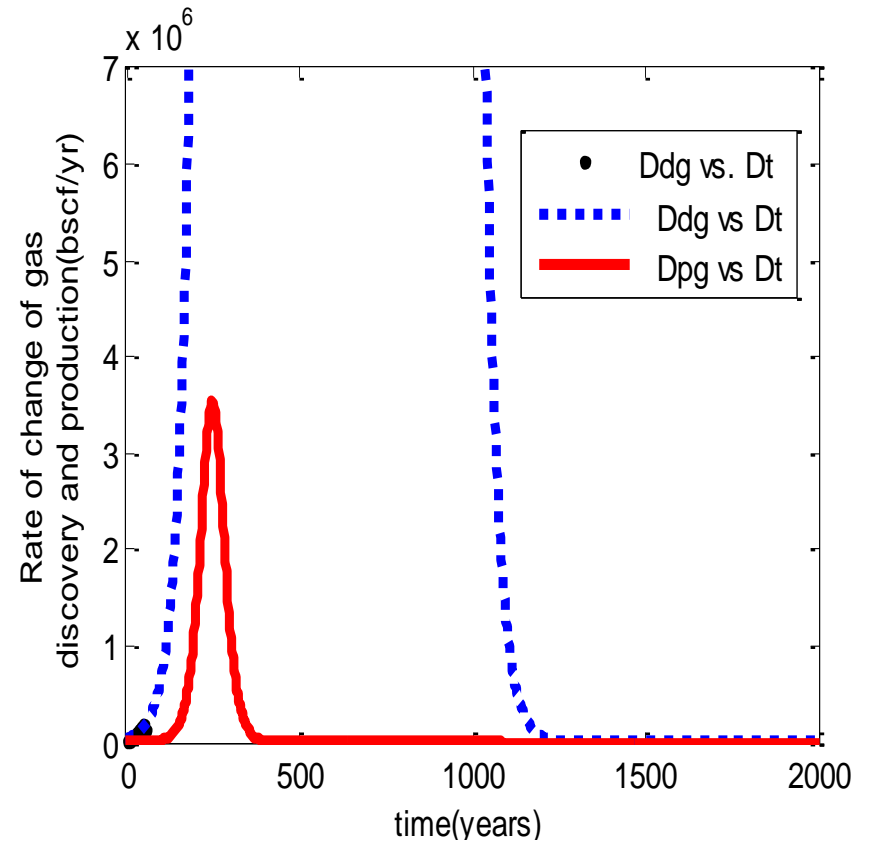


Fig 4.10(D_{GI}): Rate of change of gas discovered and produced versus time for control model 11 (Hubbert) (real intersection).

4.1.1.3 Tables of Coefficients and Statistical goodness of fits obtained for models 4-7, 9 and 11

The result tables shown below present the coefficients and statistical goodness of fits for the good models.

Table 4.1: Coefficient and statistical goodness of fit for curve-fitting of model 4

Q _{po4}		Q _{do4}		Q _{pg4}		Q _{dg4}	
F = 37.7	SSE = 1.393e ⁺⁷	F = 592.7	SSE = 8.5e ⁺⁹	F = 36.08	SSE = 2.374e ⁺⁸	F = 3546	SSE = 1.957e ⁺¹¹
R = 59.49	R ² = 0.9978	R = 1.529e ⁺⁴	R ² = 0.9983	R = 1.523e ⁺⁴	R ² = 0.9872	R =	R ² = 0.9989
V _O = -16.70	R ² Adj = 0.9978	V _O = -25.38	R ² Adj = 0.9982	V _O = -57.07	R ² Adj = 0.9971	1.116x10 ⁴	R ² Adj = 0.9989
	RMSE = 503.3		RMSE = 1.243e ⁺⁴		RMSE = 1077	V _O = -20.35	RMSE = 5.964e ⁺⁴
U: f(892) = 133442		U: f(268000) = 1.40309 e ⁺¹¹		U: f(107000) = 8.36926e ⁺⁹		U: f(179000) = 4.41262e ⁺¹¹	
$t_{\frac{1}{2}u}$: f(100) = 66710.5		$t_{\frac{1}{2}u}$: f(26000) = 7.08597e ⁺¹⁰		$t_{\frac{1}{2}u}$: f(25000) = 4.08729e ⁺⁹		$t_{\frac{1}{2}u}$: f(19000) = 2.24059e ⁺¹¹	
Int: f(119000) = 0				Int: f(151000) = 0.895302			

Table 4.2: Coefficient and statistical goodness of fit for curve-fitting of model 5

Q _{po5}		Q _{do5}		Q _{pg5}		Q _{dg5}	
F = 8.119	SSE = 2.304e ⁺⁷	F = 3.197	SSE = 7.981e ⁺⁷	F = 90.06	SSE = 8.344e ⁺⁹	F = 639.8	SSE = 3.79e ⁺¹¹
R = 19.01	R ² = 0.9964	R = 84.73	R ² = 0.9957	R = 38.27	R ² = 0.9983	R = 32.24	R ² = 0.9979
V _O = 5.464	R ² Adj = 0.9963 RMSE = 647.3	V _O = 309.2	R ² Adj = 0.9955 RMSE = 12.05	V _O = 148.3	R ² Adj = 0.9983 RMSE = 1.232e ⁺⁴	V _O = 57.45	R ² Adj = 0.9978 RMSE = 8.301e ⁺⁴
U: f(371) = 55800.8 $t_{\frac{1}{2}u}$: f(51) = 28018.9		U: f(1719) = 1.94443e ⁺⁶ $t_{\frac{1}{2}u}$: f(208) = 8.64547e ⁺⁵		U: f(738) = 5.04626e ⁺⁶ $t_{\frac{1}{2}u}$: f(108) = 2.54587e ⁺⁶		U: f(174) = 1.87086e ⁺⁷ $t_{\frac{1}{2}u}$: f(19000) = 9.26809e ⁺⁶	
Int: f(725) = 0.136817				Int: f(1194) = 1.62447			

Table 4.3: Coefficient and statistical goodness of fit for curve-fitting of model 6

Q _{po6}		Q _{do6}		Q _{pg6}		Q _{dg6}	
F = 2.585	SSE = 4.819e ⁺⁷	F = 25.69	SSE = 2.303e ⁺¹⁰	F = 0.881	SSE = 1.496 e ⁺⁸	F = 184.1	SSE = 8.809e ⁺¹¹
R = 11.32	R ² = 0.9925	R = 17.09	R ² = 0.9953	R = 23.18	R ² = 0.9919	R = 15.69	R ² = 0.9950
V _O = 149.8	R ² Adj = 0.9923	V _O = 864.6	R ² Adj = 0.9952	V _O = 1599	R ² Adj = 0.9916	V _O = 474.7	R ² Adj = 0.9948
	RMSE = 936.1		RMSE = 2.046e ⁺⁴		RMSE = 1649		RMSE = 1.266e ⁺⁵
U: f(265) = 42501.5		U: f(386) = 2.18922 e ⁺⁶		U: f(481) = 254539		U: f(308) = 1.11683e ⁺⁷	
$t_{\frac{1}{2}u}$: f(43) = 21393		$t_{\frac{1}{2}u}$: f(26000) = 1.10215e ⁺⁶		$t_{\frac{1}{2}u}$: f(85) = 126993		$t_{\frac{1}{2}u}$: f(25) = 5.69059e ⁺⁶	
Int: f(523) = 0,0133776				Int: f(483) = 0.000149588			

Table 4.4: Coefficient and statistical goodness of fit for curve-fitting of model 7

Q _{po7}		Q _{do7}		Q _{pg7}		Q _{dg7}	
F = 1.057	SSE = 8.287e ⁺⁷	F = 9.418	SSE = 4.355e ⁺¹⁰	F = 68.37	SSE = 2.349e ⁺⁸	F = 0.3102	SSE = 1.585e ⁺¹²
R = 8.113	R ² = 0.9872	R = 11	R ² = 0.9912	R = 10.44	R ² = 0.9873	R = 13.57	R ² = 0.9910
V _O = 461.4	R ² Adj = 0.9867	V _O = 2614	R ² Adj = 0.9909	V _O = 1514	R ² Adj = 0.9869	V _O = 4730	R ² Adj = 0.9907
	RMSE = 1228		RMSE = 2.808e ⁺⁸		RMSE = 2066		RMSE = 1.698e ⁺⁵
U: f(188) = 37160.5		U: f(248) = 1.57732e ⁺⁶		U: f(290) = 142928		U: f(264) = 8.48566e ⁺⁶	
$t_{\frac{1}{2}u}$: f(38) = 18675		$t_{\frac{1}{2}u}$: f(49) = 700645		$t_{\frac{1}{2}u}$: f(64) = 72646.1		$t_{\frac{1}{2}u}$: f(50) = 4.3916e ⁺⁶	
Int: f(180) = 36.6086				Int: f(224) = 3.45806			

Table 4.5: Coefficient and statistical goodness of fit for curve-fitting of model 9

Q _{po9}		Q _{do9}		Q _{pg9}		Q _{dg9}	
F = 41.65	SSE = 1.352e ⁺⁷	F = 593.5	SSE = 8.588e ⁺⁹	F = 36.1	SSE = 2.374e ⁺⁶	F = 3592	SSE = 2.04e ⁺¹¹
R = 1.567	R ² = 0.9979	R = 8228	R ² = 0.9983	R = 8001	R ² = 0.9872	R = 2169	R ² = 0.9988
e ⁺⁷	R ² Adj =	V _O = -25.29	R ² Adj = 0.9982	V _O = -57.24	R ² Adj = 0.9865	V _O = -21.48	R ² Adj = 0.9988
V _O = -16.01	0.9978	C = 1.155e ⁺⁴	RMSE =	C = 2.495e ⁺⁴	RMSE = 2099	C = 2168	RMSE =
C = 22.2	RMSE = 500.4		1.761e ⁺⁴				6.146e ⁺⁴
U: f(192000) = 1.4491e ⁺⁷		U: f(184000) = 5.63696e ⁺¹⁰		U: f(366000) = 7.2062e ⁺⁹		U: f(35000) = 1.68851e ⁺¹⁰	
$t_{\frac{1}{2}u}$: f(11000) = 7.29897e ⁺⁶		$t_{\frac{1}{2}u}$: f(17000) = 2.9093e ⁺¹⁰		$t_{\frac{1}{2}u}$: f(26000) = 3.59684e ⁺⁹		$t_{\frac{1}{2}u}$: f(3600) = 8.34732e ⁺⁹	
Int: f(5.87 e ⁺⁸) = 0.485818				Int: f(11880) = 178310			

Table 4.6: Coefficient and statistical goodness of fit for curve-fitting of the Hubbert (control) model 11

Q _{po}		Q _{do}		Q _{pg}		Q _{dg}	
F = 17.54	SSE = 6.427e ⁺⁷	F = 1215	SSE = 1.615e ⁺¹⁰	F = 2533	SSE = 8.533e ⁺⁷	F = 3129	SSE = 9.950e ⁺¹¹
Q = 3.766	R ² = 0.9900	Q = 1.42e ⁺⁶	R ² = 0.9967	Q = 1.314e ⁺⁵	R ² = 0.9954	Q = 7.805e ⁺⁶	R ² = 0.9944
e ⁺⁴	R ² Adj =	a = 73.65	R ² Adj = 0.9966	a = 115	R ² Adj = 0.9852	a = 67.57	R ² Adj = 0.9942
a = 40.9	0.9897	b = 0.08769	RMSE =	b = 0.0809	RMSE = 1246	b = 0.008975	RMSE =
b = 0.0945	RMSE = 1081		1.714e ⁺⁴				1.346e ⁺⁵
U: f(177) = 37663.7		U: f(188) = 1.41956e ⁺⁶		U: f(237) = 131351		U: f(204) = 7.80508e ⁺⁶	
$t_{\frac{1}{2}u}$: f(33) = 13661.9		$t_{\frac{1}{2}u}$: f(49) = 700645		$t_{\frac{1}{2}u}$: f(49) = 66610.4		$t_{\frac{1}{2}u}$: f(47) = 3.82075e ⁺⁶	
Int: f(184) = 3.35588				Int: f(1300) = 13618.25			

Table 4.7: Result array of the discovery and production model curve-fitting

Model	Activity Petroleum	Discovery				Production				Remark
		R ²	R ² -Adj	Ultimate Volume with time	Intersection time	R ²	R ² -Adj	Ultimate Volume with time	Intersection time	
1										Does not fit the models & cannot predict
2										
3										
4	Oil	0.9983	0.9982	U: f(268000) = 1.40309e ⁺¹¹		0.9978	0.9978	U: f(892) = 133442	f(119000) = 0	Negative Initial Volume(V ₀)
	Gas	0.9989	0.9989	U: f(179000) = 4.41262e ⁺¹¹		0.9972	0.9971	U: f(107000) = 8.36926e ⁺⁹	f(151000) = 0.895302	
5	Oil	0.9957	0.9955	U: f(1719) = 1.94443 e ⁺⁶		0.9964	0.9963	U: f(371) = 55800.8	f(725) = 0.136817	Best R ² & Positive Initial Volume(V ₀)
	Gas	0.9979	0.9978	U: f(174) = 1.87086 e ⁺⁷		0.9983	0.9983	U: f(738) = 5.04626e ⁺⁶	f(1194) = 1.62447	
6	Oil	0.9953	0.9952	U: f(386) = 2.18922 e ⁺⁶		0.9925	0.9923	U: f(265) = 42501.5	f(523) = 0.0133776	R ² Smaller than Model 5
	Gas	0.9950	0.9948	U: f(308) = 1.11683e ⁺⁷		0.9919	0.9916	U: f(481) = 254539	f(483) = 0.000149588	
7	Oil	0.9912	0.9909	U: f(248) = 1.57732e ⁺⁶		0.9872	0.9867	U: f(188) = 37160.5	f(180) = 36.6086	R ² Smaller than Model 5
	Gas	0.9910	0.9907	U: f(264) = 8.48566e ⁺⁶		0.9873	0.9869	U: f(290) = 142928	f(224) = 3.45806	
9	Oil	0.9983	0.9982	U: f(184000) = 5.63696e ⁺¹⁰		0.9979	0.9978	U: f(192000) = 1.4491e ⁺⁷	f(5.87 e ⁺⁸) = 0.485818	Negative Initial Volume(V ₀)
	Gas	0.9988	0.9988	U: f(35000) = 1.68851e ⁺¹⁰		0.9872	0.9865	U: f(366000) = 7.2062e ⁺⁹	f(11880) = 178310	
11	Oil	0.9967	0.9966	U: f(188) = 1.41956e ⁺⁶		0.9900	0.9897	U: f(177) = 37663.7	f(184) = 3.35588	Control Model
	Gas	0.9944	0.9942	U: f(204) = 7.80508e ⁺⁶		0.9954	0.9852	U: f(237) = 131351	f(1300) = 13618.25	

Table 4.8: Comparative oil/gas peaking and exhaustion dates obtained for the chosen model 5 relative to Hubbert's and Kamalu's models

Model	Oil Peak (Year)	Gas Peak (Year)	Oil Exhaustion (Year)	Gas Exhaustion (Year)
Model 5	2008	2065	2682	3151
Hubbert	1990	2006	2141	3057
Kamalu	2056	2126	2928	3014

Table 4.9: Percentage deviation of the chosen model 5 from Hubbert's and Kamalu's models based on the obtained oil/gas peaking and exhaustion dates

Percentage Deviation	Oil Peak (%)	Gas Peak (%)	Oil Exhaustion (%)	Gas Exhaustion (%)
Percentage deviation of Model 5 from Hubbert	0.9	2.9	25	3.1
Percentage deviation of Model 5 from Kamalu	2.3	2.9	8.4	4.5

4.1.2 Sensitivity/Trend test analysis results

The simulation result plots for the oil and gas production and discovery for the good models obtained (5, 6 and 7) are presented here in this section:

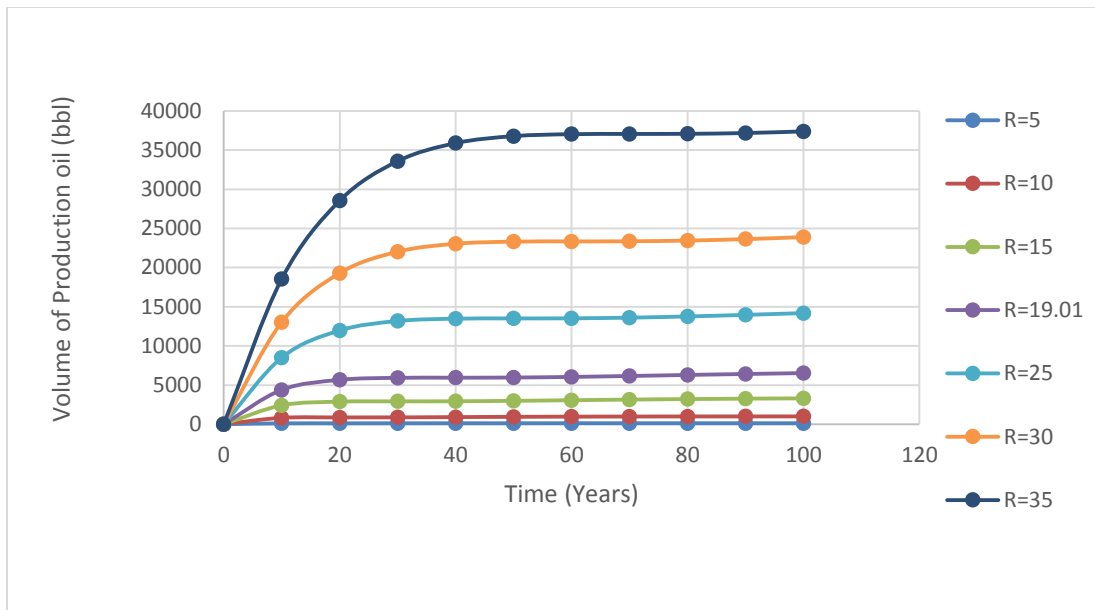


Figure 4.11A: Simulation plot for oil production for model 5.

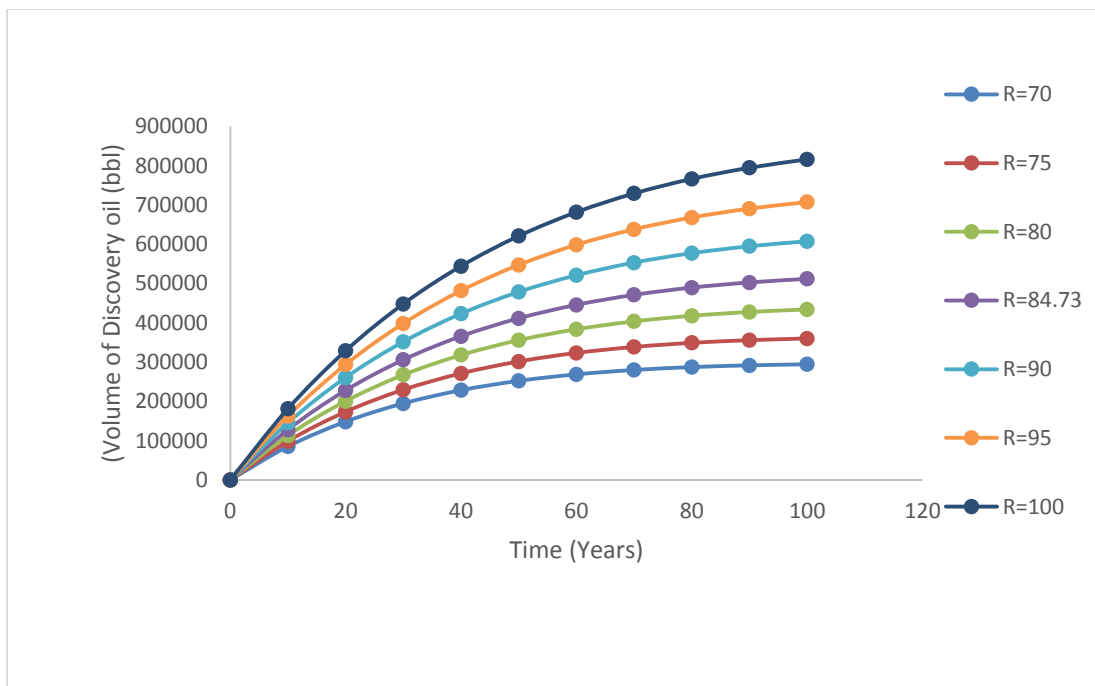


Figure 4.11B: Simulation plot for oil discovery for model 5.

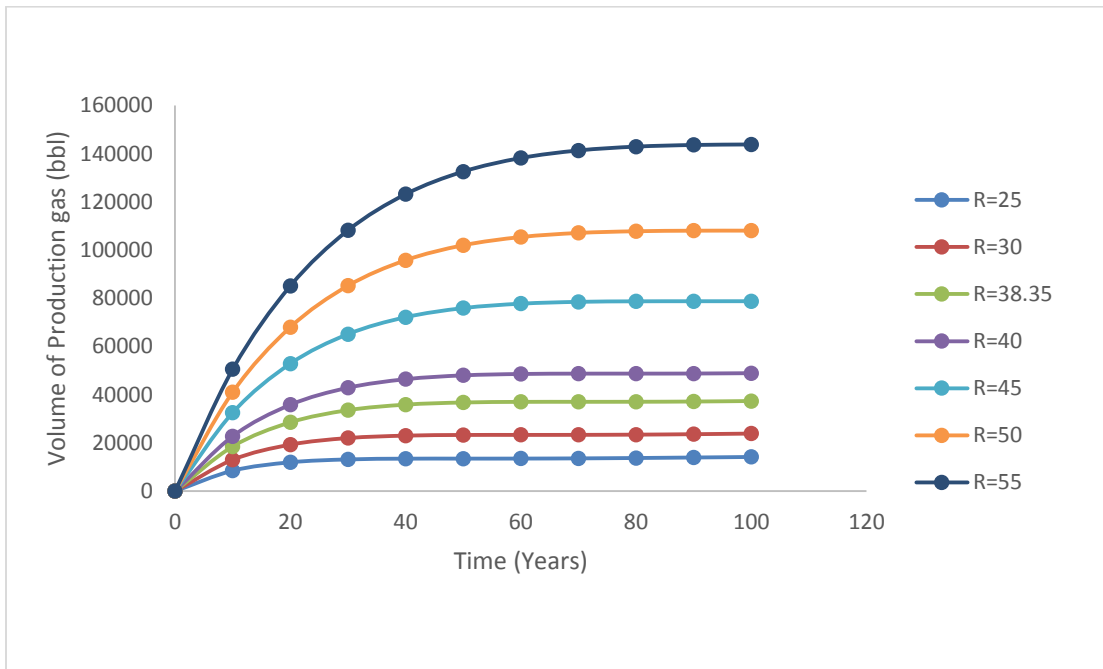


Figure 4.11C: Simulation plot for gas production for model 5.

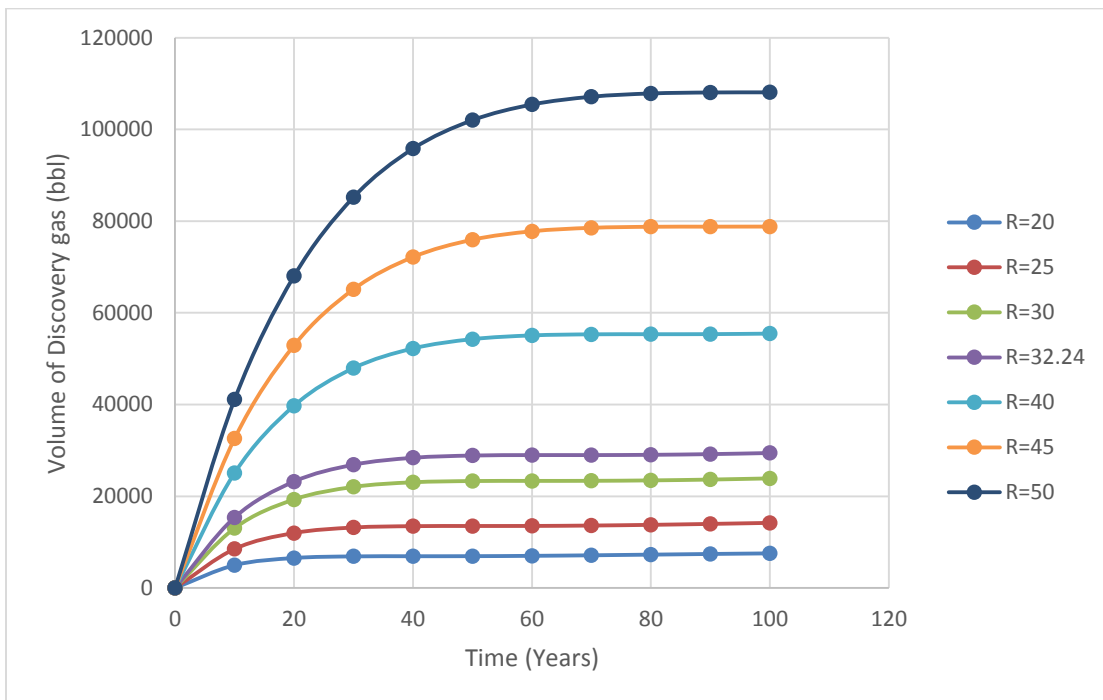


Figure 4.11D: Simulation plot for gas discovery for model 5.

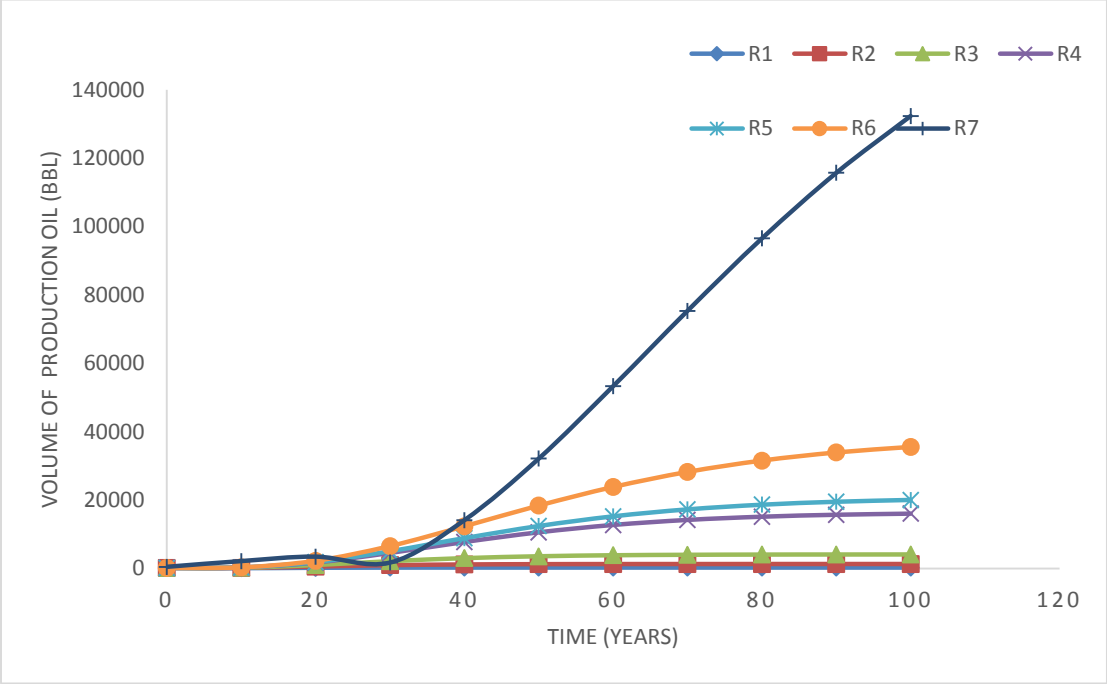


Figure 4.12A: Simulation plot for oil production for model 6.

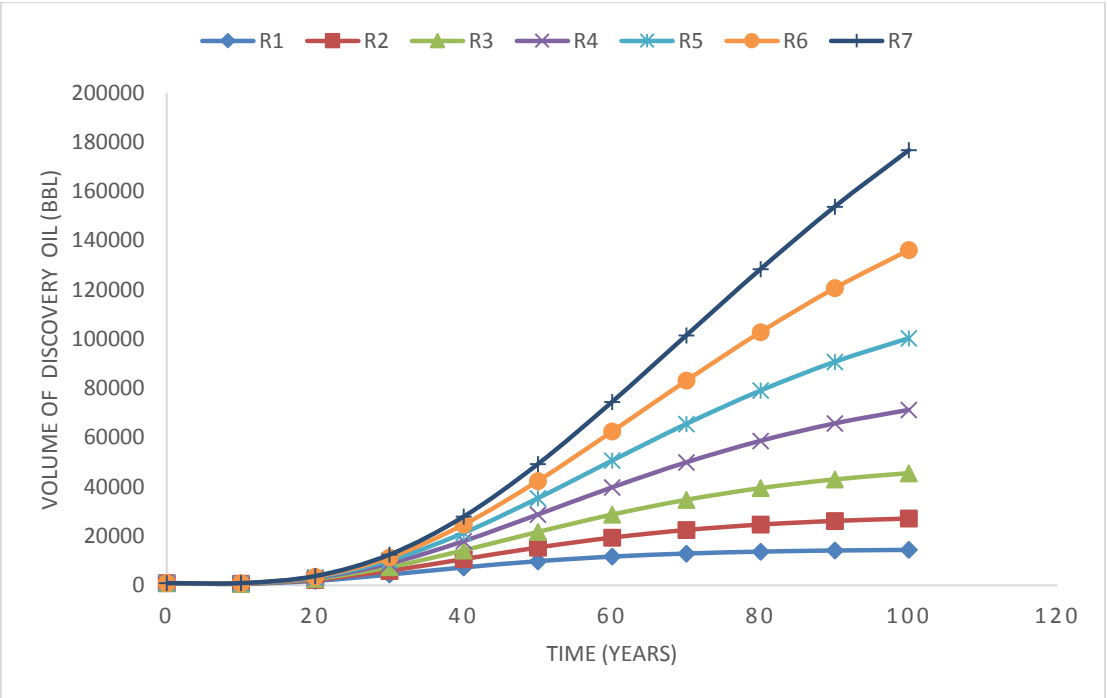


Figure 4.12B: Simulation plot for oil discovery for model 6.

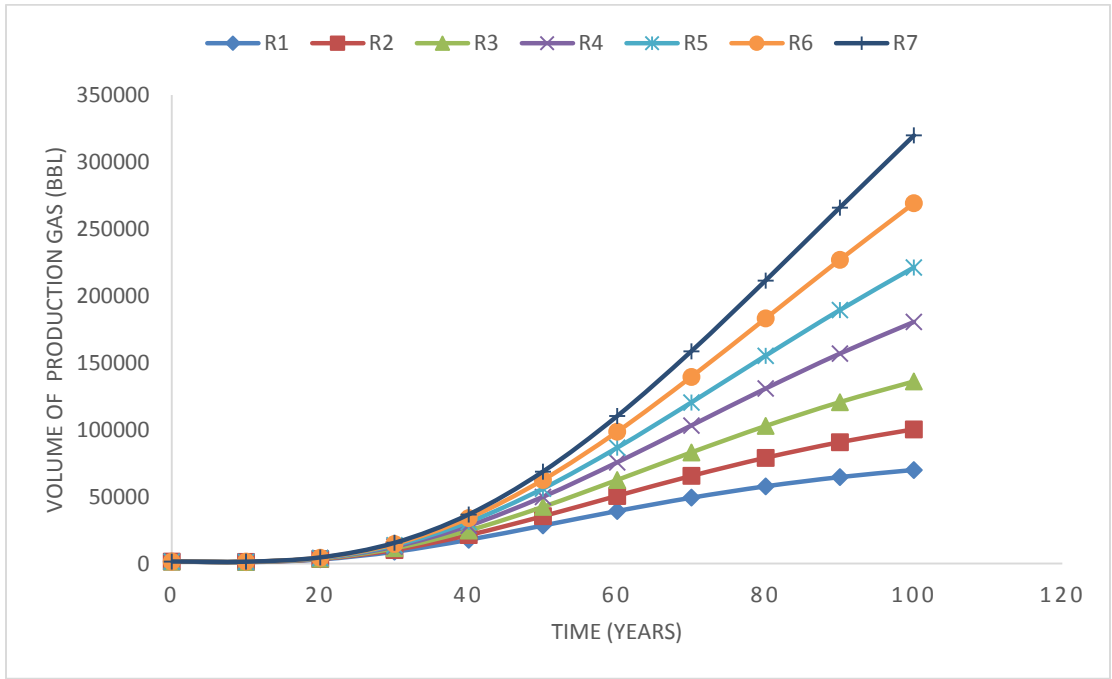


Figure 4.12C: Simulation plot for gas production for model 6.

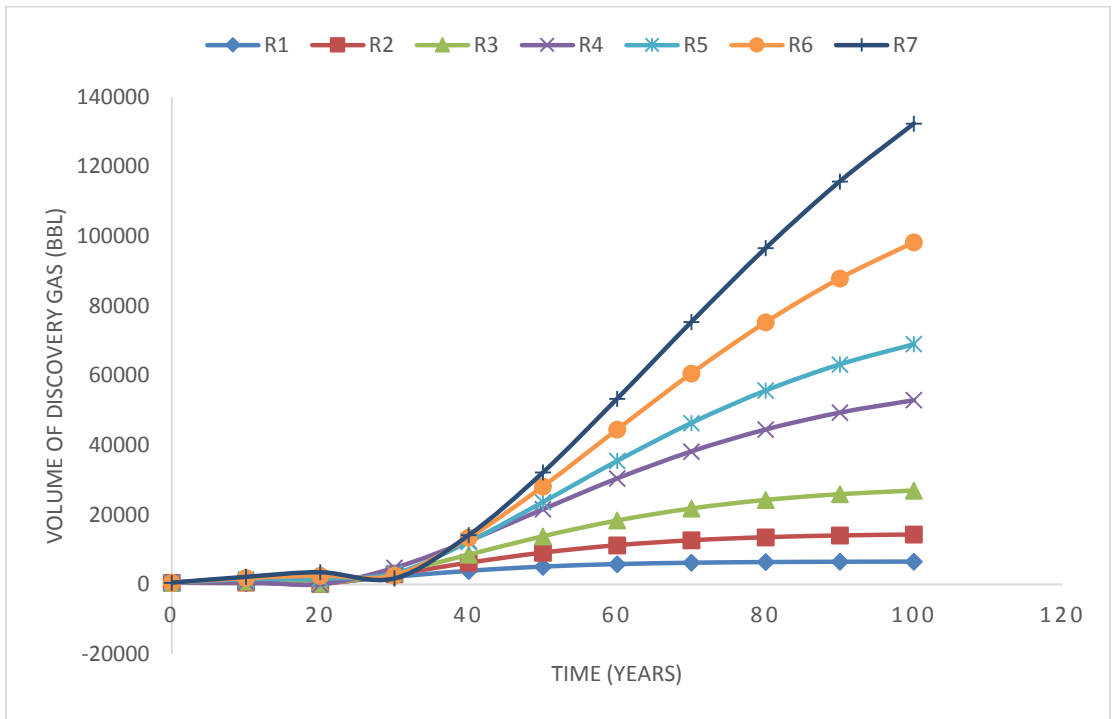


Figure 4.12D: Simulation plot for gas discovery for model 6.

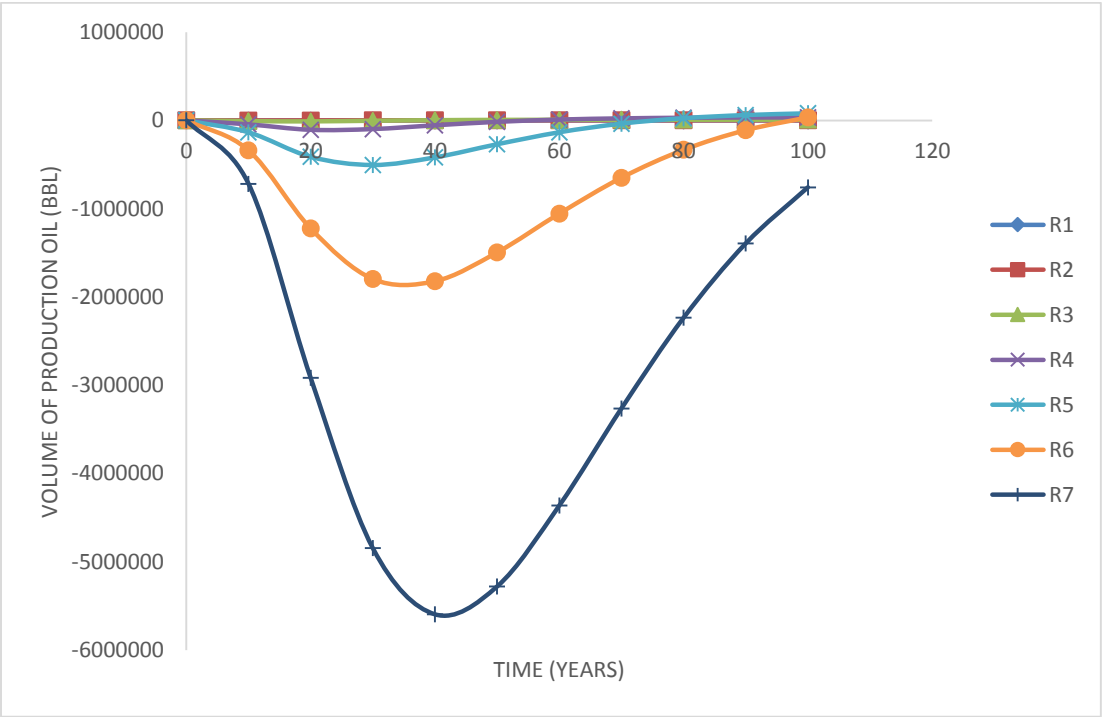


Figure 4.13A: Simulation plot for oil production for model 7.

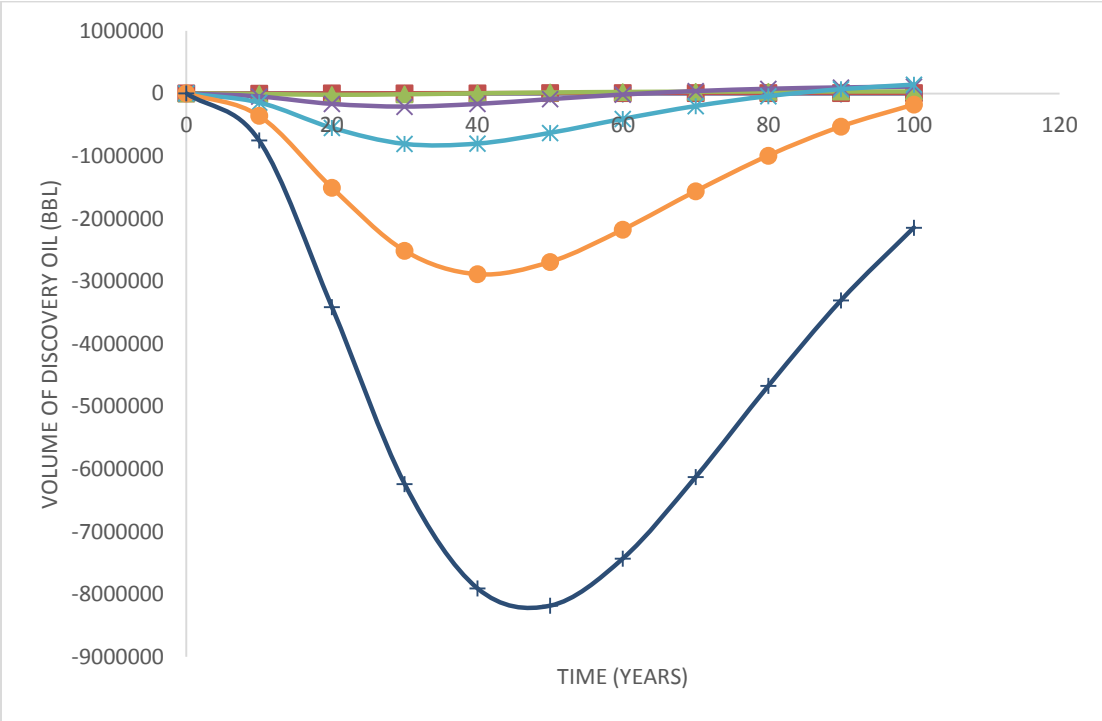


Figure 4.13B: Simulation plot for oil discovery for model 7.

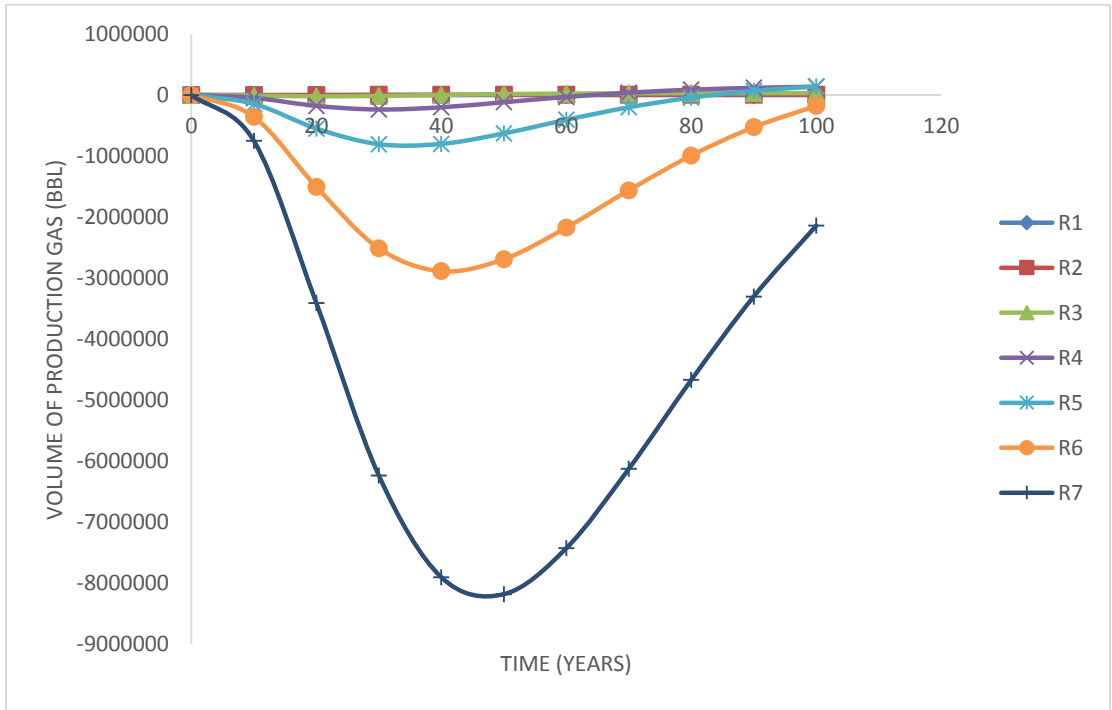


Figure 4.13C: Simulation plot for gas production for model 7.

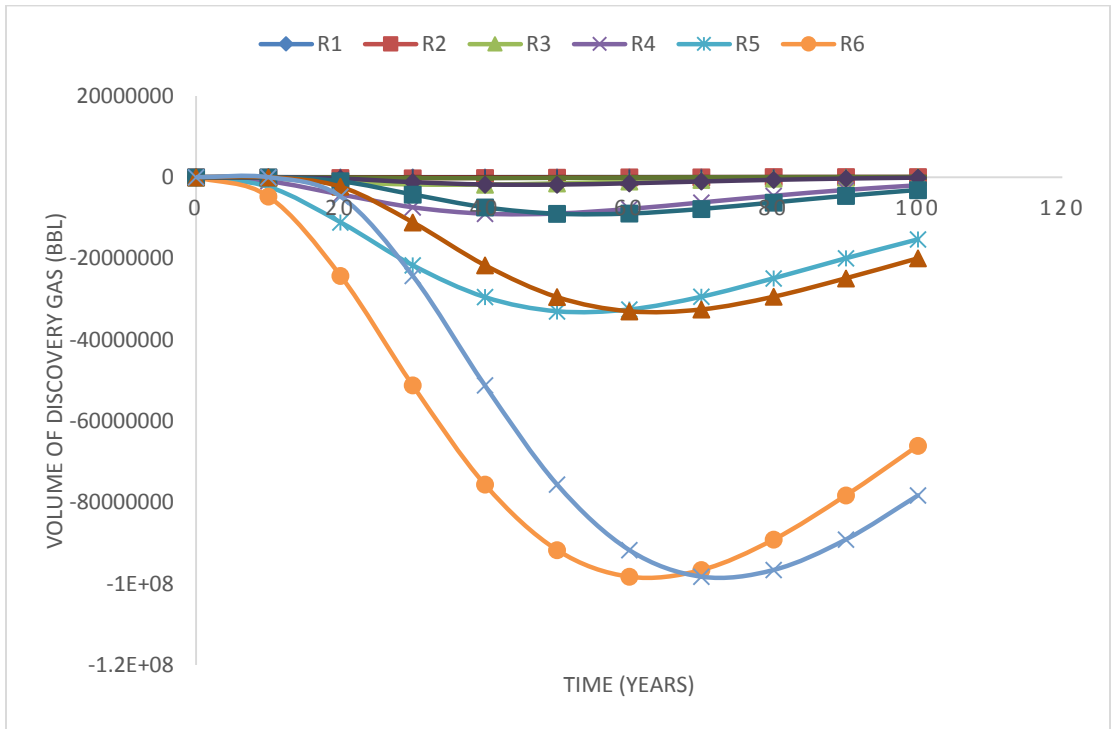


Figure 4.13D: Simulation plot for gas discovery for model 7.

4.2 Discussion

4.2.1 Models and curve-fittings of cumulative plots

The cumulative plots of the models developed are as shown in figures 4.1 to 4.10. These plots were made with the developed models to fit the cumulative oil and gas production/discovery data collected as field data for this predictive analysis. This was done for the purpose of validating the models so developed to ascertain which of them best fits the Nigerian petroleum resources data. The models which were not very useful in the validation of the field data were grouped together and treated as such. It was either a model did not fit the data or it did not predict future trends accurately, and for such models, only the cumulative oil production plots were made following the fact that the curve fitting behavior of the cumulative production and cumulative discovery are similar, for both petroleum resources under consideration (i.e for oil and gas).

The developed models with such inability to validate were models 1, 2, 3 and 10.

Models 1 and 3 gave poor fit to the data in addition to their inability to predict (see Figs 4.1 and 4.3) and so were discarded, while models 2 and 10 fitted the data but could not predict. From the foregoing, it is seen that models 1, 2, 3 and 10 have one problem or the other. A model may fit well to experimental data, but is not suitable to simulate behavior of process beyond the experimental data points. This is the case for fit to a cumulative production plot that does not taper out asymptotically or give a sigmoidal profile. In this case, except a model is sigmoidal (i.e S-shaped), it cannot predict (Hubbert, 1956). Examples of those are figures 4.2 and 4.9 (models 2 and 10); though these two figures fit very well to the data with R^2 of 0.9978 and 0.9979 respectively, they could not predict values beyond the data points, and so were discarded. Hence, for a model to be used (or at least considered), it must both fit and predict, and amongst such models which satisfied both conditions are models 4, 5, 6, 7, 9 and the Control Model (Hubbert, 1956), from which judgments were then made for the best.

Model 1 (Fig 4.1) was rejected because it has a low R^2 value of 0.7969. It can be observed from Fig 4.1 that the model does not fit and does not predict values beyond the data points, and so was

discarded. For a model to fit, the profile must run on the cumulative data scatter diagram. If it deviates, the fit is poor. The degree of deviation indicates whether to accept a model or not, for instance if the model deviated more than 2% (i.e $R^2 = 1 - 0.02 = 0.98$), then it would not be accepted.

In fig 4.2, the fitness was almost 100% (i.e $R^2 = 0.9978$) but does not follow the data scatter diagram at long run to taper out asymptotically (in this case because for a model to predict it has to obey cumulative profile since there should be growth, up to a certain point where competition sets in during its life struggle: the growth of all living things obey this natural law, and population census also obey this law). In fact, Hubbert converted Vahaust population model for American petroleum prediction which he used to predict the American petroleum resources peaking period in the 1970s, which did peak in 1971. A model should be able to simulate the behavior of the process under study beyond existing data points and such simulation result must agree with theoretical knowledge.

In figure 4.3, physically looking at the diagram, the model does not fit and does not simulate or forecast future behavior accurately. It does not fit because it does not run consistently on the scatter diagram, and does not predict because it does not taper out far beyond the experimental values.

For figure 4.9, the model gave a good fit but does not accurately simulate the future as it does not taper out asymptotically beyond the experimental values (field data).

Looking at figures 4.4A and 4.4B (for oil production and discovery respectively), both figures of model 4 fit well ($R^2 = 0.9978$ and 0.9983 respectively). They both also predicted well as they tapered out at different y-axis values. When plotted together, in one graph (fig 4.4C), a significant difference in volume can be observed between oil discovery and production, showing

the volume of oil that the rock will not let us take, which is the difference between the upper and lower limits of the y-axis (I.e discovery and production values).

In figures 4.4D and 4.4E (for gas production and discovery), both graphs of same model 4 (for gas) fitted well ($R^2 = 0.9872$ and 0.9989 respectively). The both also predicted well as they have their profiles tapered out asymptotically at different y-values. Combining the two graphs into one (fig 4.4F), the plots again shows the volume retained by the geological rocks (formation).

For model 5, figures 4.5A and 4.5B (for oil) show that both fitted adequately with $R^2 = 0.9964$ and 0.9957 respectively for production and discovery. Their profiles also simulated oil production and discovery accurately as they tapered out at different values of y-axis. When the plots were combined on one graph in figure 4.5C, the cumulative plot showed the difference in discovery and production, whose physical meaning has been interpreted to be the volume of oil the rock would not let us take during drilling.

Figures 4.5D and 4.5E (for gas) fitted well ($R^2 = 0.9983$ and 0.9979 respectively for production and discovery). Again, both figures predicted well since they tapered out accordingly as expected. In figure 4.5F, the combined plot show the difference between the discovery and production of gas, with the difference between the upper and lower limits showing the amount of gas the rock retains.

Considering models 6, 7 and 9, the plots of figures 4.6A and 4.6B, 4.7A and 4.7B, and 4.8A and 4.8B, respectively for oil production and discovery, all show that the plots fitted well looking at their respective R^2 values ($\geq 98\%$). Also, their profiles predicted well as they tapered out along the x-axis at different y-axis values. And the respective combined plots of figures 4.6C, 4.7C and 4.8C have shown the difference in the oil discovery and production for these models, clearly exposing the volumes of oil retained in each case.

Figures 4.6D and 4.6E, 4.7D and 4.7E, and 4.8D and 4.8E, are the respective plots for gas production and discovery, for models 6, 7 and 9. It is seen from these plots that they adequately

fitted (if we look at their respective R^2 values that ranged from 0.9872 to 0.9988), and their profiles also predicted very well, tapering out asymptotically at different values of y-axis. Figures 4.6F, 4.7F and 4.8F show the combined plots that reveal the difference in the gas discovery and production for the models under consideration, from which gas amounts retained by the geological rocks can be determined.

The Hubbert model was used as the control model in fitting the field data from DPR, and the cumulative plots were made for it too. Figures 4.10A and 4.10B show that both gave accurate fits for oil production and discovery respectively, with respective $R^2 = 0.9900$ and 0.9967 . Their profiles also accurately simulate future behavior as they tapered out as expected, and the combined cumulative plot of figure 4.10C again show the difference in oil discovery and production which is the volume of oil retained by the rocks.

Similarly, figures 4.10D and 4.10E are the plots of gas production and discovery respectively, and these show that they gave accurate fits too (with $R^2 = 0.954$ and 0.9944 respectively). Also their profile is seen to accurately simulate future behavior as they tapered out along the x-axis at different y-axis values. Again the combined plot of figure 4.10F show us the amount of gas retained, by showing the difference in the gas discovery and production volumes.

Now, comparing the control model (Hubbert) with the chosen model 5, it is generally seen that the chosen model 5 even fitted the field data (from DPR) better than the Hubbert model, as it produced better R^2 values for 3 out of the 4 that can form the basis of comparison (i.e, for oil discovery: 0.9957 against 0.9967, for oil production: 0.9964 against 0.9900, for gas discovery: 0.9979 against 0.9944, and for gas production: 0.9983 against 0.9954, respectively for model 5 against the Hubbert model). See table 4.7 for these comparative R^2 values. This also corroborates other remarks leading to the selection of model 5.

4.2.2 Dumb-bell plots, intersections and peak determination

Figure 4.4 (D_O) show an apparent intersection of discovery and production of oil so that the enlarged intersection is shown in figure 4.4 (D_{OI}). It is seen that from the enlarged intersection point, the years shifted up as the two graphs align together and move as one, for future values to be easily read off.

This intersection is $f(119,000) = 0$ (Table 4.1) meaning the reserve will finish in 119,000 years from 1957. And in the case of the gas, i.e figure 4.4 (D_G) and figure 4.4 (D_{GI}), the intersection occurred at $f(151,000) = 0.895302$ (Table 4.1), i.e the gas reserve will finish in 151,000 years from 1957. This would have been a good result except that in the model 4, gas has a negative initial reserve (V_O).

In model 5, figures 4.5 (D_O) and (D_{OI}) show that the intersection occurred at $f(725) = 0.136817$ (table 4.2) giving us 725 years from 1957, just as the intersection of the gas section of the same model in figures 4.5 (D_G) and (D_{GI}) is $f(1194) = 1.62447$ (table 4.2), which 1,194 years from 1957. This model is a good model since it has both positive initial reserve (V_O) as well as a comparatively high R^2 .

In model 6, figures 4.6 (D_O) and (D_{OI}) show that the intersection occurred at $f(523) = 0.0133776$ (table 4.3), which is 523 years from 1957. The gas part of the model, figures 4.6 (D_G and D_{GI}), intersect at $f(483) = 0.000149588$ (table 4.3), which is 483 years from 1957. Again, this model is a good model as it has given us a positive initial reserve (V_O) even as its R^2 is not as high as model 5.

In model 7, there are no duplicate plots since the two plots can be contained in one graph. Figure 4.7 (D_{OI}) showed an intersection of $f(180) = 36.6086$ (table 4.4), which is 180 years from 1957. The gas counterpart of the model 7, figure 4.7 (D_{GI}) intersected at $f(224) = 3.45806$ (table 4.4),

which is 224 years from 1957. This model also gave a positive initial reserve (V_0) but its R^2 although reasonable enough but comparatively lower than that of model 5.

There was no analysis with model 8 since it is a generalized model, and hence had no result.

In model 9, which does not also have duplicate plots, figure 4.8 (D_{OI}) gave an intersection of $f(5.87e^{+8}) = 0.485818$ (Table 4.5). This big value merely means that it is for a very long time.

The intersection of the gas counterpart of model 9, i.e figure 4.8 (D_{GI}) at $f(11880) = 178310$ (Table 4.5), i.e 11880 years from 1957. This model is not a good model. Just like model 4, it has negative initial reserve (V_0) even as R^2 is promising to be a good one.

When the Hubbert (control) model was considered, which do not also have a duplicate plot, figure 4.10 (D_{OI}) produced an intersection of $f(184) = 3.35588$ as seen in Table 4.6 (i.e a value of 184 years from 1957. For the gas, figure 4.10 (D_{GI}) intersected at $f(1300) = 13618.25$ (See Table 4.6), which is 1300 years from 1957. However, here in this work, the Hubbert model has been forgone for model 5 for the aforementioned reason that the preferred model 5 produced better fit to the field data which formed the core of this analysis for Nigerian petroleum resources depletion.

In the comprehensive table (Table 4.7), the R^2 , R^2 -Adj, the ultimate volume with time and intersection time, and, remark columns for oil and gas are shown. From this array of values, it is self-deciding to see which the best model is. The first three models could neither fit nor predict, and for the remaining five models, the judgment will be based on the sound theory and the R^2 of their plots. So, models 4 and 9 from reality of nature cannot qualify since they have negative initial reserves. It is said that, in the oil industry, oil is to be discovered before production. But in this case, not only that there is no oil but it is outside the reservoirs (in the rock). That is the meaning of negative initial reserve.

In models 6 and 7, though naturally okay with positive initial reserve like model 5, their R^2 were not the best. Model 5 meets the natural reality of positive initial reserve as well as best R^2 for the remaining models.

Again, in justifying the selection of model 5, it is very much in order because oil will likely finish before gas 725 years against 1194 year, that is if this is put in the present dispensation, the oil will finish in the year 2682AD and gas 3151AD.

Also, from the dumb-bell plots and statistical summary table (figures 4.5 D_{OI} and D_{GI} , and table 4.2) for the selected model 5, it is found that the peaking periods for the oil and gas resources respectively are 51 years and 108 years from 1957, which are 2008AD and 2065AD, for oil and gas, with volumes of 28,018.9MMB and 2.54587×10^9 bscf respectively.

Comparing these results with those for the control model 11 (Hubbert) which shows (see table 4.8 for the comparative dates) that the Nigerian oil reserve had peaked in the year 1990AD (as against 2008AD obtained in this work) with volume of 13661.9MMB and gas reserve had peaked in 2006AD (as against 2065AD obtained) with volume of 6.66104×10^{10} bscf, and that oil will finish in the year 2141AD (as against 2682AD obtained) and gas will follow sooth in the year 3057AD (as against 3151AD obtained). From these results it is certain that oil had peaked before gas and will also finish before gas. And the comparison show that the obtained results are within a close range with those of Hubbert model except for the oil exhaustion time, which may be due to some unseen error in the analysis. Based on the use of a statistical tool for comparison, the aforementioned can be established having a percentage deviation values of 0.9%, 2.9% and 3.1% for oil peak time, gas peak time and gas exhaustion time respectively (table 4.9).

Also, the result of Kamalu's prediction (Kamalu, 2010) corroborates what has been obtained in this work, as the results are seen to be in very close range too. From his work, (Kamalu, 2010) found out the peak and exhaustion periods to be within the range of years 2032 – 2080 and year 2928 (2682AD obtained in this work) respectively for crude oil; and within years 2099 – 2153

and year 3014 (3151AD obtained in this work) respectively for natural gas. He noted that non-associated gas production peak occurred between 67 to 73 years after that of crude oil, and its exhaustion occurred about 86 years after that of oil. According to him, Nigeria's crude oil is expected to peak in the year 2056 (as against 2008AD obtained in this work); and natural gas in the year 2126 (as against 2065AD obtained), which is about 70years time gap (see table 4.8). Again, on the basis of the use of a statistical tool for this comparison, the established time relationships can be corroborated with the obtained percentage deviation values of 2.3%, 2.9%, 8.4% and 4.5% for oil peak time, gas peak time, oil exhaustion time and gas exhaustion time respectively (table 4.9).

However, Kamalu (2010) stated that no one model can be used effectively to predict peaking and exhaustion times for both crude oil and natural gas. He said that a particular model predicts best for a particular petroleum component. According to him, the values of the petroleum resources depletion models' constants are specific to characteristic Nigerian petroleum resources (crude oil and natural gas) production data. He developed two models (the two best models; one for crude oil, one for natural gas) which were more superior to the control model, just like the case in this work (i.e showing significantly better correlation coefficient values than that of Hubbert model), when applied on Nigeria petroleum production data.

Again in confirming the efficacy of the developed models, plotting the derivatives of the cumulative data (dumbbell data) for the various models as generated by the MATLAB software with the analytical derivatives of the developed models gives a perfect fit of R^2 approximately equal to unity showing conformity of the Hubbert theory with the physical explanation of the model performance viz-a-viz the data analysis and the ability of the models developed to predict into the far future beyond the available petroleum data range (i.e 1957 - 2014).

4.2.3 Meaning of input transfer function

The qualified input function for the selected model 5 is $Q(s) = \frac{1}{s[s+\frac{1}{R}]^2}$. This input function is made up of two parts i.e a product of $\frac{1}{s}$ (unit step) and $\frac{1}{[s+\frac{1}{R}]^2}$ (an exponential function which in time domain is, $te^{-\frac{t}{R}}$).

$Q(s) = \frac{1}{s}$ has single pole at $s = 0$

$Q(s) = \frac{1}{[s+\frac{1}{R}]^2}$ has a repeated pole at $s = -\frac{1}{R}$

Hence, the assumption of one input flow rate into the one composite reservoir is justified.

The input having the component $\frac{1}{s}$, means that the composite reservoir assumed is dull for a while and stopped for a long period. This is the characteristics of vibrations and disturbances.

4.2.4 Sensitivity analysis

Looking at table 4.10A and figure 4.11A, there was a very sharp increase in oil production as predicted by model 5, in the first ten years of operation for all constant R values tested. The rate of increase dropped drastically after the first ten years for all R (valve constant) values tested. Simulation results show an increase in oil production to 108.82MMB, 818.07 and 2415.15 barrels for constant R = 5, 10 and 15 respectively at the tenth year: and an increase to 113.65, 865.40 and 2882.2 barrels at the twentieth year. These show differences in increase of 103, 812 and 2410 in the first ten years, i.e between zero and ten years, as compared to differences in increase of 5, 46 and 466 at the end of the twentieth year for constant R = 5, 10 and 15 respectively. This drop in rate of increase with time is seen throughout the period of simulation of the model (or of production) for all constant R values tested.

From table 4.10B and figure 4.11B, the observed trends/behavior for the oil discovery is similar to that observed for oil production except that the initial magnitude of increase is larger and subsequent magnitude of decrease is also smaller for all constant R values tested.

Again, similar trends were observed for the gas production. From table 4.10C and figure 4.11C, it was seen that the rate of increase in gas production as predicted by the chosen model 5 were more for the first ten years, for the various R values, and this rate of increase in gas production reduced progressively between the tenth and twentieth year, and the subsequent intervals of 10 years.

For the gas discovery, the trend is not otherwise. Table 4.10D and figure 4.11D again shows a steep slope of volume of gas discovery, for all tested R values, for the first ten years, and a continual decrease in the rate of increase of the gas discovery volume in the subsequent intervals of ten years.

These models predicted an increase in the rate of production in the first ten years of operation, after which there was a gradual decline in the production rate until the well eventually stopped production.

This trend is consistent with typical well behavior where the production rate rises (increases), reaches the maximum and begins to decline (decreases). This behavior is as a result of reduced well pressures as production continues.

The effect of the valve restriction/opening is also seen in the oil production as predicted by the models. Oil production increases in the rate in which the valve restriction/opening is increased (R values).

The typical application of this trend test for the selected model 5 is that the model can be used to determine the life span (exhaustion) or production period of the reserve at various R values.

Tables 4.12 (A – D) and figures 4.13 (A – D), were all not subjected to any considerations for the basic reasons that the simulation tables and plots produced negative values which had no

physical meaning viz-a-viz the petroleum resources production and discovery volumes under discuss, and the added fact that we had already made a selection of model 5 based on the remarks for the selection of the various models developed, where in out of the models found to have positive initial reserve (models 5, 6 and 7), model 5 proved to be the best based on their respective R^2 values, especially for gas, while model 7 may be preferred for oil as it is closer to the control.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

In this work, models were developed from material balance of Nigerian petroleum around a composite reserve. Process control concepts were introduced to obtain the transfer functions so that as input functions were varied, new models were obtained, to determine which input has the best impact. Hubbert oil depletion concept was employed for the peak determination, dumbbell intersection and establishment of petroleum depletion, and this was because Hubbert's work on petroleum resources depletion, peaking and exhaustion stands out. The model development from this work can be deployed to the study of any growth and natural resource depletion.

The Nigerian petroleum data were obtained from the Department of Petroleum Resources (DPR) as field data for fifty seven (57) years giving fifty seven (57) data points, and MATLAB software version 7.9 was employed in the mathematical computations, analysis, curve fittings and plotting of graphs, all of which served to validate the developed models with the data obtained. The results obtained were juxtaposed with the Hubbert model (model 11) as the control model for the same data points, and afterwards validation and sensitivity analysis were performed on the set of the good models obtained (models 5, 6 & 7) which still turned out to justify the chosen model 5 to be superior (to an extent) to the Hubbert model. The chosen model 5 developed in this work is

$V_{(t)} = R^3(1 - e^{-\frac{1}{R}t}) + (V_0 - R^2t - \frac{Rt^2}{2}) e^{-\frac{1}{R}t}$ for the cumulative production and discovery of oil/gas

resources, and $P_{(t)} = (\frac{V_0}{R} - \frac{t^2}{2}) e^{-\frac{1}{R}t}$ for the annual production and discovery of oil/gas resources

respectively, which were established for the input function $Q(s) = \frac{1}{s[s + \frac{1}{R}]^2}$.

With the use of the foregoing model, it has been found that the Nigerian oil reserve has peaked in the year 2008AD with a volume of 28,019MMB and the gas reserve will peak in the year 2065AD with a volume of 2.54587×10^9 bscf, and that the oil reserve will be exhausted in the year

2682AD and the gas will follow thereafter in the year 3151AD. This result is considered accurate to R^2 of 0.9957 - 0.9963 for oil and 0.9979 - 0.9983 for gas, as shown for model 5 (table 4.2).

Comparing these results with those for the control model (Hubbert) which shows that the Nigerian oil reserve had peaked in the year 1990AD (as against 2008AD obtained in this work) with volume of 13661.9MMB and gas reserve had peaked in 2006AD (as against 2065AD obtained) with volume of 6.66104×10^{10} bscf, and that oil will be exhausted in the year 2141AD (as against 2682AD obtained) and gas will follow in the year 3057AD (as against 3151AD obtained). From these results it is certain that oil had peaked before gas and will also be exhausted before gas.

From the validation by comparison done, it is seen that the obtained results clearly show the efficacy of the models developed, and their reliability in the prediction of Nigeria's petroleum resources depletion, and from the sensitivity analysis carried out, it is seen that the developed model predicted an increase in the rate of production in the first ten years of operation, after which there was a gradual decline in the production rate until the well eventually stopped production. This trend is consistent with typical well behavior where the production rate rises (increases), reaches the maximum and begins to decline (decreases). This behavior is as a result of reduced well pressures as production continues. The effect of the valve restriction/opening was also seen in the oil production as predicted by the model. Oil production increases in the rate in which the valve restriction/opening is increased (R values).

Findings from this study can be used by Nigerian government for planning, diversification of the economy and international bargaining and positioning in OPEC.

5.2 Recommendations

The following recommendations are put forward:

- i. Nigerian researchers in the energy specialty should study more of fossil fuel depletion predictions, especially for petroleum resources in order to create awareness and make

room for planning. This will help cushion the effect of possible and sudden petroleum resources exhaustion.

- ii. More research should pay attention on improving this investigation by studying non-conventional petroleum reserves depletion, production, peaking and exhaustion.
- iii. Nigerian researchers should as a matter of urgency begin to study updated technology of oil-fuelled industrial processes and mechanical systems to gas-fuelled due to the prediction that gas will last a little longer than oil.
- iv. Other alternative technologies such as the gas-to-liquid (GTL) technology should be given attention since the country's gas reserve is enormous.

5.3 Contribution to Knowledge

Findings from this research have added to existing knowledge in the following ways:

- i. The literature survey undertaken revealed that comprehensive information on fossil fuel resources depletion in Nigeria is not available. This work has done this. Such data are necessary to guide national economic planning and development. This is even more urgent for a mono-economy country like Nigeria. The country depends on export of crude oil for 90% of foreign exchange earnings.
- ii. In this work, simple mathematical models which are capable of predicting fossil fuel (crude oil and natural gas) resources depletion in our country Nigeria have been successfully developed, with pronounced and proven efficacy.

The models include:-

For the cumulative production and discovery of oil/gas resources:

$$\text{Model 5: } V_{(t)} = R^3(1 - e^{-\frac{1}{R}t}) + (V_0 - R^2t - \frac{Rt^2}{2}) e^{-\frac{1}{R}t}$$

$$\text{Model 6: } V_{(t)} = R^4(1 - e^{-\frac{1}{R}t}) + (V_0 - R^3t + \frac{R^2t^2}{2} - \frac{Rt^3}{6}) e^{-\frac{1}{R}t}$$

$$\text{Model 7: } V_{(t)} = R^5(1 - e^{-\frac{1}{R}t}) + (V_0 - R^4t + R^3\frac{R^3t^2}{2} - \frac{R^2t^3}{6} - \frac{Rt^4}{24}) e^{-\frac{1}{R}t}$$

For the annual production and discovery of oil/gas resources:

$$\text{Model 5: } P_{(t)} = \left(\frac{V_0}{R} - \frac{t^2}{2}\right)e^{-\frac{1}{R}t}$$

$$\text{Model 6: } P_{(t)} = \left(\frac{V_0}{R} - \frac{t^3}{6}\right)e^{-\frac{1}{R}t}$$

$$\text{Model 7: } P_{(t)} = \left(\frac{V_0}{R} - \frac{t^4}{24}\right)e^{-\frac{1}{R}t}$$

- iii. The developed models also have the imminent ability to analyze and predict the depletion of other natural resources which have a cumulative growth tendency. Also the models are well suited in the analysis of human population of any country, where the growth rate and death rate of humans are analogous to the petroleum resources discovery and production respectively, and the population density at any point in time as the reserve, although this does not imply that the world human population will exhaust like fossil fuel.
- iv. The results generated from the developed mathematical models have revealed the earliest possible times of peaking (i.e 2008AD for oil production at volume of 28019MMB and 2065AD for gas production at volume of 2.54587×10^9 bscf) and exhaustion of Nigeria's crude oil and natural gas. Thus providing useful information for national economic planning and development.
- v. This study revealed the availability of large quantity of natural gas resources in Nigeria and perhaps fifth to the quantity of Russian natural gas in the world. Russia supplies most of the gas used in the entire Europe through Ukraine. Nigeria is capable of supplying the entire West Africa her natural gas needs for decades.
- vi. The finding has also provided the basis for the short-, mid- or long-term establishment of downstream petrochemical industries because of availability of their basic raw materials.

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APPENDIX A

Official Response for Petroleum Resources Data (DPR, 2016)

MINISTRY OF PETROLEUM RESOURCES

DEPARTMENT OF PETROLEUM RESOURCES
7 KOFO ABAYOMI STREET VICTORIA ISLAND LAGOS

File No: 12650

File Ref: 01-2790000

Website: www.ppr.gov.ng



Ref No: PI/RE/3042/UNC/Vol.3/704

Date: 21st March, 2016

Ag. Head
Department of Chemical Engineering
Federal University Technology Owerri
Imo State.

Dear Sir,

Re: DISCOVERY AND PRODUCTION DATA ON OIL AND CONDENSATE

Reference is made to your letter dated 13th January, 2016 on the above subject matter.

Please find attached the requested data which should be forwarded to the concerned student. You are to ensure that the data is treated as confidential and a copy of the final project should be made available to the **Department of Petroleum Resources** on completion of the project.

The transmitted data should be in the custody of the University for re-use by other students.

Failure to comply with the above will dampen our enthusiasm in responding to your request in the future.

We acknowledge the receipt of payment with Remita Retrieval Reference (RRR) number F99918726 via the Remita platform dated 10th March, 2016 for the sum of one thousand naira only (N10,000.00), being the statutory fee for this application.

Yours faithfully,

A.M. Abba
for: Director, Petroleum Resources

APPENDIX B

Oil, Gas and Condensate Reserve 1958 – 2014 (DPR, 2016)

Oil, Gas and Condensate Reserve 1958 - 2014

Year	Oil Reserves	Condensate Reserves	Daily Average Oil & Condensate Prod Rate	Oil + cond Prod	Net Addition to Oil & Condensate Reserves	Total Gas Reserves
	(MMB)	(MMB)	bopd	(MMB)	(MMB)	(BSCF)
1956			0.00			
1957			0.00			
1958	184.00		5,139.94	1.88	184.00	2,260.00
1959	442.00		11,221.06	4.10	258.00	2,950.00
1960	1,152.00		17,461.91	6.37	710.00	3,270.00
1961	1,648.00		46,033.45	16.80	496.00	4,550.00
1962	1,773.00		67,463.44	24.62	145.00	4,380.00
1963	1,431.00		76,476.72	27.91	362.00	4,822.00
1964	2,211.00		120,541.70	44.00	780.00	6,370.00
1965	2,290.00		272,206.35	99.36	79.00	10,930.00
1966	2,630.00		417,617.80	152.43	340.00	13,030.00
1967	3,258.00		319,330.09	116.56	628.00	13,960.00
1968	3,566.00		142,214.46	51.91	308.00	15,120.00
1969	3,963.00		537,751.26	196.28	397.00	17,210.00
1970	6,939.00		1,084,501.72	395.84	2,976.00	44,550.00
1971	8,251.00		1,531,077.07	558.84	1,312.00	51,204.00
1972	8,580.00		1,822,727.72	665.30	329.00	60,744.00
1973	9,224.00		1,970,902.13	719.38	644.00	60,684.00
1974	9,465.00		2,255,673.22	823.32	241.00	58,322.00
1975	10,332.00		1,808,625.62	660.15	867.00	75,598.00
1976	10,865.00		2,076,872.27	758.06	533.00	76,706.00
1977	10,965.00		2,098,781.69	766.06	100.00	84,356.00
1978	10,976.00		1,907,738.92	696.32	11.00	99,962.00
1979	12,428.00		2,316,338.79	845.46	1,452.00	95,085.00
1980	12,750.00		2,082,514.30	760.12	322.00	99,369.00
1981	13,202.00		1,439,153.67	525.29	452.00	93,421.00
1982	14,161.00		1,289,420.22	470.64	959.00	94,021.00
1983	15,459.00		1,235,510.24	450.96	1,298.00	95,000.00
1984	15,108.00		1,390,375.36	507.49	-351.00	94,150.00
1985	14,682.00	1,219.00	1,498,873.04	547.09	793.00	96,728.00
1986	13,733.00	1,228.78	1,468,298.63	535.93	-939.22	96,096.00
1987	14,051.00	1,130.16	1,175,030.14	428.89	219.38	95,391.00
1988	14,554.00	1,298.36	1,449,334.25	529.01	671.20	95,141.00
1989	14,046.00	1,645.00	1,716,852.05	626.65	-161.36	97,762.00
1990	16,636.00	2,890.51	1,804,906.85	658.79	3,835.51	103,564.00
1991	17,362.00	2,820.22	1,893,101.37	690.98	655.71	97,947.92
1992	17,542.61	2,939.74	1,956,767.12	714.22	300.13	106,992.10
1993	17,701.00	3,285.39	2,058,575.34	751.38	504.04	104,954.47
1994	19,783.00	3,169.17	2,007,665.75	732.80	1,965.78	107,720.70
1995	19,783.37	3,562.41	2,038,156.16	743.93	393.61	112,574.44
1996	21,414.96	3,519.03	2,252,315.17	822.10	1,588.21	114,705.47
1997	22,532.72	3,687.37	2,358,572.90	860.88	1,286.10	129,888.94
1998	24,198.67	3,982.92	2,171,057.08	792.44	1,961.50	136,330.61
1999	25,251.54	4,078.77	2,125,477.19	775.80	1,148.72	136,642.04
2000	25,878.49	3,955.63	2,319,552.39	846.64	503.81	147,562.62
2001	27,667.31	4,240.95	2,364,679.33	863.11	2,074.14	171,464.56
2002	29,189.12	5,160.20	2,041,063.54	744.99	2,441.06	176,602.22
2003	30,165.72	5,089.15	2,291,234.97	836.30	905.55	186,748.48
2004	31,020.00	4,856.00	2,499,869.12	912.45	621.13	187,896.60
2005	31,329.00	4,891.00	2,520,128.19	919.85	344.00	186,293.56
2006	32,535.69	4,959.12	2,222,156.32	811.09	1,274.81	184,867.82
2007	32,942.57	5,183.10	2,212,897.07	807.71	630.86	192,651.48
2008	33,266.50	5,202.05	2,001,052.67	730.38	342.88	183,020.75
2009	31,813.97	5,351.67	2,153,589.61	786.06	-1,302.91	182,554.46
2010	31,219.00	5,313.97	2,524,035.62	921.27	-632.67	182,816.65
2011	31,169.60	5,077.81	2,495,260.27	910.77	-285.55	182,752.95
2012	32,233.24	4,905.86	2,535,608.74	925.50	891.68	180,724.43
2013	31,813.51	5,257.32	2,333,232.88	851.63	-68.26	180,324.38
2014	31,870.00	5,578.24	2,215,556.16	808.68	377.41	187,379.88
2015						

OR *J.S.F.* OP ✓ GR

APPENDIX C

Nigerian Petroleum Cumulative Production from 1958 – 2014 (Crude Oil)

Year	Y_r t_p	Cumulative Oil Production, Y_{po} (MMB)	Cumulative Oil Reserve, Y_{ro} (MMB)	Cumulative Oil Discovery, Y_{do} (MMB); Q_{do} = Q_{po} + Q_{ro}
1958	1	1.88	184.00	186
1959	2	5.98	626.00	632
1960	3	12.35	1778.00	1791
1961	4	29.15	3426.00	3455
1962	5	53.77	5179.00	5233
1963	6	81.16	6610.00	6692
1964	7	125.68	8821.00	8945
1965	8	225.04	11111.00	11336
1966	9	377.47	13741.00	14119
1967	10	494.03	16999.00	17493
1968	11	545.94	20565.00	21111
1969	12	742.22	24528.00	25270
1970	13	1138.06	31467.00	32605
1971	14	1696.90	39718.00	41415
1972	15	2362.20	48298.00	50660
1973	16	3081.58	57522.00	60604
1974	17	3904.90	66987.00	70892
1975	18	4565.05	77319.00	81884
1976	19	5323.11	88184.00	93507
1977	20	6089.17	99149.00	105238
1978	21	6785.49	110125.00	116911

1979	22	7630.95	122553.00	130184
1980	23	8391.07	135303.00	143694
1981	24	8916.36	148505.00	157421
1982	25	9387.00	162666.00	172053
1983	26	9837.96	178125.00	187963
1984	27	10345.45	193233.00	203579
1985	28	10892.54	207915.00	218808
1986	29	11428.47	221648.00	233077
1987	30	11857.36	235699.00	247556
1988	31	12386.37	250253.00	262639
1989	32	13013.02	264299.00	277312
1990	33	13671.81	280935.00	294607
1991	34	14362.79	298297.00	312660
1992	35	15077.01	315839.61	330917
1993	36	15828.39	333540.61	349369
1994	37	16561.19	353323.61	369885
1995	38	17305.12	373106.98	390412
1996	39	18127.22	394521.94	412649
1997	40	18988.10	417054.66	436043
1998	41	19780.54	441253.33	461034
1999	42	20556.34	466504.87	487061
2000	43	21402.98	492383.36	513791
2001	44	22266.09	520050.67	542317
2002	45	23011.08	549239.79	572251
2003	46	23847.38	579405.51	603253

2004	47	24759.83	610425.51	635185
2005	48	25679.68	641754.51	667434
2006	49	26490.77	674290.20	700781
2007	50	27298.48	707232.77	734531
2008	51	28028.86	740499.27	768528
2009	52	28814.92	772313.24	801128
2010	53	29736.19	803532.24	833268
2011	54	30646.96	834701.84	865349
2012	55	31572.46	866935.08	898508
2013	56	32424.09	898748.59	931173
2014	57	33232.77	930618.59	963851

APPENDIX D

Nigerian Petroleum Cumulative Production from 1958 – 2014 (Gas)

Year	$Y_r t_p$	Cumulative Gas Production, Y_{pg} (bscf)	Cumulative Gas Reserve, Y_{rg} (bscf)	Cumulative Gas Discovery, Y_{dg} (bscf); $Q_{dg} = Q_{pg} + Q_{rg}$
1958	1	1.61	2260.00	2262
1959	2	6.55	5210.00	5217
1960	3	57.50	8480.00	8538
1961	4	68.44	13030.00	13098
1962	5	85.62	17410.00	17496
1963	6	107.73	22232.00	22340
1964	7	144.06	28602.00	28746
1965	8	223.50	39532.00	39756
1966	9	326.17	52562.00	52888
1967	10	419.20	66522.00	66941
1968	11	470.83	81642.00	82113
1969	12	616.56	98852.00	99469
1970	13	900.46	143402.00	144303
1971	14	1358.69	194606.00	195965
1972	15	1963.34	255350.00	257313
1973	16	2736.12	316034.00	318770
1974	17	3695.71	374355.00	378051
1975	18	4354.54	449954.00	454309
1976	19	5105.92	526660.00	531766
1977	20	5880.18	611015.00	616896
1978	21	6632.62	710978.00	717611
1979	22	7607.96	806063.00	813671

1980	23	8486.78	905432.00	913919
1981	24	9094.25	998853.00	1007947
1982	25	9617.95	1092874.00	1102492
1983	26	10155.00	1187874.00	1198029
1984	27	10809.00	1282024.00	1292833
1985	28	11464.10	1378752.00	1390216
1986	29	12125.90	1474848.00	1486974
1987	30	12732.20	1570239.00	1582971
1988	31	13448.00	1665380.00	1679928
1989	32	14376.70	1763142.00	1777519
1990	33	15404.50	1866706.00	1882171
1991	34	16576.90	1964653.92	1981231
1992	35	17759.80	2071646.02	2089406
1993	36	18952.20	2176600.49	2195530
1994	37	20122.80	2284321.19	2304444
1995	38	21363.80	2396895.63	2418260
1996	39	22702.40	2511601.10	2534303
1997	40	23966.80	2641490.00	2665457
1998	41	25218.40	2777820.65	2803039
1999	42	26597.40	2914462.69	2941060
2000	43	28318.20	3062025.31	3090343
2001	44	30261.30	3233489.87	3263751
2002	45	32012.50	3410092.09	3442105
2003	46	33905.80	3596840.57	3630747
2004	47	36015.98	3784737.17	3820753

2005	48	38151.32	3971030.73	4009182
2006	49	40441.21	4155898.55	4196340
2007	50	43048.08	4348550.13	4391598
2008	51	45628.48	4531570.78	4577199
2009	52	47856.63	4714125.24	4761982
2010	53	50676.31	4896941.89	4947618
2011	54	53642.96	5079694.84	5133338
2012	55	55609.00	5260419.27	5317028
2013	56	59420.98	5440743.65	5446686
2014	57	62469.53	5628123.53	5690593

APPENDIX E

Derivatives for Oil and Gas Production and Discovery – Hubbert’s Model

Time (years)	Dp _o (MMB)	Dd _o (MMB)	Dp _g (MMB)	Dd _g (MMB)
0	82.92	1645.31	90.843	10067.0
5.7	137.39	2665.71	142.62	16468.9
11.4	222.49	4272.15	222.613	26605.3
17.1	347.29	6728.37	344.343	42121.0
22.8	511.95	10310.5	525.245	64590.6
28.5	694.07	15152.60	784.311	94336.2
34.2	840.69	20947.80	1134.62	128367.0
39.9	889.06	26624.4	1568.07	158732.0
45.6	814.73	30428.70	2035.05	174490.0
51.3	654.52	30813.70	2435.04	168598.0
57.0	472.71	27607.60	2644.80	143796.0

APPENDIX F

Dumb-bell Data for Models 4, 5, 6 and 7

Year	DQ _{po4}	DQ _{do4}	DQ _{pg4}	DQ _{dg4}	DQ _{po5}	DQ _{do5}	DQ _{pg5}	DQ _{dg5}
0	10.5804	0.9830	0.1356	6.4968	-2.333	349.0060	-11.6674	-1.1404 e ⁺³
5.7	204.5960	3.3782e ⁺³	205.7230	2.0209 e ⁺⁴	96.0005	959.7870	37.6476	7.7540 e ⁺³
11.4	363.6080	6.7529 e ⁺⁴	411.1580	4.0391 e ⁺⁴	288.3760	4.0851 e ⁺³	171.3880	2.8892 e ⁺⁴
17.1	491.6080	1.0124 e ⁺⁴	616.4410	6.0553 e ⁺⁴	481.9600	8.1985 e ⁺³	372.4530	5.4367 e ⁺⁴
22.8	593.1830	1.3495 e ⁺⁴	821.5680	8.0694 e ⁺⁴	635.4210	1.2708 e ⁺⁴	625.9920	8.1425 e ⁺⁴
28.5	672.0940	1.6862 e ⁺⁴	1.0265 e ⁺³	100814	735.9590	17201	919.1620	106872
34.2	731.6300	2.0227 e ⁺⁴	1.2314 e ⁺³	120913	785.4330	2.1404 e ⁺⁴	1.2409 e ⁺³	129128
39.9	774.6840	2.3589e ⁺⁴	1.4360 e ⁺³	140993	792.2420	2.5146 e ⁺⁴	1.5818 e ⁺³	147393
45.6	803.7600	2.6949 e ⁺⁴	1.6405 e ⁺³	161051	766.7920	2.8331 e ⁺⁴	1.9336 e ⁺³	161393
51.5	821.0570	3.0306 e ⁺⁴	1.8449 e ⁺³	181089	719.1290	3.0919 e ⁺⁴	2.2897 e ⁺³	171226
57	828.4830	3.3661 e ⁺⁴	2.0491 e ⁺³	201107	657.8680	3.2907 e ⁺⁴	2.6443 e ⁺³	177177

Year	DQ _{po6}	DQ _{do6}	DQ _{pg6}	DQ _{dg6}	DQ _{po7}	DQ _{do7}	DQ _{pg7}	DQ _{dg7}
0	-34.1950	-1.3000 e ⁺³	-60.7660	-5.5687 e ⁺³	0	0	0	0
5.7	27.5630	-363.1960	-26.2570	-79.0480	1.88	186	1.61	2262
11.4	220.7560	2.5880 e ⁺³	95.8730	19293	5.98	632	6.55	5217
17.1	468.3080	7.3915 e ⁺³	322.0700	4.9732 e ⁺⁴	13.35	1791	57.50	8538
22.8	677.2740	1.3020 e ⁺⁴	628.1750	8.3767 e ⁺⁴	29.15	3455	68.44	13098
28.5	802.2420	1.8449 e ⁺⁴	976.4260	114644	53.77	5233	85.62	17496
34.2	839.1940	2.2965 e ⁺⁴	1.3296 e ⁺³	138230	81.68	6692	107.73	22340
39.9	806.1330	2.6197 e ⁺⁴	1.6576 e ⁺³	152912	125.68	8947	144.06	28746
45.6	727.6900	2.8056 e ⁺⁴	1.9592 e ⁺³	-152912	225.04	11336	223.50	39756
51.5	626.4650	2.8642 e ⁺⁴	2.1621 e ⁺³	157421	377.47	14119	326.11	52888
57	519.5450	2.8162 e ⁺⁴	2.3213 e ⁺³	150231	494.03	17493	419.20	66941

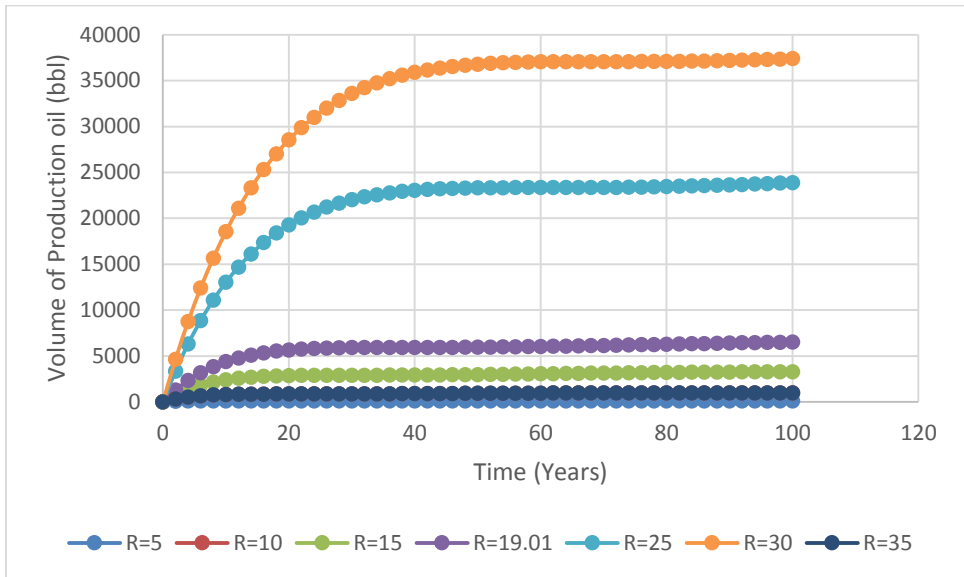
APPENDIX G

Detailed Trend Test Analysis for the Selected Model 5

Detailed simulation table for oil production against time

t	R=5	R=10	R=15	R=19.01	R=25	R=30	R=35
0	5.464	5.464	5.464	5.464	5.464	5.464	5.464
2	71.68542	333.1143	793.6448	1307.318	3374.205	4634.288	333.1143
4	98.24875	547.845	1391.601	2356.003	6315.685	8756.297	547.845
6	107.0681	684.688	1840.278	3195.253	8877.775	12420.45	684.688
8	108.942	768.8041	2172.647	3861.978	11103.51	15671.43	768.8041
10	108.8226	818.0704	2415.148	4387.25	13031.55	18550.03	818.0704
12	108.7129	845.0247	2588.883	4797.161	14696.59	21093.42	845.0247
14	109.2176	858.3212	2710.606	5113.552	16129.7	23335.48	858.3212
16	110.3445	863.8135	2793.54	5354.648	17358.74	25307.05	863.8135
18	111.8973	865.3581	2848.05	5535.606	18408.61	27036.18	865.3581
20	113.6528	865.4042	2882.196	5668.978	19301.55	28548.36	865.4042
22	115.4294	865.4256	2902.186	5765.116	20057.41	29866.76	865.4256
24	117.1033	866.2331	2912.751	5832.513	20693.88	31012.39	866.2331
26	118.6033	868.1989	2917.443	5878.099	21226.68	32004.27	868.1989
28	119.8987	871.4149	2918.889	5907.493	21669.8	32859.67	871.4149
30	120.9856	875.8044	2918.983	5925.214	22035.64	33594.16	875.8044
32	121.877	881.1971	2919.054	5934.868	22335.16	34221.85	881.1971
34	122.5948	887.3807	2919.993	5939.3	22578.05	34755.45	887.3807
36	123.1637	894.1333	2922.36	5940.726	22772.88	35206.42	894.1333
38	123.609	901.2434	2926.461	5940.845	22927.16	35585.07	901.2434
40	123.9535	908.5219	2932.423	5940.933	23047.48	35900.66	908.5219
42	124.2176	915.8068	2940.236	5941.921	23139.64	36161.52	915.8068
44	124.4182	922.9654	2949.802	5944.461	23208.66	36375.09	922.9654
46	124.5696	929.8931	2960.96	5948.985	23258.95	36548.04	929.8931
48	124.6831	936.5113	2973.511	5955.746	23294.28	36686.3	936.5113
50	124.7676	942.7643	2987.24	5964.86	23317.96	36795.16	942.7643
52	124.8302	948.6158	3001.924	5976.337	23332.79	36879.34	948.6158
54	124.8764	954.0459	3017.345	5990.107	23341.19	36943	954.0459
56	124.9103	959.0479	3033.294	6006.04	23345.2	36989.81	959.0479
58	124.9351	963.6253	3049.579	6023.967	23346.55	37023.03	963.6253
60	124.9532	967.7898	3066.024	6043.689	23346.69	37045.5	967.7898
62	124.9663	971.5585	3082.472	6064.993	23346.81	37059.71	971.5585
64	124.9758	974.9528	3098.786	6087.657	23347.9	37067.83	974.9528
66	124.9827	977.9967	3114.849	6111.459	23350.75	37071.76	977.9967
68	124.9876	980.7155	3130.561	6136.181	23355.98	37073.09	980.7155
70	124.9912	983.1352	3145.84	6161.615	23364.08	37073.24	983.1352
72	124.9937	985.2814	3160.619	6187.563	23375.41	37073.37	985.2814
74	124.9955	987.1793	3174.847	6213.841	23390.21	37074.48	987.1793
76	124.9968	988.8527	3188.485	6240.279	23408.65	37077.41	988.8527
78	124.9978	990.3243	3201.507	6266.723	23430.82	37082.83	990.3243
80	124.9984	991.6153	3213.894	6293.032	23456.72	37091.29	991.6153

82	124.9989	992.7452	3225.638	6319.083	23486.33	37103.23	992.7452
84	124.9992	993.7319	3236.739	6344.767	23519.56	37118.99	993.7319
86	124.9994	994.592	3247.201	6369.989	23556.3	37138.8	994.592
88	124.9996	995.3402	3257.036	6394.668	23596.38	37162.85	995.3402
90	124.9997	995.9899	3266.258	6418.734	23639.64	37191.21	995.9899
92	124.9998	996.5531	3274.886	6442.13	23685.9	37223.94	996.5531
94	124.9999	997.0406	3282.94	6464.811	23734.93	37261.03	997.0406
96	124.9999	997.4619	3290.444	6486.74	23786.53	37302.41	997.4619
98	124.9999	997.8255	3297.422	6507.89	23840.48	37348	997.8255
100	125	998.1389	3303.899	6528.24	23896.56	37397.68	998.1389



Detailed simulation plot for oil production against time

Table G1: Simulation table for oil production for model 5, as shown in table 4.2

t	R=5	R=10	R=15	R=19.01	R=25	R=30	R=35
0	5.464	5.464	5.464	5.464	5.464	5.464	5.464
10	108.8226	818.0704	2415.148	4387.25	8506.512	13031.55	18550.03
20	113.6528	865.4042	2882.196	5668.978	11976.66	19301.55	28548.36
30	120.9856	875.8044	2918.983	5925.214	13179.44	22035.64	33594.16
40	123.9535	908.5219	2932.423	5940.933	13480.95	23047.48	35900.66
50	124.7676	942.7643	2987.240	5964.860	13511.13	23317.96	36795.16
60	124.9532	967.7898	3066.024	6043.689	13527.64	23346.69	37045.50
70	124.9912	983.1352	3145.840	6161.615	13611.00	23364.08	37073.24
80	124.9984	991.6153	3213.894	6293.032	13765.45	23456.72	37091.29
90	124.9997	995.9899	3266.258	6418.734	13968.65	23639.64	37191.21
100	125	998.1389	3303.899	6528.240	14194.19	23896.56	37397.68

Detailed simulation table for oil discovery against time

t	R=70	R=75	R=80	R=84.73	R=90	R=95	R=100
0	5.464	5.464	5.464	5.464	5.464	5.464	5.464
2	19054.54	21914.56	24974.57	28053.44	35354.61	39214.62	21914.56
4	37038.18	42678.31	48718.42	54800.54	69238.69	76878.76	42678.31
6	54008.74	62349.3	71289.8	80299.74	101711	113051.3	62349.3
8	70016.26	80977.97	92739.48	104602.1	132823.1	147784	80977.97
10	85108.56	98612.64	113116.2	127756.7	162624.8	181127.1	98612.64
12	99331.35	115299.6	132466.9	149811	191164.3	213129	115299.6
14	112728.3	131083.3	150836.6	170810.5	218488.4	243837	131083.3
16	125341	146006.3	168268.6	190799.1	244642	273296.4	146006.3
18	137209.4	160109.2	184804.4	209819.2	269668.8	301551.6	160109.2
20	148371.5	173431.2	200483.9	227911.4	293610.9	328645.3	173431.2
22	158863.5	186009.6	215345.7	245115	316509.1	354618.8	186009.6
24	168720.2	197880.1	229426.3	261467.5	338402.7	379512.2	197880.1
26	177974.6	209077.2	242761.4	277005.4	359329.8	403364.5	209077.2
28	186658.2	219633.6	255384.6	291763.6	379327.3	426213.1	219633.6
30	194801.2	229580.6	267328.8	305775.6	398430.6	448094.5	229580.6
32	202432.2	238948.4	278625	319073.9	416674.1	469043.8	238948.4
34	209578.6	247765.6	289303.4	331689.4	434091	489095.2	247765.6
36	216266.5	256059.9	299392.8	343652.1	450713.3	508281.5	256059.9
38	222520.9	263857.5	308920.6	354990.8	466572	526634.9	263857.5
40	228365.3	271183.5	317913.5	365733	481697.1	544186	271183.5
42	233822.4	278062	326396.8	375905.5	496117.3	560964.9	278062
44	238913.7	284515.9	334394.9	385533.8	509860.7	577000.4	284515.9
46	243659.7	290567.4	341931.2	394642.3	522954	592320.5	290567.4
48	248079.8	296237.3	349027.9	403254.8	535423.4	606952.2	296237.3
50	252192.7	301545.7	355706.6	411393.9	547293.9	620921.7	301545.7
52	256016	306511.7	361987.8	419081.3	558589.7	634254.2	306511.7
54	259566.5	311153.7	367891.1	426338	569334.2	646974.3	311153.7
56	262860.1	315489.1	373435.6	433184.2	579549.9	659105.6	315489.1
58	265912.2	319534.5	378639	439639	589258.6	670670.8	319534.5
60	268737.1	323305.9	383518.9	445721	598481.4	681692.2	323305.9
62	271348.5	326818.4	388091.6	451447.9	607238.4	692191.1	326818.4
64	273759.5	330086.6	392373.1	456836.9	615549.1	702188.1	330086.6
66	275982.5	333124.1	396378.5	461904.3	623432.4	711703.1	333124.1
68	278029.1	335944.1	400122.2	466665.8	630906.5	720755.5	335944.1
70	279910.7	338559.3	403618.2	471136.5	637988.8	729363.7	338559.3
72	281637.7	340981.4	406879.6	475330.7	644696.2	737545.8	340981.4
74	283220.2	343221.9	409919.1	479262.4	651045	745319.2	343221.9
76	284667.6	345291.6	412748.9	482944.8	657050.9	752700.5	345291.6
78	285989.1	347200.8	415380.5	486390.6	662728.8	759706.1	347200.8
80	287193.1	348959.3	417824.9	489612	668093.5	766351.4	348959.3
82	288287.7	350576.4	420092.7	492620.7	673158.7	772651.6	350576.4
84	289280.6	352061.1	422193.9	495427.9	677938.2	778621.2	352061.1
86	290179.1	353421.6	424138.2	498044.2	682444.7	784274.3	353421.6

88	290989.9	354666.1	425934.7	500479.9	686690.9	789624.4	354666.1
90	291719.5	355802.2	427592.1	502744.7	690688.7	794684.5	355802.2
92	292374	356837	429118.9	504848.2	694449.8	799467.4	356837
94	292959.3	357777.4	430522.9	506799.2	697985.2	803985.1	357777.4
96	293480.7	358630	431811.7	508606.3	701305.7	808249.3	358630
98	293943.3	359400.9	432992.5	510277.7	704421.6	812271.4	359400.9
100	294352.1	360095.9	434072.3	511821.3	707342.8	816062.3	360095.9

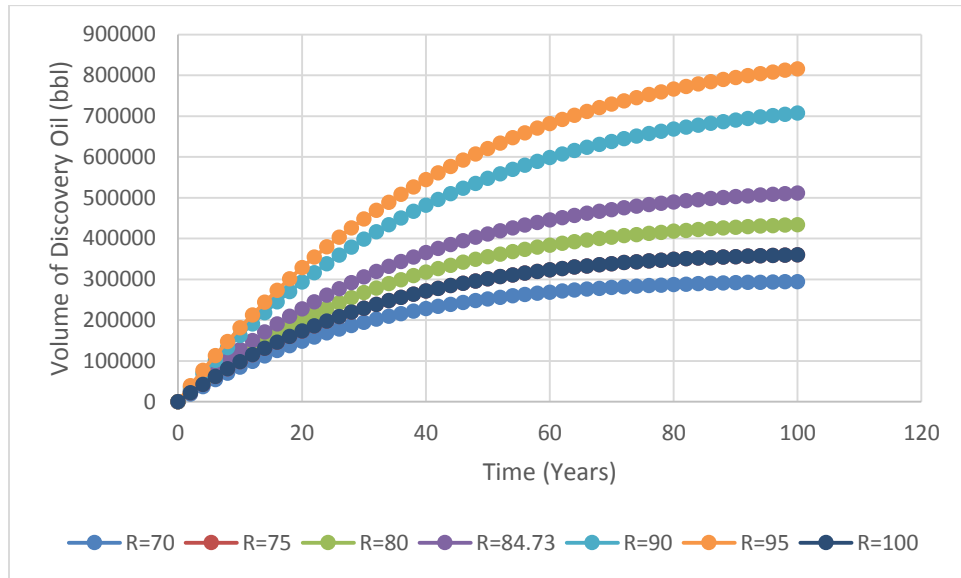


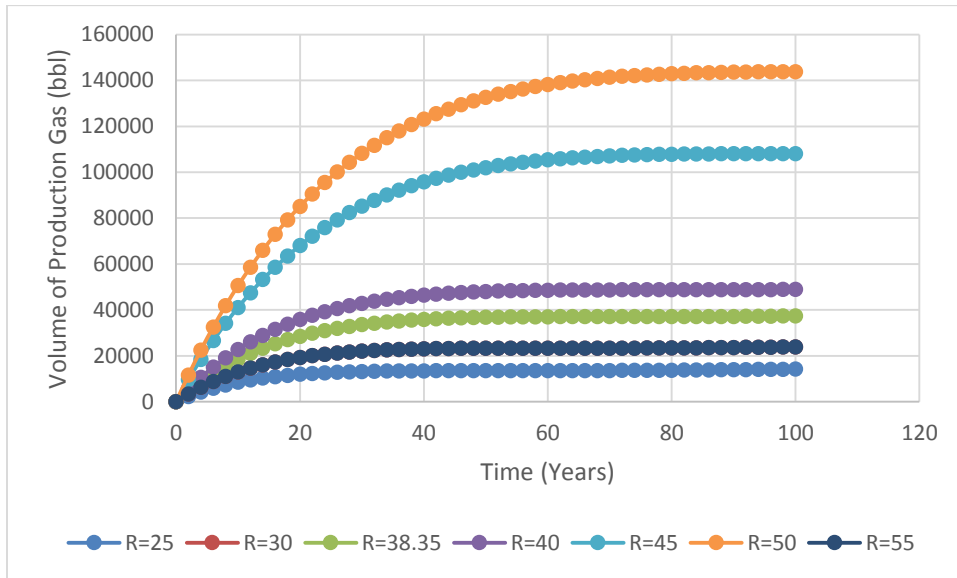
Table G2: Simulation table for oil discovery for model 5, as shown in table 4.2

t	R=70	R=75	R=80	R=84.73	R=90	R=95	R=100
0	5.464	5.464	5.464	5.464	5.464	5.464	5.464
10	85108.56	98612.64	113116.2	127756.7	145122.2	162624.8	181127.1
20	148371.5	173431.2	200483.9	227911.4	260573.0	293610.9	328645.3
30	194801.2	229580.6	267328.8	305775.6	351750.2	398430.6	448094.5
40	228365.3	271183.5	317913.5	365733.0	423160.5	481697.1	544186.0
50	252192.7	301545.7	355706.6	411393.9	478559.8	547293.9	620921.7
60	268737.1	323305.9	383518.9	445721.0	521068.9	598481.4	681692.2
70	279910.7	338559.3	403618.2	471136.5	553272.1	637988.8	729363.7
80	287193.1	348959.3	417824.9	489612.0	577301.8	668093.5	766351.4
90	291719.5	355802.2	427592.1	502744.7	594910.0	690688.7	794684.5
100	294352.1	360095.9	434072.3	511821.3	607529.6	707342.8	816062.3

Detailed simulation table for gas production against time

t	R=25	R=30	R=38.35	R=40	R=45	R=50	R=55
0	5.464	5.464	5.464	5.464	5.464	5.464	5.464
2	2314.091	3374.205	4634.288	5590.421	9614.438	11674.47	3374.205
4	4274.84	6315.685	8756.297	10615.2	18477.42	22517.66	6315.685
6	5934.11	8877.775	12420.45	15129.33	26645.37	32586.43	8877.775
8	7332.715	11103.51	15671.43	19178.41	34166.13	41929.32	11103.51
10	8506.512	13031.55	18550.03	22804.36	41084.57	50592.12	13031.55
12	9486.956	14696.59	21093.42	26045.75	47442.79	58618.03	14696.59
14	10301.6	16129.7	23335.48	28937.98	53280.25	66047.82	16129.7
16	10974.53	17358.74	25307.05	31513.59	58633.95	72919.92	17358.74
18	11526.77	18408.61	27036.18	33802.42	63538.53	79270.54	18408.61
20	11976.66	19301.55	28548.36	35831.84	68026.46	85133.85	19301.55
22	12340.11	20057.41	29866.76	37626.94	72128.13	90542	20057.41
24	12630.96	20693.88	31012.39	39210.67	75871.98	95525.3	20693.88
26	12861.15	21226.68	32004.27	40604.06	79284.62	100112.3	21226.68
28	13041.02	21669.8	32859.67	41826.31	82390.92	104329.7	21669.8
30	13179.44	22035.64	33594.16	42894.98	85214.16	108203	22035.64
32	13284.05	22335.16	34221.85	43826.08	87776.04	111755.8	22335.16
34	13361.37	22578.05	34755.45	44634.22	90096.86	115010.5	22578.05
36	13416.94	22772.88	35206.42	45332.71	92195.56	117988	22772.88
38	13455.48	22927.16	35585.07	45933.64	94089.8	120708.1	22927.16
40	13480.95	23047.48	35900.66	46448.03	95796.07	123189.4	23047.48
42	13496.7	23139.64	36161.52	46885.86	97329.71	125449.2	23139.64
44	13505.49	23208.66	36375.09	47256.2	98705.03	127503.8	23208.66
46	13509.63	23258.95	36548.04	47567.25	99935.33	129368.5	23258.95
48	13511	23294.28	36686.3	47826.42	101033	131057.7	23294.28
50	13511.13	23317.96	36795.16	48040.42	102009.5	132584.8	23317.96
52	13511.24	23332.79	36879.34	48215.28	102875.7	133962.4	23332.79
54	13512.3	23341.19	36943	48356.43	103641.3	135202.3	23341.19
56	13515.04	23345.2	36989.81	48468.77	104315.6	136315.5	23345.2
58	13520.03	23346.55	37023.03	48556.66	104907.2	137312.3	23346.55
60	13527.64	23346.69	37045.5	48624.01	105424	138202.3	23346.69
62	13538.16	23346.81	37059.71	48674.31	105873.3	138994.6	23346.81
64	13551.72	23347.9	37067.83	48710.68	106261.8	139697.5	23347.9
66	13568.4	23350.75	37071.76	48735.86	106595.8	140318.8	23350.75
68	13588.18	23355.98	37073.09	48752.3	106881.2	140865.9	23355.98
70	13611	23364.08	37073.24	48762.14	107123.1	141345.6	23364.08
72	13636.73	23375.41	37073.37	48767.27	107326.5	141764.1	23375.41
74	13665.23	23390.21	37074.48	48769.32	107495.9	142127.3	23390.21
76	13696.32	23408.65	37077.41	48769.74	107635.5	142440.7	23408.65
78	13729.79	23430.82	37082.83	48769.76	107749	142709.4	23430.82
80	13765.45	23456.72	37091.29	48770.44	107839.9	142938.1	23456.72
82	13803.06	23486.33	37103.23	48772.67	107911.4	143131	23486.33
84	13842.4	23519.56	37118.99	48777.22	107966.4	143292.3	23519.56
86	13883.26	23556.3	37138.8	48784.72	108007.6	143425.7	23556.3

88	13925.41	23596.38	37162.85	48795.69	108037.3	143534.5	23596.38
90	13968.65	23639.64	37191.21	48810.53	108057.8	143622.1	23639.64
92	14012.76	23685.9	37223.94	48829.57	108070.9	143691.1	23685.9
94	14057.55	23734.93	37261.03	48853.06	108078.6	143744.5	23734.93
96	14102.83	23786.53	37302.41	48881.17	108082.4	143784.5	23786.53
98	14148.43	23840.48	37348	48913.99	108083.7	143813.6	23840.48
100	14194.19	23896.56	37397.68	48951.6	108083.8	143833.6	23896.56



Detailed simulation plot for gas production against time

Table G3: Simulation table for gas production for model 5, as shown in table 4.2

t	R=25	R=30	R=38.27	R=40	R=45	R=50	R=55
0	5.464	5.464	5.464	5.464	5.464	5.464	5.464
10	8506.512	13031.55	18550.03	22804.36	32575.45	41084.57	50592.12
20	11976.66	19301.55	28548.36	35831.84	52898.12	68026.46	85133.85
30	13179.44	22035.64	33594.16	42894.98	65136.06	85214.16	108203.0
40	13480.95	23047.48	35900.66	46448.03	72164.69	95796.07	123189.4
50	13511.13	23317.96	36795.16	48040.42	75942.77	102009.5	132584.8
60	13527.64	23346.69	37045.50	48624.01	77781.84	105424.0	138202.3
70	13611.00	23364.08	37073.24	48762.14	78540.98	107123.1	141345.6
80	13765.45	23456.72	37091.29	48770.44	78766.82	107839.9	142938.1
90	13968.65	23639.64	37191.21	48810.53	78793.31	108057.8	143622.1
100	14194.19	23896.56	37397.68	48951.6	78812.28	108083.8	143833.6

Detailed simulation table for gas discovery against time

t	R=20	R=25	R=30	R=32.24	R=40	R=45	R=50
0	5.464	5.464	5.464	5.464	5.464	5.464	5.464
2	1453.921	2314.091	3374.205	3913.996	7754.399	9614.438	2314.091
4	2633.6	4274.84	6315.685	7359.642	14837.12	18477.42	4274.84
6	3588.771	5934.11	8877.775	10390.8	21304.08	26645.37	5934.11
8	4357.122	7332.715	11103.51	13051.3	27202.26	34166.13	7332.715
10	4970.661	8506.512	13031.55	15380.83	32575.45	41084.57	8506.512
12	5456.513	9486.956	14696.59	17415.26	37464.42	47442.79	9486.956
14	5837.601	10301.6	16129.7	19187	41907.09	53280.25	10301.6
16	6133.247	10974.53	17358.74	20725.33	45938.77	58633.95	10974.53
18	6359.68	11526.77	18408.61	22056.64	49592.28	63538.53	11526.77
20	6530.492	11976.66	19301.55	23204.7	52898.12	68026.46	11976.66
22	6657.02	12340.11	20057.41	24190.91	55884.64	72128.13	12340.11
24	6748.678	12630.96	20693.88	25034.51	58578.14	75871.98	12630.96
26	6813.25	12861.15	21226.68	25752.77	61003.05	79284.62	12861.15
28	6857.137	13041.02	21669.8	26361.18	63182.03	82390.92	13041.02
30	6885.568	13179.44	22035.64	26873.62	65136.06	85214.16	13179.44
32	6902.786	13284.05	22335.16	27302.48	66884.62	87776.04	13284.05
34	6912.204	13361.37	22578.05	27658.84	68445.71	90096.86	13361.37
36	6916.542	13416.94	22772.88	27952.57	69836	92195.56	13416.94
38	6917.94	13455.48	22927.16	28192.46	71070.91	94089.8	13455.48
40	6918.057	13480.95	23047.48	28386.31	72164.69	95796.07	13480.95
42	6918.154	13496.7	23139.64	28541.03	73130.49	97329.71	13496.7
44	6919.167	13505.49	23208.66	28662.73	73980.44	98705.03	13505.49
46	6921.763	13509.63	23258.95	28756.82	74725.73	99935.33	13509.63
48	6926.395	13511	23294.28	28828.03	75376.68	101033	13511
50	6933.344	13511.13	23317.96	28880.52	75942.77	102009.5	13511.13
52	6942.75	13511.24	23332.79	28917.94	76432.73	102875.7	13511.24
54	6954.649	13512.3	23341.19	28943.46	76854.56	103641.3	13512.3
56	6968.994	13515.04	23345.2	28959.81	77215.62	104315.6	13515.04
58	6985.672	13520.03	23346.55	28969.39	77522.66	104907.2	13520.03
60	7004.531	13527.64	23346.69	28974.22	77781.84	105424	13527.64
62	7025.381	13538.16	23346.81	28976.06	77998.8	105873.3	13538.16
64	7048.018	13551.72	23347.9	28976.37	78178.71	106261.8	13551.72
66	7072.221	13568.4	23350.75	28976.41	78326.25	106595.8	13568.4
68	7097.769	13588.18	23355.98	28977.2	78445.71	106881.2	13588.18
70	7124.441	13611	23364.08	28979.59	78540.98	107123.1	13611
72	7152.021	13636.73	23375.41	28984.27	78615.59	107326.5	13636.73
74	7180.303	13665.23	23390.21	28991.8	78672.72	107495.9	13665.23
76	7209.092	13696.32	23408.65	29002.59	78715.28	107635.5	13696.32
78	7238.205	13729.79	23430.82	29016.95	78745.87	107749	13729.79
80	7267.475	13765.45	23456.72	29035.11	78766.82	107839.9	13765.45
82	7296.746	13803.06	23486.33	29057.21	78780.25	107911.4	13803.06
84	7325.881	13842.4	23519.56	29083.32	78788.03	107966.4	13842.4
86	7354.753	13883.26	23556.3	29113.45	78791.84	108007.6	13883.26

88	7383.254	13925.41	23596.38	29147.56	78793.16	108037.3	13925.41
90	7411.284	13968.65	23639.64	29185.57	78793.31	108057.8	13968.65
92	7438.76	14012.76	23685.9	29227.35	78793.45	108070.9	14012.76
94	7465.611	14057.55	23734.93	29272.78	78794.6	108078.6	14057.55
96	7491.776	14102.83	23786.53	29321.67	78797.63	108082.4	14102.83
98	7517.204	14148.43	23840.48	29373.84	78803.3	108083.7	14148.43
100	7541.856	14194.19	23896.56	29429.09	78812.28	108083.8	14194.19

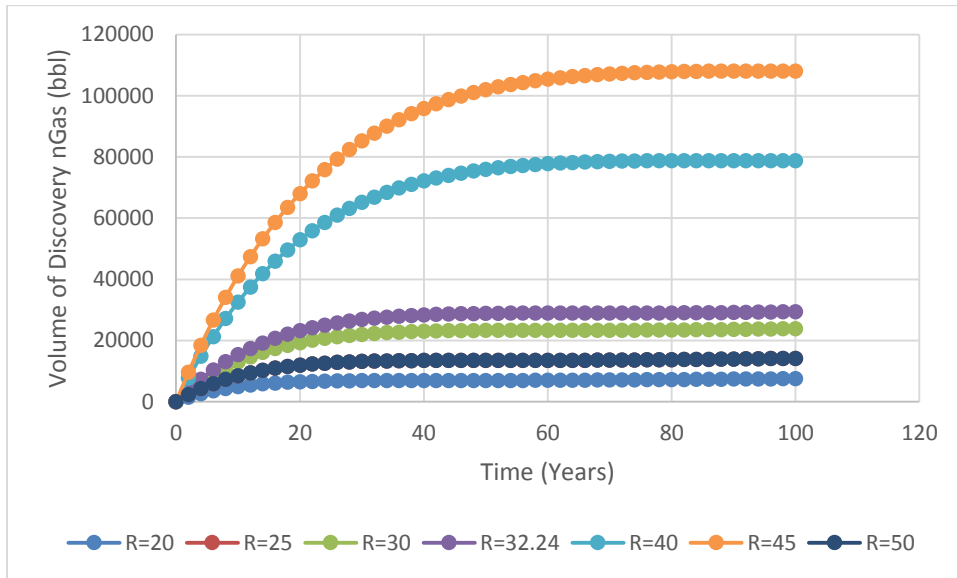


Table G4: Simulation table for gas discovery for model 5, as shown in table 4.2

t	R=20	R=25	R=30	R=32.24	R=40	R=45	R=50
0	5.464	5.464	5.464	5.464	5.464	5.464	5.464
10	4970.661	8506.512	13031.55	15380.83	25064.22	32575.45	41084.57
20	6530.492	11976.66	19301.55	23204.70	39742.09	52898.12	68026.46
30	6885.568	13179.44	22035.64	26873.62	47942.12	65136.06	85214.16
40	6918.057	13480.95	23047.48	28386.31	52229.87	72164.69	95796.07
50	6933.344	13511.13	23317.96	28880.52	54260.40	75942.77	102009.5
60	7004.531	13527.64	23346.69	28974.22	55076.01	77781.84	105424.0
70	7124.441	13611.00	23364.08	28979.59	55312.25	78540.98	107123.1
80	7267.475	13765.45	23456.72	29035.11	55339.28	78766.82	107839.9
90	7411.284	13968.65	23639.64	29185.57	55357.84	78793.31	108057.8
100	7541.856	14194.19	23896.56	29429.09	55463.61	78812.28	108083.8

Table G5: Simulation table for oil production for model 6, as shown in table 4.3

t	R=4	R=6	R=8	R=11.32	R=12	R=14	R=16
0	149.8	149.8	149.8	149.8	149.8	149.8	474.7
10	74.35684	142.6878	199.6685	269.7113	281.1138	310.3057	2152.624
20	189.1627	558.7541	1005.264	1723.987	1858.601	2225.610	3455.478
30	240.9416	953.5358	2118.012	4525.483	5039.198	6530.270	1708.990
40	253.3608	1165.445	3011.463	7731.740	8859.906	12332.43	14117.66
50	255.6026	1252.266	3562.784	10568.39	12410.64	18393.91	32134.80
60	255.9459	1282.611	3853.823	12720.61	15241.43	23830.43	53319.30
70	255.9932	1292.163	3992.379	14195.07	17279.59	28235.77	75371.73
80	255.9992	1294.954	4053.670	15134.35	18644.27	31556.25	96546.37
90	255.9999	1295.726	4079.335	15701.01	19509.64	33926.94	115732.2
100	256	1295.931	4089.633	16028.72	20035.71	35549.68	132369.4

Table G6: Simulation table for oil discovery for model 6, as shown in table 4.3

t	R=11	R=13	R=15	R=17.09	R=19	R=21	R=23
0	864.6	864.6	864.6	864.6	864.6	864.6	864.6
10	552.0329	627.6734	689.8456	743.7522	785.3106	822.5989	854.7670
20	1774.971	2200.967	2581.680	2933.835	3219.182	3485.409	3722.788
30	4330.681	5861.587	7350.136	8820.742	10074.68	11292.61	12416.25
40	7233.272	10607.33	14162.84	17907.20	21263.33	24655.31	27892.45
50	9750.053	15324.63	21648.43	28719.03	35364.26	42341.73	49222.32
60	11615.42	19338.52	28696.41	39751.92	50613.75	62439.02	74471.02
70	12862.38	22420.90	34688.21	49921.31	65514.05	83076.96	101485.8
80	13637.08	24621.58	39425.31	58640.00	79058.09	102794.9	128381.1
90	14092.77	26109.47	42972.45	65720.62	90720.91	120643.0	153750.6
100	14349.68	27073.81	45518.86	71233.93	100350.4	136134.4	176698.0

Table G7: Simulation table for gas production for model 6, as shown in table 4.3

t	R=17	R=19	R=21	R=23.18	R=25	R=27	R=29
0	1599	1599	1599	1599	1599	1599	1599
10	1149.444	1219.178	1278.768	1334.508	1375.065	1414.474	1449.367
20	3146.030	3475.501	3768.756	4052.776	4265.296	4476.524	4667.413
30	8885.236	10226.10	11468.61	12714.24	13672.61	14646.85	15545.48
40	17816.79	21352.79	24764.63	28306.41	31109.35	34024.79	36770.63
50	28447.47	35417.11	42409.63	49918.85	56027.55	62526.08	68773.52
60	39276.74	50644.97	62481.20	75612.10	86581.42	98508.36	110205.9
70	49234.10	65532.50	83103.15	103207.4	120434.2	139563.2	158690.7
80	57747.62	79068.99	102811.2	130779.7	155330.9	183148.6	211487.1
90	64641.67	90727.35	120653.1	156879.1	189412.4	226989.8	265960.9
100	69993.81	100354.2	136140.7	180568.4	221330.0	269277.8	319859.2

Table G8: Simulation table for gas discovery for model 6, as shown in table 4.3

t	R=9	R=11	R=13	R=15.69	R=17	R=19	R=21
0	474.7	474.7	474.7	474.7	474.7	474.7	474.7
10	327.7912	571.9908	850.4285	710.5209	1473.990	1808.144	2152.624
20	398.5115	153.4039	287.5144	28.38984	1641.123	2501.189	3455.478
30	2176.441	2892.598	3315.633	4736.966	3144.631	2571.368	1708.990
40	3882.909	6211.237	8524.034	12503.16	12232.75	13419.77	14117.66
50	5086.978	9108.993	13820.98	21559.52	23635.79	28139.73	32134.80
60	5808.076	11244.24	18336.91	30412.57	35443.62	44474.99	53319.30
70	6197.330	12659.07	21789.83	38169.16	46343.51	60581.33	75371.73
80	6392.801	13530.18	24239.90	44471.73	55654.19	75253.64	96546.37
90	6485.852	14038.29	25885.74	49317.24	63171.89	87877.22	115732.2
100	6528.36	14322.60	26945.86	52887.71	68986.92	98276.86	132369.4

Table G9: Simulation table for oil production for model 7, as shown in table 4.4

t	R=2	R=4	R=6	R=8.11	R=10	R=12	R=14
0	461.4	461.4	461.4	461.4	461.4	461.4	461.4
10	23.48368	-392.453	-7725.89	-44379	-134660	-338280	-717682
20	31.48438	245.4352	-10790	-106483	-412407	-1223622	-2915919
30	31.98842	816.3486	-3559.89	-97774.3	-504641	-1796008	-4844550
40	31.99982	984.1872	2777.152	-55263.9	-418568	-1821152	-5593920
50	32	1017.665	5945.078	-15963.4	-269854	-1494812	-5278458
60	32	1023.105	7180.813	9499.875	-134191	-1055363	-4362017
70	32	1023.884	7597.931	23302.58	-36488.6	-649092	-3261974
80	32	1023.986	7725.88	30001.42	25162.92	-332792	-2234585
90	32	1023.998	7762.537	33014.29	60841.51	-110700	-1392598
100	32	1024	7772.515	34295.7	80254.08	34652.71	-757546

Table G10: Simulation table for oil discovery for model 7, as shown in table 4.4

t	R=5	R=7	R=9	R=11	R=13	R=15	R=17
0	1514	1514	1514	1514	1514	1514	1514
10	514.7994	657.5304	-7365.78	-46897.5	-143412	-356538	-749129
20	791.5633	1614.554	-20037.9	-166233	-547500	-1503323	-3408192
30	966.2635	4330.605	-12596.2	-208878	-804499	-2510948	-6235531
40	1013.291	6307.93-	3222.367	-166484	-799792	-2886103	-7902517
50	1022.327	7247.332	16553.37	-89661.8	-627871	-2689444	-8180840
60	1023.766	7605.804	24825.59	-16251.8	-404309	-2171909	-7427220
70	1023.97	7725.392	29177.99	39060.49	-199784	-1559445	-6125722
80	1023.996	7761.815	31239.25	75547.04	-41313.7	-990473	-4668544
90	1024	7772.201	32146.24	97608.98	69664.32	-525424	-3300839
100	1024	7775.019	32524.17	110146.4	142178.2	-175218	-2140947

Table G11: Simulation table for gas production for model 7, as shown in table 4.4

t	R=4	R=6	R=8	R=10.44	R=12	R=16	R=18
0	2614	2614	2614	2614	2614	2614	2614
10	605.0929	865.2935	-7050.63	-45059.3	-142934	-355999	-748540
20	798.975	1653.796	-19947.6	-176724	-547292	-1503059	-3407877
30	966.8719	4338.016	-12570.3	-235946	-804409	-2510819	-6235362
40	1013.341	6309.329	3229.779	-199509	-799752	-2886040	-7902427
50	1022.331	7247.597	16555.49	-116296	-627854	-2689414	-8180791
60	1023.767	7605.854	24826.2	-29704	-404301	-2171894	-7427194
70	1023.97	7725.401	29178.16	39894.09	-199781	-1559437	-6125709
80	1023.996	7761.817	31239.3	88528.87	-41312.3	-990469	-4668536
90	1024	7772.201	32146.25	119582.7	69664.93	-525422	-3300835
100	1024	7775.019	32524.18	138186	142178.5	-175217	-2140945

Table G12: Simulation table for gas discovery for model 7, as shown in table 4.4

t	R=7	R=9	R=11	R=13.57	R=15	R=17
0	4730	4730	4730	4730	4730	4730
10	-6730.78	-75460.6	-334686	-993929	-2340371	-4744767
20	-10495	-210542	-1219793	-4241312	-1.1E+07	-2.4E+07
30	-3470.48	-226806	-1792703	-7401860	-2.2E+07	-5.1E+07
40	2802.947	-160622	-1818716	-8988795	-2.9E+07	-7.6E+07
50	5952.113	-81232.7	-1493195	-8937718	-3.3E+07	-9.2E+07
60	7182.641	-20462.3	-1054364	-7808248	-3.3E+07	-9.8E+07
70	7598.388	17573.77	-648506	-6208941	-2.9E+07	-9.7E+07
80	7725.99	38696.6	-332461	-4572929	-2.5E+07	-8.9E+07
90	7762.563	49518.53	-110519	-3134660	-2E+07	-7.8E+07
100	7772.521	54748.25	34749.88	-1981130	-1.5E+07	-6.6E+07

APPENDIX H

MATLAB Program Code for the Sensitivity Analysis

```
clear
clc
Vo1=5.464;
Vo2=309.2;
Vo3=148.3;
Vo4=57.45;
%R1=[5 10 15 19.01 25 30 35]';
%R2=[70 75 80 84.73 90 95 100]';
%R3=[25 30 35 38.35 45 50 55]';
%R4=[20 25 30 32.24 40 45 50]';
R1=35;
R2=100;
R3=55;
R4=50;
t=[0:2:100]';
format short
Vt_po=100000
Vt_do=100000
Vt_pg=100000
Vt_dg=100000
n=51
for i=1:n

Vt_po(i)=(R1.^3).*(1-exp(-t(i)/R1))+(Vo1+((R1.^2).*t(i))-
R1.*((t(i).^2)/2)).*exp(-t(i)/R1);
Vt_do(i)=(R2.^3).*(1-exp(-t(i)/R2))+(Vo1+((R2.^2).*t(i))-
R2.*((t(i).^2)/2)).*exp(-t(i)/R2);
Vt_pg(i)=(R3.^3).*(1-exp(-t(i)/R3))+(Vo1+((R3.^2).*t(i))-
R3.*((t(i).^2)/2)).*exp(-t(i)/R3);
Vt_dg(i)=(R4.^3).*(1-exp(-t(i)/R4))+(Vo1+((R4.^2).*t(i))-
R4.*((t(i).^2)/2)).*exp(-t(i)/R4);

    %end
end
%time=[0 2000 4000 6000 8000 10000 12000 14000 16000]';
filename = 't5abcdata.xlsx';
V1=Vt_po';
V2=Vt_do';
V3=Vt_pg';
V4=Vt_dg';
table=[t,V1,V2,V3,V4]
xlRange='A2'
sheet=7
xlswrite(filename,table,sheet,xlRange)
```