

**DETERMINATION OF CRITICAL DESIGN PARAMETERS OF
CLASSICAL RECTANGULAR PLATES UNDER UNIFORMLY
DISTRIBUTED LATERAL LOAD**

BY

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CERTIFICATION

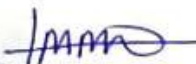
This is to certify that this work, "determination of critical design parameters of classical rectangular plates under uniformly distributed lateral load" was done by ILESANMI ADEWALE (Reg No. 20164998298), in partial fulfillment of the requirements for the award of the degree of Masters of Engineering (M. Eng) in Civil Engineering (Structural Engineering), Federal University of Technology, Owerri.


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DEDICATION

This thesis is dedicated to my parents, Mr. Ilesanmi Oshuntoye & Mrs. Ilesanmi Henrietta, my kin, Mrs. Oyerinde Oluwabukola & family, Mr. Ilesanmi Oluwasegun & family, and also Nnabue Abigail, Engr. E. I. Ndunaka & family, Nnadozie Uchenna & family, Obolo Samuel & family, Koboye Ebikebunah, Innocent Uzoemeka Sky, Ononiwu Justise, WAILE Innovations team.

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ABSTRACT

This study investigated the critical design parameters of classical rectangular plates under uniformly distributed lateral load. The study used Kirchhoff's hypotheses on total strain energy, work energy principle, kinematics, stress deflection relationship, and constitutive relationship to derive the third-order total potential energy functional for an isotropic rectangular thin plate with small deflection. External work was substituted into the third-order total potential energy functional, and the general equation of a classical rectangular plate under pure bending was obtained. The plate general equation was minimized with respect to deflection to obtain the equilibrium of forces governing equation of thin rectangular plates. The resistant forces were solved with Split-deflection approach and the solution gave the general polynomial deflection equation. Satisfying the boundary conditions of the plate with respect to the general polynomial deflection equation gave the peculiar deflection equation. The general polynomial deflection equation was simplified and substituted into the plate governing equation to obtain the amplitude of the deflection function. A close integral was performed on the shape function for various boundary conditions with respect to the general stiffness equation, which gave the peculiar stiffness and non-dimensional deflection coefficients of a plate with varying aspect ratios ranging from $(0.1 \leq \alpha \leq 2.0)$ with 0.1 intervals. The plate boundary conditions considered in this work are SSSS, CCCC, CSCS, CCSS, CSSS and CCCS (where S stands for simply supported edge and C stands for clamped edge). Limit state conditions, such as ultimate limit state of stress and serviceability limit state of deflection were satisfied and the critical design parameters for thickness (t_c) and lateral imposed load (q_{ic}) were obtained. Numerical examples were performed with the critical design equations, and results were presented for critical design thickness suitable to withstand a given set of loads and critical design-imposed loads a given thickness can withstand for several boundary conditions such as SSSS, CCCC, CSCS, CCSS, CSSS, and CCCS. The aspect ratios considered for these examples are 1, 1.5, and 2. The percentage difference results from the center deflection for various boundary conditions with varying aspect ratios of the present study were around equivalent with respect to those obtained from Ibearugbulem et al. (2014), and the results are within the acceptable limit in engineering. With the presented critical design parameters (q_{icD} , t_{cD} , q_{icE} , t_{cE}), it was concluded that this can be used in the determination of a suitable plate thickness from a specified lateral load and also the critical lateral imposed load a specified plate thickness can withstand under specified conditions of operation.

Keywords: Stiffness; Flexural Rigidity; Aspect Ratio; Pure Bending; Shape Function; Maximum Deflection; Critical Thickness; Critical Impose Load; Strain Energy; Deflection Limit State; Elastic Limit State.

CHAPTER ONE

INTRODUCTION

1.1 Background Information

Engineers are regularly concerned about the analysis of continuum structures, for example, plates, dam walls, folded-plate and shell structures, where the structural surface is continuous. Plate is one of the continuum structures generally used in buildings, bridges, automobiles, hydraulic structures, pavements, containers, airplanes, missiles, ships, instruments, machine parts, table tops, street manhole covers, side panel, roof deck, tank bottom and so forth.

A plate is a structural component limited by two parallel planes called faces, and a cylindrical surface, called an edge or boundary. The division between the plane appearances is referred to as the thickness (t) of the plate, which it is common to isolate the thickness into equivalent parts by a plane parallel to its faces. This plane is known as the center plane (or basically, the mid-plane) of the plate as appeared in Figure (1.1), where a and b are principal measurements, and t is the thickness (Yamaguchi, 1999).

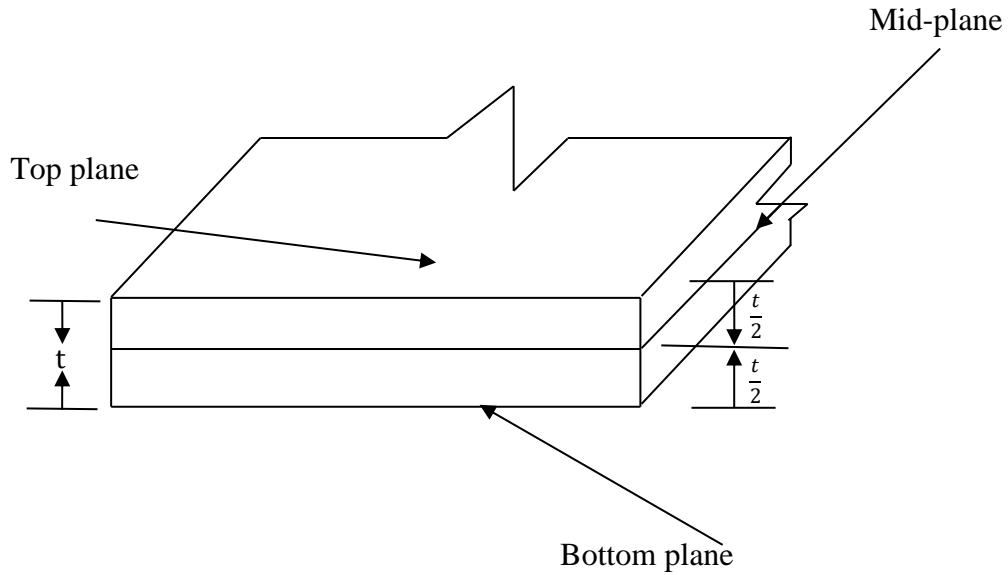


Figure 1.1a: A classical rectangular plate

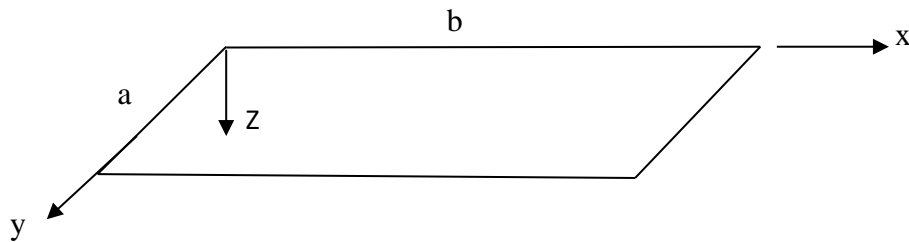


Figure 1.1b: A classical rectangular plate mid plane.

The investigations of plates are into three groups: thin plates with small deflection, thin plates with large deflections and thick plates. As indicated by the definition applied to thin plate, the proportion of the thickness (t) to the smaller span length (a) should be less than $1/20$. In this project, we shall consider only small deflections of thin plates, which is a consistent magnitude of deformation found in plate structure. It is expected, except if generally indicated, that plate materials are homogeneous and isotropic. A *homogenous* material presents identical properties all

through and when the material is the same in all directions, the material is called *isotropic* (Ventsel and Krauthammer, 2001).

A plate resembles a straight beam and it opposes the applied load by the means of bending into two directions and twisting moments. The applied forces on a plate are opposite to the plate plane. Pure bending is a state of stress where a bending moment is applied to a plate without the synchronous presence of axial, stress, or torsional force as appeared in Figure (1.2).

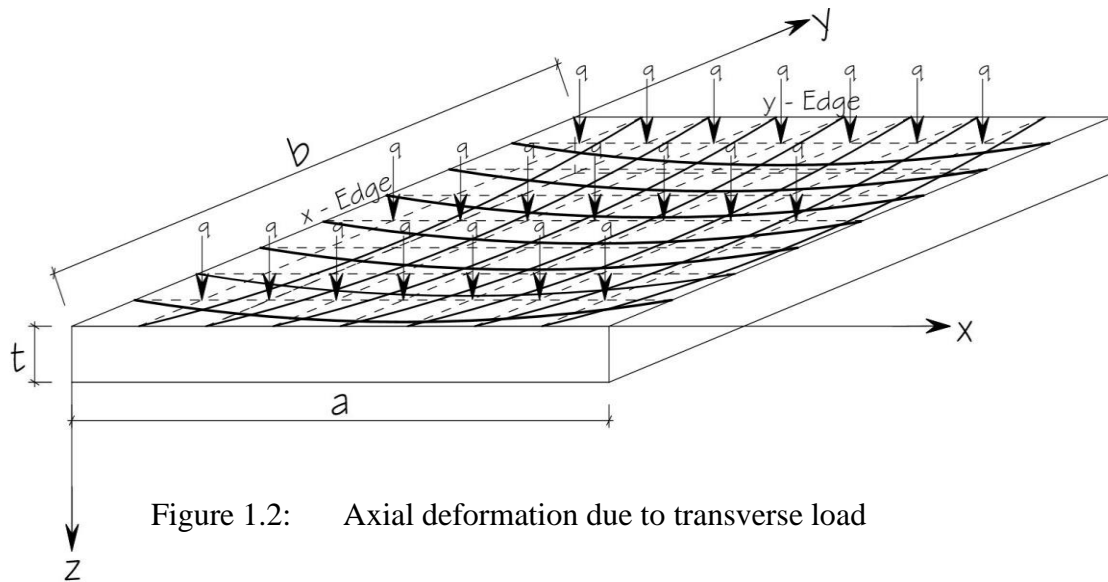


Figure 1.2: Axial deformation due to transverse load

The aim of plate hypothesis is to compute the deformation and stresses in a plate exposed to load bending. To diminish the full three-dimensional solid mechanics issue to a two-dimensional issues the Kirchoff assumptions are utilized (Mansfield, 2005).

Basically, these assumptions are used to reduce the equations of three-dimensional theory of elasticity to two dimensions (Szilard, 2004; Jawad, 2004).

Kirchoff assumptions of classical rectangular thin plates are stated below:

- i. The line normal to the neutral axis before bending remains straight after bending.
- ii. The thickness of the plate is small compared to its lateral dimensions. The normal stress in the transverse direction can be neglected compared to the normal stresses in the plane of the plate. The normal stress in thickness. i.e., $\sigma_z = 0$. This assumption converts the 3D problem into a 2D problem.
- iii. The transverse shearing strains are assumed to be zero. i.e., shear strains γ_{yz} and γ_{xz} will be zero. Thus, thickness of the plate does not change during bending.
- iv. The plate material is linear elastic and follows Hooke's law.
- v. The plate material is homogeneous and isotropic. Its elastic deformation is characterized by Young's modulus E and Poisson's ratio μ .

These assumptions expressed above are known as the Love-Kirchhoff theories and they enable us to diminish the elasticity equations to one differential equation, portraying the plate-bending problem. Classical plates are thin plates that obey Kirchhoff-Love assumptions and plane stress conditions (Ventsel and Krauthammer, 2001).

We are concerned about the connections of external forces or moment to strain, stress and deflections. External lateral forces acting on a plate might be considered as *surface forces* and *body forces*. A surface force is of the concentrated type when it acts at a point and it might likewise be distributed subjectively over a limited territory, while body forces follow up on volumetric elements of the plate.

To analyze stress and displacement for a plate subjected to external lateral load requires consideration of basic conditions i.e., physical laws, material properties, geometry and surface forces.

Three approaches are employed to analyze the stability of thin plates which are: equilibrium (Euler) approach, energy (approximate) approach and numerical approach. Energy approach will be adopted in this study.

Utilization of trigonometric function in the assumed shape functions ruled research in classical plate analysis prior to the year 2012 when Ibearugbulem evolved polynomial shape function (Ibearugbulem, 2012). Afterward, Ibearugbulem et al. (2014) affirmed that the expected polynomial shape function is exact shape function by integration of the governing differential equation of the classical plate. In this present work, the polynomial shape function will be adopted.

In structural design, design constraints are frequently referred to as limit state, which means limit states are conditions of breaking point failure. Failure being characterized as any state that makes the structure condition never again satisfies its plan criteria, such as; fitness for use, structural integrity, durability, and so on.

All activities liable to happen during a structure design life are considered during the Limit state design (LSD) method and this helps to guarantee that the structure stays fit for use with proper degrees of unwavering quality.

LSD involves assessing the subjected loads on a structure, picking the sizes of members to check, and choosing the suitable plan criteria. LSD requires two head criteria to be fulfilled: the ultimate limit state (ULS) and the serviceability limit state (SLS).

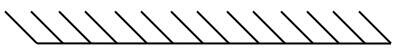
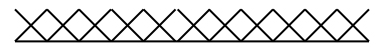
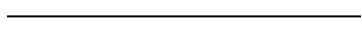
In this investigation, the serviceability limit state of deflection ($W_{\max} < W_a$) and ultimate limit state of elastic ($\bar{U} \leq U_0$) will be satisfied to obtain the suitable design parameters for classical rectangular plates under uniformly distributed lateral load.

From the literature, it has been discovered that there is no exploration by past researchers on the determination of critical design parameters of classical rectangular plates under uniformly distributed lateral load, a reason why the results presented in this study represent a novelty element brought by this research, which will be an advantage for plate designs. With this research a solution to critical load which a known plate thickness can withstand and also the critical thickness of a plate that can withstand a specified loading, can be known under specified conditions of operation.

For the purpose of description, the following notation will be adopted for the plate problems analyzed in this study.

The rectangular plate has four edges and the numbering of the edges is shown in Figure (1.3)

Some of the boundary conditions of the edges of a rectangular plate are:

- S designates simply support 
- C designates clamped support 
- F designates free support 

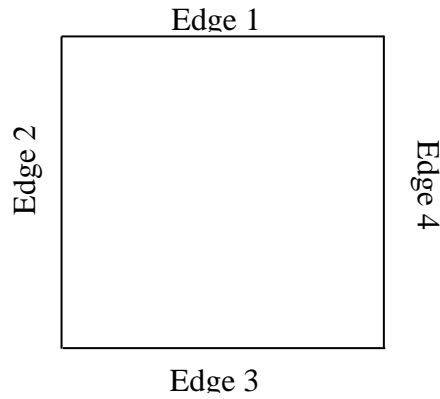
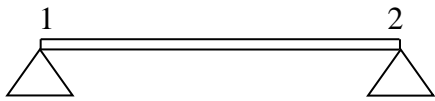
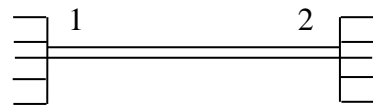


Figure 1.3: Rectangular plate with edge numbering

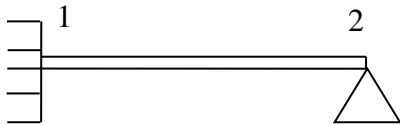
Two main boundary conditions are used namely simply supported (S) and clamped (C). Plate is formed by grids of beams and these beams are orthogonal to each other as shown in Figure (1.4), where 1 and 2 are the ends numbering. In combining them, the end conditions of beam in x-direction (horizontal) are placed first before placing the end conditions of beam in y-direction (vertical), which means the plate are labeled anticlockwise. The combinations of the three independent beams generated a total of 6 boundary conditions of plate necessary for the purpose of this research as shown in Figure (1.5).



Simply supported edge (SS) plate on both ends

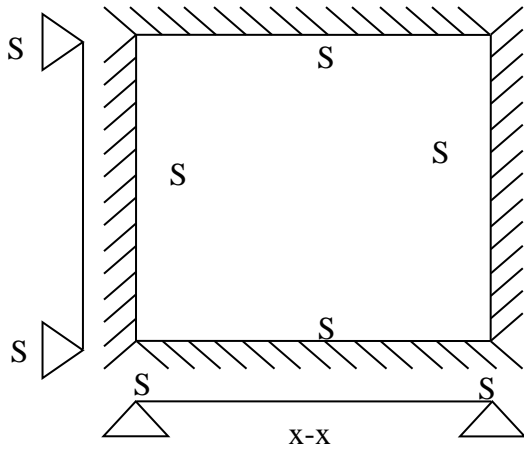


Clamped supported edge (CC) on both ends

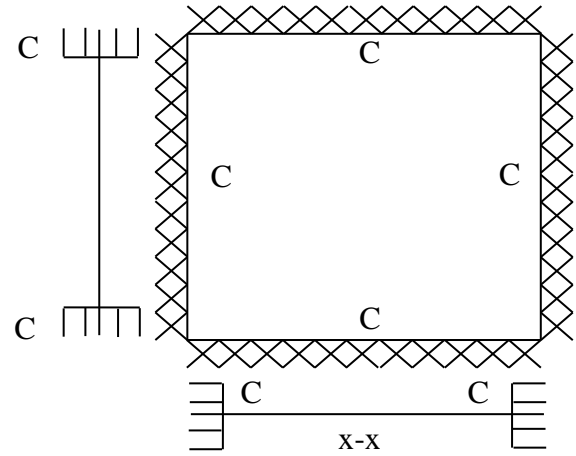


Clamped at one edge and simply supported at the other edge

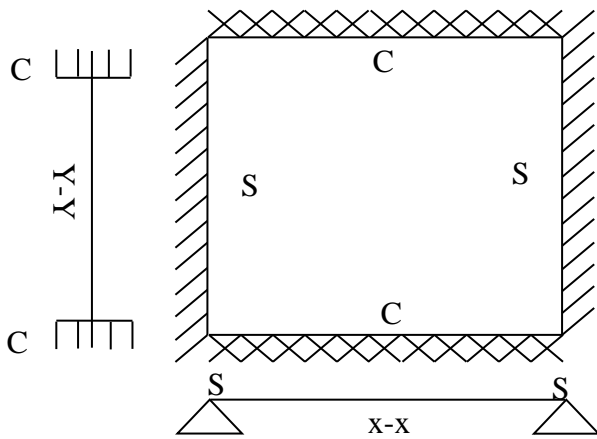
Figure 1.4: Independent Beams of different boundary conditions



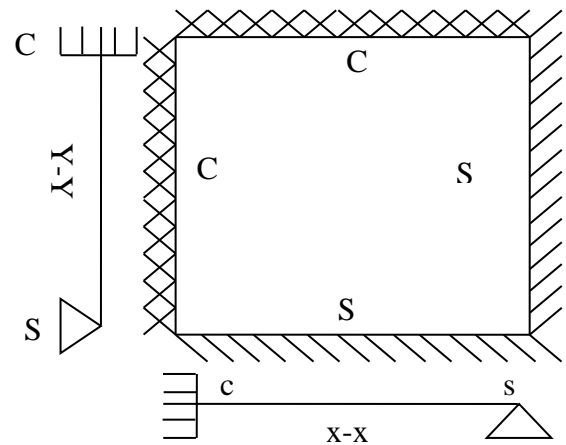
Boundary condition for SSSS plate.



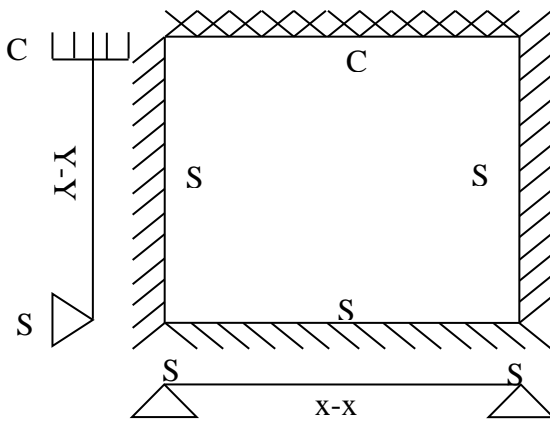
Boundary condition for CCCC plate.



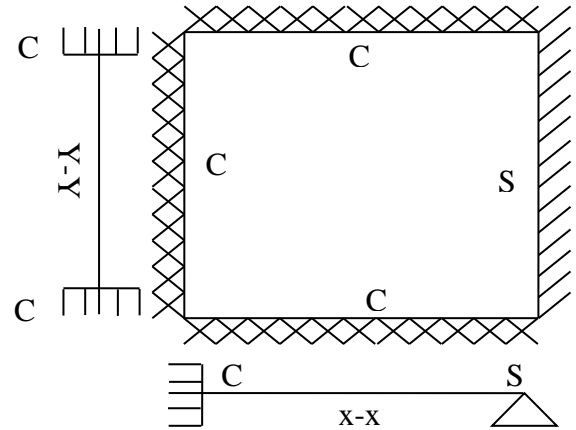
Boundary condition for CSCS plate.



Boundary condition for CCSS plate.



Boundary condition for CSSS plate.



Boundary condition for CCCS plate.

Figure 1.5: Classical rectangular plates of different edge conditions.

1.2 Problem Statement

Plate theory which is the calculation of deformation and stresses in a plate when subjected to lateral loads, are the major problem solved by many researchers over the years. Several methods have been proposed on pure bending analysis for isotropic rectangular plate using energy functional, such as second order (Ritz energy function) and fourth order (Galerkin and work error energy functional) making plate analysis easily achievable, but gaps have been left behind on the designing function of a plate, making the determination of plate sections and suitable lateral imposed load that is most efficient under specified conditions of operation a major problem.

This stated gap is worth filling, hence the need for this present research work “Determination of critical design parameters of classical rectangular plates under uniformly distributed lateral load” is an advantage, giving solutions to design function of classical rectangular plates, whereby a plate thickness can be designed easily from a specified lateral load and also, the lateral load a specified plate thickness can withstand can also be designed with the critical design parameters proposed in this study.

1.3 Aim and Objectives of Study

The aim of this study is to determine the critical design parameters of classical rectangular plates under uniformly distributed lateral load.

To achieve this aim, the following objectives are to:

- i. Formulate a third order energy functional of a classical rectangular plate under pure bending;
- ii. Determine the deflection equation of a classical rectangular plate under pure bending;

- iii. Determine the stiffness coefficient of classical rectangular plates of various boundary conditions, with varying aspect ratios;
- iv. Determine the critical (Limit) design parameters for classical rectangular plate;
- v. Evaluate the determined critical design parameters for classical rectangular plates through numerical approach.

1.4 Justification of Study

This study “determination of critical design parameters of classical rectangular plates under uniformly distributed lateral load” will be beneficial in the following ways;

- i. The study will address issues for design functions of classical rectangular plates, creating a critical (Limit) design parameters whereby a plate thickness can be designed easily from a specified lateral load and also, the lateral load a specified plate thickness can withstand can also be designed.
- ii. It will contribute to the existing literatures on classical rectangular plates under pure bending with various edge conditions.
- iii. It will create an alternative energy approach of obtaining the deflection solutions of classical rectangular plate with various boundary conditions and varying aspect ratios using third order energy functional.
- iv. It will be a useful tool for engineers and designers.

1.5 Scope of the Study

This study is strictly on the determination of critical design parameters of classical rectangular plates under uniformly distributed lateral load. The type of plate considered in this study is an

isotropic homogenous thin rectangular plate with small deflection, other type of plates like orthotropic thin rectangular plates with small deflection, non-homogenous thin plates, thin plates with large deflections and thick plates are not considered.

Third order energy functional was the method used for the pure bending analysis, considering the deflections, lateral load, thickness, yield strength, ultimate limit state of stress and serviceability limit state of deflection, other methods like Equilibrium method, Numerical methods, Ritz energy function, Galerkin and work error energy functional are not considered. The plate boundary conditions considered are CCCC, SSSS, CSCS, CCCS, SSSC and CCSS and the solution used for the plate governing differential equations was the orthogonal polynomial deflection equation, deflection equations such as the closed-form solution, solutions by double trigonometric series (Navier's approach), solutions by single trigonometric series (Levy's method) are not considered.

Only pure bending analysis is considered, other aspects of plate analysis like the vibration analysis and buckling analysis are not done in this work. On aspect ratio, only aspect ratios based on the ratio of the longer dimension to the shorter dimension of the plate $\left(\frac{b}{a}\right)$ are considered in this study and it ranges from $(0.1 \leq \alpha \leq 2.0)$ with 0.1 intervals. The reverse case of aspect ratio $\left(\frac{a}{b}\right)$ formulation was not considered in this study.

The stages involved in the course of executing the project were stated as thus. Writing audit and web search were made to know the degree past researchers had gone toward this path and this was done in chapter two under the title "literature review".

In chapter three, the third order energy functional was derived from first principle and deflection equation of classical rectangular plate under pure bending was determined. The deflection was

used to satisfy several boundary conditions of a plate, obtaining the peculiar deflection equation of the considered boundary conditions. Also, we move further in getting the stiffness of several boundary conditions of a plate before the critical parameter was solved. Satisfying the SLS of deflection and ULS of elasticity, the critical design parameters for Lateral impose load and thickness was obtained, this equation was used in solving for the critical lateral load a specified plate thickness can withstand and vice versa and all this was handled in chapter three under the title “methodology”. Results of the formulated third-order energy functional of a classical rectangular plate, peculiar deflection equation for the considered boundary conditions, stiffness coefficients for the considered boundary conditions, values of non-dimensional deflection coefficients for the considered boundary conditions, critical design parameters, were presented and numerical examples were formulated with the determine critical (Limit) design parameters of classical rectangular plate, presenting tables for suitable thicknesses and lateral imposed loads for classical rectangular plates under specified condition of operation, discussion of all these presented results was summarized, and all these were handled in chapter four under the title “results and discussions”.

Finally, conclusions, recommendation and contribution to knowledge were discussed under the title “conclusion.” in chapter five.

CHAPTER TWO

LITERATURE REVIEW

2.1 Definition of Plate

A plate is a structural element which is described into two key properties. Firstly, its geometric configuration is a three dimensional solid whose thickness is exceptionally smaller compared to other dimensions (length and width). Secondly, the impacts of the loads applied on it create stresses whose resultants are, in practical terms, exclusively normal to the element's thickness (Steele and Balch, 2009; Chennamsetti, 2012; Megson, 2007; Kelly, 2013; Shufrin et al., 2008).

Ventsel and Krauthammer (2001), regards thin plates as structural members bounded by two parallel planes, called faces, and a cylindrical surface, called an edge or boundary. The generators of the cylindrical surface are perpendicular to the plane faces. The separation between the plane appearances is known as the thickness (t) of the plate and it is expected that the plate thickness is little compared with other characteristic measurements of the faces (length, width, diameter, etc.).

The plane isolating the plate thickness similarly and parallel to its surface is alluded to as mid plane or center surface as shown in Figure (2.1) (Ugural, 1999; Ezeh, et al., 2013; Ventsel and Krauthammer, 2001; Timoshenko and Gere, 2016).

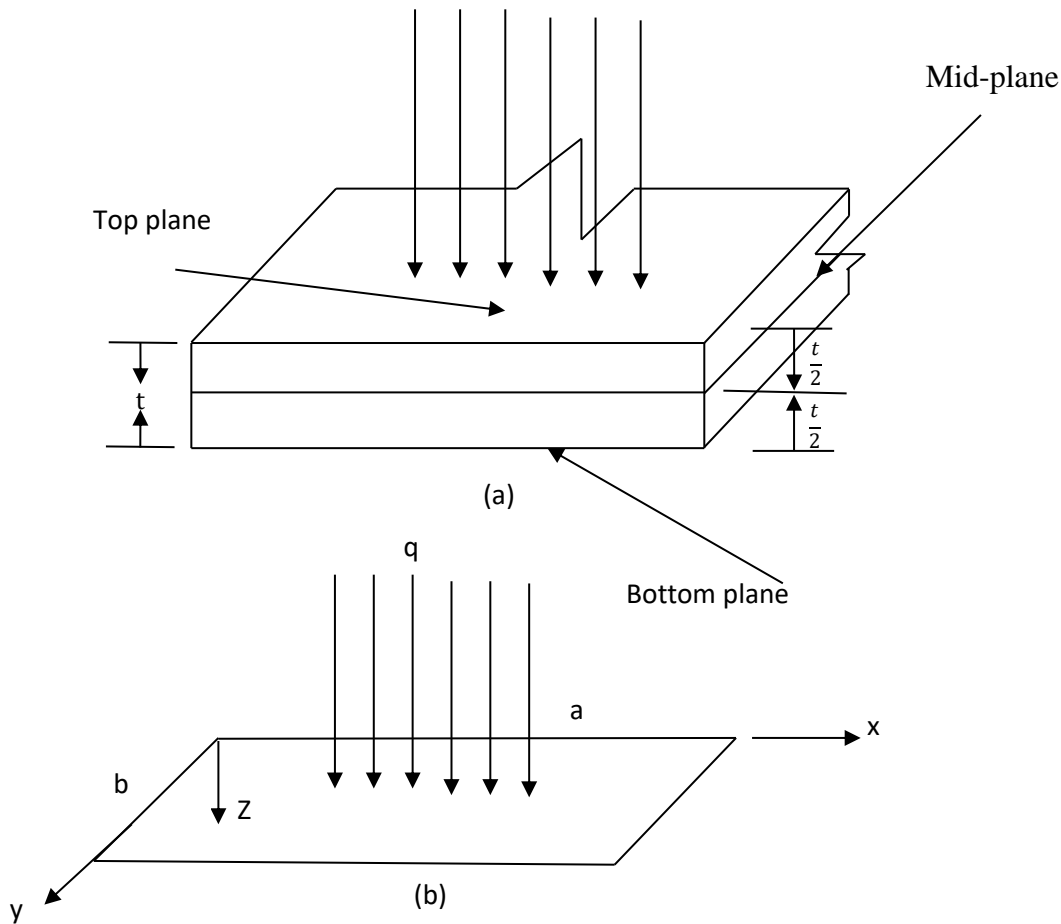


Figure 2.1: A lateral loaded classical rectangular plate.

2.2 History of Plates

The history as introduced in this research will be credited to Ventsel and Krauthammer (2001), Sathyamoorthy (1998), Elzein (1991), Szilard (2004).

Euler was the first to work on a plate problem mathematically where he introduced a free vibration analysis of plate problems in 1776. Chladni, a physicist from Germany discovered the different methods of free vibrations in an investigation on flat plates, where he conveyed powder equitably to shape a standard example after acceptance of the free vibration analysis in 1776. Bernoulli endeavored to legitimize hypothetically, on the consequences of these acoustic analyses and his answers depended on the past work bringing about the Euler Bernoulli's bending beam hypothesis, he likewise displayed a plate as an arrangement of commonly opposite strips at right angle to each

other, where each strip is viewed as a beam, yet the governing differential equation as unmistakable from current methodologies didn't contain the middle term.

Germain a French mathematician created differential condition of a plate that came up short on the warping term, Lagrange later looked into the work and amended the outcomes from Germain by including the missing term, making him the first to exhibit the general equation of a plate appropriately.

Cauchy and Poisson were first to detail the issue of plate bending with the guideline of hypothesis of elasticity general conditions, where they got the governing differential equation for deflections that matches totally with the known Germain–Lagrange general equation of a plate.

Poisson in 1829, extended the Germain–Lagrange plate condition to the arrangement of a plate under static loading, where the flexural rigidity (D) of the plate was set equivalent to a consistent term, he additionally recommended three boundary conditions for any point on a free boundary. The boundary conditions inferred by Poisson and an inquiry regarding the number and nature of these conditions had been the subject of a lot of discussions and were the subject of further examinations.

According to Ventsel and Krauthammer (2001), Elzein (1991), Navier was the first to derive a satisfactory equation on plate bending, where he considered the flexural rigidity defined in terms of a single elasticity constant and furthermore following Poisson's prior investigation, a requirement of three natural boundary conditions. He additionally presented an "accurate" strategy which changed the differential condition into logarithmic articulations by utilization of Fourier trigonometric arrangement.

Kirchhoff in 1850, chipped away at a significant proposition on the hypothesis of thin plates where he expressed the suspicions generally acknowledged in the plate bending hypothesis, derived the potential fundamental plate energy expression of theory and applied the virtual work strategy to get the differential equation for plates with the flexural rigidity characterized as far as both Young's modulus and Poisson ratio. Moreover, he understood that the three natural boundary conditions proposed by Poisson are incompatible with the fourth order nature of the differential equation and contracted them into two boundary conditions (Ventsel and Krauthammer, 2001; Elzein, 1991).

Kirchhoff and Clebsh examined the large deflection theory of plates and Foppl in 1907 was the first to utilize the idea of stress function so as to rearrange the initially confused conditions researched by Kirchhoff and Clebsh on large deflection theory of plates (Elzein, 1991).

Timoshenko and Woinowsky-Krieger (1970) made a critical commitment to the hypothesis and utilization of plate bending analysis, among Timoshenko's various significant commitments are solutions of circular plates considering large deflections and the formulation of elastic stability problems. Timoshenko and Woinowsky-Krieger (1970) published a fundamental monograph that represented a profound analysis of various plate bending problems.

Timoshenko and Woinowsky-Krieger (1970) stated that Nadai made broad hypothetical and trial examinations related with a check of the precision of Kirchhoff's plate theory, where he treated various kinds of singularities in plates because of a concentrated force application, point support effects, and so forth.

The general conditions for large deflections of thin plates were simplified by Foppl who utilized the stress function acting at the center plane of a plate, the final form of the differential equation of the large-deflection hypothesis was created by von Karman and he likewise examined the postbuckling behavior of plates. Timoshenko and Woinowsky-Krieger (1970) stated that Hencky

(1947) made a commitment to the hypothesis of large deformations and the general theory of elastic stability of thin plates.

Timoshenko and Woinowsky-Krieger (1970) stated that Huber built up an inexact hypothesis of orthotropic plates and solved plates exposed to nonsymmetrical distributed loads and edge moments.

Lagrange acquired the governing differential equation for small deflection for thin plate bending analysis based on Kirchhoff's assumptions and it is stated as (Ventsel and Krauthammer, 2001):

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{P}{D} \quad 2.1$$

Hence, the Navier's equation as presented by Kirchhoff and the Von Karman equations are still considered to be efficient and accurate representations of the small and large deflection theories of plates, respectively (Ventsel and Krauthammer, 2001).

2.3 Previous works on thin plates

Damanpack et al, (2012), investigated the boundary element method applied to the bending analysis of thin functionally graded plates where they assumed the material properties are graded through the thickness direction of the plate according to a power law distribution. The neutral surface position for such plate is determined and the classical plate theory based on the exact neutral surface position is employed to extract the equilibrium equations. Similar investigations were done to assess the adequacy of the proposed technique for bending analysis of isotropic and functionally graded plates subjected to the transverse loads. Then, an arrangement parametric investigation was performed to analyze the impacts of the intensity of functionally graded material, boundary conditions and geometrical parameters on the deformation and stress of functionally graded plates.

Study of dynamic analysis of isotropic SSSS plate using Taylor series shape function in Galerkin's functional was carried out by Ezeh, et al, (2013), where the exceptional shape function for SSSS plate in vibration was acquired and it was utilized on Galerkin's functional to determine the fundamental natural frequency equation for SSSS plate. The dimensionless frequency parameter from the investigation was compared with those from past research works for aspect ratios ranging from 0.1 to 1.0 and it demonstrates that an average percentage difference of 0.0604% exist between the solutions from present examination and those from from past research works which gave a conclusion that the Taylor series formulated shape function used to formulate the peculiar shape function for SSSS plate is a close approximation of the definite shape function and the Galerkin's functional as derived is a good variational method.

Okafor and Udeh (2015) contemplated direct technique for investigation of an isotropic rectangular plate utilizing orthogonal polynomials and direct variation principle (in view of Ritz method) and it was used to acquire the total potential energy of the plate by employing the static elastic theory of plate, shape function of fourth order for the plate under uniformly distributed load for various boundary conditions was formulated using the characteristic orthogonal polynomials (COP). Analyses of results obtained was compared with exact results presented in the monograph of Timoshenko and Woinowsky – Krieger, given a conclusion that the characteristic orthogonal polynomial based derived shape functions for rectangular plates are acceptable in approximating the deformed shape of thin rectangular plates of various boundary conditions and direct variation principle based on Ritz method can be utilized in certainty to satisfactorily analyze real time rectangular thin plates of various boundary conditions under lateral loadings.

Ibearugbulem, et al. (2016a) studied split-deflection method of classical rectangular plate analysis, the deflection was split in two components which are x and y and polynomial deflection and

trigonometric deflection were respectively used for the x and y components of the deflection for SSSS plate. The deflection components were substituted into this direct governing equation and the coefficient of the deflection was obtained together with the maximum central deflection of the plate. The studied maximum central deflection was compared with previous scholars studies such as Navier's and Levi's. It was observed that the value from the present study makes lower bound and upper bound differences of 0.37% and 2.08% with the values from Navier and Levi respectively which shows that this present method is reliable, Ibearugbulem, et al, (2016b) still moved forward to present a study to buckling analysis of rectangular plate by the use of split-deflection method where the study formulated the total potential energy functional from theory of elasticity principles, using this assumption and by direct variation, the energy functional was minimized and equation for critical buckling load was obtained and compared with the values from previous study giving a conclusion that the present method is sufficient and reliable for classical plate theory (cpt) buckling analysis of rectangular plates.

Studies on buckling analysis of isotropic rectangular plate with one free edge and other edges simply supported (SSFS) were carried out by Ibearugbulem, et at, (2014a). The study was completed through a theoretical formula based on polynomial series and the application of Ritz method and the polynomial series was truncated at the fifth term, which satisfied all the boundary conditions of the plate that resulted to a particular shape function for SSFS plate. The shape function was substituted into the total potential energy functional, which was subsequently minimized and the critical buckling load was obtained. The critical buckling load from Timoshenko and that from present study were compared with the aspect ratios ranging from 0.1 to 2.0 and the graph of critical buckling load against aspect ratio was plotted. It was discovered that for aspect ratios of 0.5 and 1.0, the percentage differences between the critical buckling load of the

present study, that of Tomoshenko and Ibearugbulem were -2.9576%, -5.4079% and -8.7594%, -13.672% respectively. The actual buckling load for 0.5 and 1.0 aspect ratios of the present study were $42.15 D^2/b^2$ and $12.97D^2/b^2$ respectively. The behavior of the graph shows that as aspect ratio increases from 0.1 to 2.0, the critical buckling load decreases.

Ezeh, et al, (2013) studied pure bending analysis of thin rectangular flat plates using ordinary finite difference method, in the study; a numerical analysis using Ordinary Finite Difference Method (OFDM) on pure bending of thin rectangular flat plates was implemented. The analysis was accomplished through a theoretical transformation of the partial differential equations for the plate into finite difference form to fit the chosen grid pattern. The finite difference forms were evaluated at the grid points in order to obtain a set of simultaneous linear equations. After using the boundary conditions, SSSS and CCCC respectively, the unknown functional values in form of deflections were determined. Visual Basic (VB) software was developed to simplify the determination of the deflections. The resulting maximum deflections W_{max} , were compared with exact solutions from previous studies within the range of aspect ratios of 1.0 to 2.0. Hence, Ordinary finite difference method (OFDM) provides a very simple and approximate solution for thin rectangular flat plates. Emmanuel, et al, (2018) studied pure bending analysis of isotropic thin rectangular plates using third-order energy functional. The Rayleigh Ritz energy method of direct variation approach for plate analysis is adopted using the principles of theory of elasticity to derive the third order energy functional method, giving an alternative energy method for thin plate analysis under pure bending. The central deflection of the various plate with aspect ratio (ranging from 1.0 to 2.0 at the increment of 0.1) from the studies were compared with the values of Szilard (2004) and Timoshenko and Krieger (1959).

Shooshtari and Razavi (2015) studied nonlinear free and forced vibration of a transversely isotropic rectangular magneto-electro-elastic (MEE) thin plate with simply supported boundary based on the thin plate theory along with the Von Karman's nonlinear strains. The partial differential equation method is transformed to an ordinary differential equation using Galerkin method. Nosier and Ruhi (2006) presented a semi-analytical solution for analysis of laminated cylindrical panels with piezoelectric layers. Mehidtabar et al. (2013) studied the magneto-thermo-elastic problem of a functionally graded truncated conical shell. Some analytical or numerical studies have been carried out to study the MEE structures which include studies on the static deformation (Pan, 2001; Pan 2005), free vibration (Anniger, 2007; Dong, 2008; Chang, 2013; Lang, 2013), and on the linear dynamic response (Tsai, 2008; Wu, 2009; Wu, 2010; Milazzo, 2012), but in the nonlinear field, only a few studies on the non-linear static response of MEE plates are available. Xue et al. (2011) studied the large deflection of a rectangular MEE plate based on the Von Karman plate theory.

Application of a direct variational principle in elastic stability analysis of thin rectangular flat plates was studied by Ibearugbulem (2012), where he used the energy approach in the form of direct variational principle (Rayleigh-Ritz method) for buckling analysis of thin rectangular plates with various boundary conditions using Taylor series shape functions. The buckling loads from the study were compared with those of earlier researches and the results showed that the average percentage differences recorded for SSSS, CCCC, CSCS, CSSS, and SSFS plates are 0.069%, 3.54%, 3.071%, 6.25% and 4.14% respectively. Oguaghamba (2015) also studied on analysis of elastic buckling and post buckling strength of isotropic rectangular plate, where he determined the exact particular empirical solution equation for twelve different boundary conditions of elastic

buckling and post buckling strength for a thin rectangular plate under in-plane loading on one direction using minimized energy functional

Osadebe and Aginam (2011) studied bending analysis of isotropic rectangular plate with all edges clamped where they developed Ritz mathematical and variational method and applied it in the analysis of uniformly loaded clamped isotropic rectangular plate, making a conclusion that the developed model can conveniently be applied to thin rectangular plate of variable thickness.

Adah, et al, (2016) studied development of polynomial based program for pure bending analysis of SSSS rectangular thin isotropic plate where the study was aimed at developing a polynomial based program for analysis of SSSS rectangular plate using polynomial shape functions derived for SSSS plate. Total potential energy functional equation derived based on Ritz energy equation was used to obtain expression for the coefficient of amplitude deflection which was used to compute other parameters. The program is written based on the derived expressions using Matlab programming language in which provision is made for input by user. The values of coefficients of deflection, moments and shear forces were obtained from running the program, and were compared with those of earlier studies.

Oguaghamba et al, (2014) studied yield stress of CCSS thin rectangular plate in postbuckling load regimes, where the exact displacement and stress profile of the plate was obtained by applying the direct integration theory to the Kirchhoff's linear governing differential equation and von Karman's non linear governing capability equation respectively.

Shuohui, et al, (2013), studied Free Vibration analyses of FGM (functionally graded materials) thin plates by IGA (isogeometric analysis) based on classical plate theory and physical neutral surface and the objective of the work was to provide an efficient and accurate numerical simulation approach for the non-homogeneous thin plates and shells. Higher order basis functions can be

easily obtained in IGA, thus the formulation of CPT (classical plate theory) based on the IGA can be simplified. For the FGM thin plates, material property gradient in the thickness direction is unsymmetrical about the mid-plane, so effects of mid-plane displacements cannot be ignored, whereas the CPT neglects mid-plane displacements. To eliminate the effects of mid-plane displacements without introducing new unknown variables, the physical neutral surface is introduced into the CPT. The approximation of the deflection field and the geometric description are performed by using the NURBS (nonuniform rational B-spline) basis functions. Compared with the first-order shear deformation theory, the present method has lower memory consumption and higher efficiency. Several numerical results show that the present method yields highly accurate solutions.

2.4 Type of plates and Theories

A plate opposes transverse loads by methods of bending, the flexural properties of a plate depend extraordinarily upon its thickness in examination with different measurements which makes the investigation of plate characterized into three groups, dependent on length (width) to thickness proportion $\frac{a}{t}$ or $\frac{b}{t}$ whichever is the smaller, where 'a' and 'b' are the width and breadth of a plate in plane respectively while 't' is the plate thickness (Ventsel and Krauthammer, 2001; Chakraverty, 2009; Reddy, 2004 and Ugural, 1999; Timoshenko and Woinowsky-Krieger, 1970).

According to Ventsel and Krauthammer (2001) the three groups are expressed below:

The first group is referred to as thick plates having ratios $\frac{a}{t} \leq 8 \dots 10$. The investigation of such plates incorporates all the components of stresses, strains and displacements, as the case of solid bodies, utilizing the general equations of three-dimensional elasticity.

The second group represents plates with ratios $\frac{a}{t} \geq 80 \dots 100$ and this class of plates is alluded to as membranes and they are without flexural rigidity. Membranes carry the horizontal loads by axial tensile forces N and central shear forces acting in the plate middle surface. These forces are called membrane forces; they produce projection on a vertical axis and this balance a lateral load applied to the plate-membrane. Their moment resistance is of negligible order.

The third group is the most extensive group which represents an intermediate type of plate, so-called thin plate with small deflection $8 \dots 10 \leq \frac{a}{t} \leq 80 \dots 100$. Depending on the value of the ratio $\frac{w}{t}$, the ratio of the maximum deflection of the plate to its thickness, the part of flexural and membrane forces here might be extraordinary.

- i. Stiff Plates: A thin plate is classified as stiff plate, if $\frac{w}{t} \leq 0.2$. Stiff plates are flexurally rigid thin plates. They convey loads two dimensionally, generally by internal bending and twisting moments, and by transverse shear forces. As for these plates, as indicated to Kirchhoff's Theory, the middle plane deformation and the membrane forces are negligible. Generally speaking, the term "plate" is understood to mean a stiff plate, unless otherwise specified.
- ii. Flexible plates: when the plate deflections are beyond a certain level, $\frac{w}{t} > 0.3$, then, the lateral deflections will be accompanied by stretching of the middle surface. Such thin plates are referred to as flexible plates. These plates represent a combination of stiff plates and membranes, and carry external loads by the combined action of internal moments, shear forces, and membrane (axial) forces. When the magnitude of the maximum deflection is

considerably greater than the plate thickness, the membrane action predominates. So, if $\frac{w}{t} > 5$, the flexural stress can be neglected compared with the membrane stress.

2.4.1 Theory of Thin Plates with Small Deflections

When plates are exposed to applied forces, say pure bending forces which are usually perpendicular to the plane of the plate, they oppose the forces by means of bending in two directions. Therefore, deformations and stresses are induced or developed in the plate; the thickness of plate governs the bending induced by the applied load. This is because the flexural properties of a plate depend extraordinarily upon its thickness compared with other dimensions. Plate hypothesis are aimed at determining the deformations and stresses created in the plate due to applied loads (Timoshenko and Woinowsky-Krieger, 1959; Reddy, 2007; Szilard, 2004; Ventsel and Krauthammer, 2001).

An acceptable inexact hypothesis of bending of the plate by lateral loads can be created by Kirchhoff's hypotheses, which are essential in the advancement of linear, elastic, small-deflection theory for the bending of thin plates. These speculations re-expressed here are from Chennamsetti (2012), Ventsel and Krauthammer (2001), Timoshenko and Woinowsky-Krieger (1970), Ugural (1999), Kelly (2013), Kharde, et al, (2013), Johnson 2010; Megson 2007:

- a) The material of the plate is elastic, homogenous, and isotropic.
- b) The plate is initially flat.
- c) The deflection (the normal component of the displacement vector) of the mid-plane is small compared with the thickness of the plate. The slope of the deflected surface is therefore very small and the square of the slope is a negligible quantity in comparison with unity.

- d) The straight lines, initially normal to the middle plane before bending, remain straight and normal to the middle surface during the deformation, and the length of such elements is not altered. This means that the vertical shear strains γ_{xy} and γ_{yz} are negligible and the normal strain ϵ_z may also be omitted. This assumption is referred to as the “hypothesis of straight normals.”
- e) The stress normal to the middle plane, σ_z , is small compared with the other stress components and may be neglected in the stress-strain relations.
- f) Since the displacements of the plate are small, it is assumed that the middle surface remains unstrained after bending.

These points are the foundation for plate bending theory that is usually referred to as the classical or Kirchhoff's plate theory.

A homogenous material displays identical properties throughout and isotropic material is when the properties are same in all direction, according to Ugural (1999).

According to Szilard (2004), since plates have both isotropic and orthotropic mechanical properties and furthermore made out of layered materials, there are varieties of plate hypotheses applied for their examination. These plate speculations can be classed by their stress-strain relationship as linear elastic plate theories and non linear elastic, plastic and viscoelastic theories. Linear elastic theories are based on assumptions of Hooke's law where stress – strain relationship is linear while others are based on complex stress – strain relationship. Special case of viscoelastic plate theory which is concerned with dynamic conditions only, the other theories are further broken down into

statics and dynamics of plates, depending on whether the applied load is static or dynamic in nature (Chia, 1980; Shufrin et al., 2009; Ventsel and Krauthammer, 2001; Hoa et al., 2011).

2.5 Boundary Conditions

As indicated by Szilard (2004), Jawad (2004), Birman (2011), Connor and Faraji (2012), Ibearugbulem (2012), Mansfield (2005), Ugural (1999), Statically plates have free, simply supported and fixed boundary conditions, including elastic supports and elastic restraints, or, in some cases, even point supports.

Boundary conditions are known conditions on the surface of the plate which must be defined and satisfied in order to obtain the solution of the governing differential equation for the deflection of thin plate bending analysis. The conditions that define the shape or geometry of the continuum at the boundaries are referred to as Essential boundary conditions – deflection and rotation are examples of this. The boundary conditions that define the mechanical behavior of the continuum are referred to as non-essential boundary conditions, and they include bending moment and shear force (Szilard, 2004; Mansfield, 2005; Ugural, 1999).

Generally, the boundary conditions of plate bending can be classified as follows:

- i) Geometrical Boundary conditions (At fixed Edges): At this condition, the deflection and the slope of the deflected plate surface are zero and this is generally referred to as geometrical.
- ii) Statical boundary condition (At Free-Edges): The edge moment and the transverse shear force (v) are zero.
- iii) Mixed Boundary condition (At simply supported edges): The deflection and the bending moment are zero.

Regardless of whether one uses indirect or direct method for the solution, it is important that the proper boundary conditions are considered in the analysis. Considering the plate edges at $x = 0$ and $x = a$, boundary conditions for the two other edges at $y = 0$ and $y = b$ can be obtained by simply replacing x with y and a with b . The boundary conditions with their mathematical expressions are summarized in the Table (2.1).

Table 2.1: Summary of boundary conditions

Type of support at $x = a$	Mathematical Expressions
Simple support	$(w)_{x=a} = 0; (M_x)_{x=a} = \left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right)_{x=a} = 0$
Fixed edge	$(w)_{x=a} = 0; \left(\frac{\partial w}{\partial x} \right)_{x=a} = 0$
Free-edge	$(M_x)_{x=a} = \left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right)_{x=a} = 0$ $(V_x)_{x=a} = \left[\frac{\partial^3 w}{\partial x^3} + (2 - \mu) \frac{\partial^3 w}{\partial x \partial y^2} \right]_{x=a} = 0$
Partially fixed edge	$(w)_{x=a} = 0$ $\left(\frac{\partial w}{\partial x} \right)_{x=a} = -(p <)^{-1} D \left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right)_{x=a}$
Elastic support	$(M_x)_{x=a} = \left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right)_{x=a} = 0$ $(w)_{x=a} p^{-1} D \left[\frac{\partial^2 w}{\partial x^2} + (2 - \mu) \frac{\partial^3 w}{\partial x \partial y^2} \right]_{x=a}$
Elastic support and restraint	$(w)_{x=a} p^{-1} D \left[\frac{\partial^3 w}{\partial x^3} + (2 - \mu) \frac{\partial^3 w}{\partial x \partial y^2} \right]_{x=a}$ $\left(\frac{\partial w}{\partial x} \right)_{x=a} = -(p <)^{-1} D \left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right)_{x=a}$

2.6 Solutions of Governing Differential Equations

Numerically, the differential equation of plates is classified as a linear partial differential equation of the fourth order having constant coefficients, and it's called the *biharmonic equation* (Birman, 2011; Szilard, 2004; Jawad, 2004; Mansfield, 2005; Ugural, 1999).

Its homogeneous form is stated as:

$$\nabla^2 \nabla^2 w = 0 \quad 2.2$$

2.6.1 Closed-Form Solution

According to Szilard (2004), Closed-form solutions of plate problems are rare. The general solution of the governing differential equation of the plate is usually obtained as the sum of the solution of the homogeneous Equation (2.2) and a particular solution. While obtaining a particular solution is relatively easy, finding suitable functions for the biharmonic equation presents considerable difficulties. For certain combinations of boundary and loading conditions, the rigorous solution has not yet been found.

Thus, the most general form of the rigorous solution of the governing differential equation can be written as:

$$w(x, y) = w_H(x, y) + w_p(x, y) \quad 2.3$$

Where W_H represents the solution of the homogeneous equation and W_P is a particular solution of the non-homogeneous differential equation of the plate.

2.6.2 Solutions by Double Trigonometric Series (Navier's Approach)

As indicated by Birman (2011), Szilard (2004), Jawad (2004), Mansfield (2005), Ugural (1999), Navier's answer is now and then called the *forced* solution of the differential equations since it “forcibly” changes the differential equation into an arithmetical condition, in this way impressively encouraging the necessary numerical activities.

Onah et al, (2018) and Johnarry (2011) utilized the Double Trigonometric Series (Navier's Approach) to get arrangement of the differential equation on their examinations.

The deflections are expressed by a double sine series

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad 2.4$$

For simply supported rectangular plates, Navier's answer offers extensive numerical focal points, since the solution of the governing fourth-order partial differential equation is reduced to solution of an algebraic equation.

The system of use of Navier's technique is outlined as follows:

- i) Expand the lateral load into double Fourier (sine) series
- ii) Express the deflections also in double sine series.
- iii) Find the deflections
- iv) The substitution of the deflections, w , into the expressions for internal moments, shears and edge forces gives the required quantities.
- v) Carry out all operations for a specific (m, n) th component; the final results are obtained by adding the terms.

The convergence of the series is usually fast in the case of distributed loads. The convergence, however, becomes slow for concentrated and discontinuous loads.

2.6.3 Solutions by Single Trigonometric Series (Levy's Method)

According to Birman (2011), Szilard (2004), Jawad (2004), Mansfield (2005), Ugural (1999), Levy's technique utilizes a single trigonometric series, is more general than Navier's solution, the previous doesn't have an altogether broad character either since in its unique structure it tends to be applied, just if:

- i. The two opposite edges of the plate are simply supported (in the solution presented the simple supports were assumed to be at $x = 0$ and $x = a$) and

- ii. The shape of the loading function is the same for all sections parallel to the direction of the other two edges (parallel to the X axis).

Levy recommended the answer to be expressed in terms of homogeneous and specific parts each of which comprises of a single Fourier series where the unknown function is determined from the boundary conditions. The solution is expressed as:

$$w = w_h + w_p \quad 2.5$$

The homogeneous solution is written as

$$w_h = \sum_{m=1}^{\infty} f_m(y) \sin \frac{m\pi x}{a} \quad 2.6$$

Where $f_m(y)$ indicates function of 'y' only. This equation also satisfies a simply supported boundary condition at $x = 0$ and $x = a$.

It ought to be noted, however, that the convergence of Levy's solution is incredibly quick, even in the case of concentrated or line loads.

For some plate issues the assumption $b \rightarrow \infty$ tends to be too limiting in the derivation of particular solutions for the Levy method, since the class of loads that can be accommodated is considerably restricted. To conquer this restriction, we may utilize the deflection functions of simply supported plates (obtained by Navier's method) for a particular solution.

Thus, in the case of identical boundary conditions at $y = \pm \frac{b}{2}$, Levy's solution of the differential equation of the plate can be represented by

$$w(x, y) = \sum_{m=1}^{\infty} W_m \sin \frac{m\pi x}{a} + \sum_{m=1}^{\infty} \left(A_m \cosh \frac{m\pi y}{a} + B_m \frac{m\pi y}{a} + \sinh \frac{m\pi y}{a} \right) \sin \frac{m\pi x}{a} \quad 2.7$$

Miltic and Miletic (2016) used the Single Trigonometric Series (Levy's Method) solution of differential equation on the study of the buckling analysis of a rectangular plate elastically clamped along the longitudinal edges.

2.6.4 Orthogonal Polynomial Shape Functions.

Orthogonal polynomial shape function offered a simpler and progressively precise methodology for thin plate analysis, and Split-deflection approach can be used to obtain exact general orthogonal polynomial deflection equation of a rectangular plate (Ibearugbulem, 2016a). By and large, the utilization of polynomial displacement functions offers the accompanying favorable circumstances:

- i. The differentiation and integration of polynomial displacement functions are quite easy and accurate.
- ii. Polynomial displacement functions as shape functions can be easily applied to approximate solution problems of linear and nonlinear cases.
- iii. Polynomial displacement functions can be used successfully to solve thick rectangular plate of any boundary condition.

Thusly, the use of polynomial functions has now turned out to be famous in approximating plate issues. Bhat (1985) used characteristic orthogonal polynomials to evaluate the natural frequencies of rectangular plates. Also, Ibearugbulem et al. (2014) employed Taylor-Maclaurin series to formulate deflection functions for plate problems as:

$$w = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4). (b_0 + b_1Q + b_2Q^2 + b_3Q^3 + b_4Q^4) \quad 2.8$$

When,

W is deflection functions for plate problems

R is non dimensional variables along x-axis

Q is non dimensional variables along y-axis

a and b are deflection coefficients called constants

Ibearugbulem (2016c), Baffah (2007), Ezeh (2013), Ibearugbulem (2016b), Ibearugbulem (2014a), Ibearugbulem (2013), Oguaghamba (2015) used the Polynomial Shape Functions to get solution of the differential equation on their studies.

2.7 Pure Bending Analysis of thin plates

Pure bending of plates can be studied, in general perspective, using the equilibrium approach or the energy approach (Iyengar, 1988). However, three approaches were identified in structural mechanics. They are equilibrium, energy and numeric approaches (Reddy, 1984). Equilibrium approach was also regarded as Euler approach. It sums all the forces acting on a continuum to zero. This summation was referred to the governing equation. It could either be ordinary differential equation or partial differential equation (Ugural, 1999).

Energy approach, on the other hand, sums all the work (strain energy and potential energy or external work) on the continuum to be equal to total potential energy (Iyengar, 1988). The numeric methodology, contingent upon a specific strategy, will model the governing equation or the energy equation to approximate the solution of plate.

2.7.1 Equilibrium Method (Euler)

This methodology will in general discover solution of the governing differential equation by direct integration and satisfying the boundary conditions of the four edges of the plate. The integration

of this type of differential equation would yield a number of constant terms. Determination of the true values of these constant terms would prompt the finishing of the solution of the differential equation (Szilard, 1974; Vinson, 1974; Mansfield, 1964 and Donnell 1976). The boundary conditions of the continuum would help in the assurance of the constants. This methodology of examination of thin plates in elastic stability is not simple. For straightforward cases (like a rectangular plate simply supported all round, and loaded with in-plane load only along one axis) it could be employed to find definite solution of thin plates in elastic stability. Notwithstanding, for most functional cases in real time situations, this methodology could be mind boggling and some of the time incomprehensible (Reddy, 2002).

2.7.2 Energy Methods

This approach uses the total potential energy of a system to determine the unknown force or displacement at a specified point along the continuum. These energy methods yield approximate solution and the closeness between the approximate deflection function and exact deflection function determines the accuracy of these methods. Hence, there is the need for variational principle; which employs total potential functional. Szilard (2004),

ADVANTAGES OF ENERGY METHODS

Szilard (2004) stated the advantages of energy method as follows:

- i) Energy techniques are simpler, both conceptually and mathematically.
- ii) They are incredibly amazing to acquire reusable analytical solution even for plates of arbitrary shape and boundary conditions.
- iii) They give an important planning to understand the standards of finite element methods.

2.7.3 Numerical Method

Numerical methodology is a decent option in contrast to the Euler approach. A few instances of this approach include truncated double Fourier series, finite difference, finite strip, Runge- Kutta and finite element methods among others. Techniques for numerical methodologies have the limit of taking care of plates of different boundary conditions. It has been appeared from past works that in most cases, the solution from numerical approach approximate closely to the exact approach (Ventsel and Krauthammer, 2001).

Szilard (2004) expressed that the numerical techniques are the so-called *discrete* methods. That is, the continuum of the plate or its boundary is discretized either mathematically or physically. The numerical methods for the solution of various plate problems are:

- i) Finite difference methods (FDMs)
- ii) Grid work method (GWM)
- iii) Finite element method (FEM)
- iv) Finite strip method (FSM)
- v) Boundary element method (BEM)

Of these five strategies the FDM and BEM depend on mathematical discretization of the plate continuum or its boundary, respectively, while the others utilize different kinds of physical discretization procedures.

According to Ventsel and Krauthammer (2001), approximate method has been conventionally grouped into two distinct groups – indirect and direct methods.

2.7.4 Indirect Methods

According to Ventsel and Krauthammer (2001), these techniques empower us to get numerical estimations of unknown functions by, essentially, direct discretization of the governing differential

equation of the corresponding boundary value problem. They are also computer-based approaches. Finite difference method, boundary element method, Galerkin method, and method of boundary collocations are considered under this method.

2.7.5 Direct Method

Direct technique is a powerful analytical tool and it is referred to also as variational method. It manages the supposed direct techniques of calculus of variation – regularly called energy method (Szilard, 2004).

2.7.5.1 Variational Principles

Variational principles are terms, assumptions and statements that formulate properties of a functional and helps in determining numerical field of unknown function. A functional is therefore a subfunction used in determining stresses, strains, and displacement. Variational principles indicate some fundamental hypothesis concurred in a type of integral equalities connecting stresses, strains, and displacement throughout the volume of a body and based on the properties of work done by external and internal forces.

Direct methods use the variational principles for deciding numerical fields of unknown functions (deflections, internal forces, and moments), avoiding the differential equations of the plate (Ventsel and Krauthammer, 2001). Ritz method and Finite Element method are considered under this method.

2.6 Limits State Design (LSD)

According to Quimby (2008), Mosley et al, (1990), Anwar and Najam (2017), structure requirements are every now and again alluded to as limit states. Limit states are conditions of potential disappointment and failure.

Limit states take the general form of:

Request \leq Capacity

Structural limit states generally fall into two major categories which are namely ultimate (Strength) limit state and serviceability limit state.

2.6.1 Ultimate Limit States

Ultimate limit states are potential modes of structural failure. For steel members, the failure may either be yielding (permanent deformation) or rupture (actual fracture). This limit state can be written in the general form:

Required Strength \leq Nominal Strength

The required strength is the internal force derived from the analysis of the structure considered while the nominal strength is the anticipated capacity the structure is capable of supporting (a function of the stress capacity of the material and the section properties of the member)

An example of the ultimate Limit states is the stress limit state and it can be expressed as ($\bar{U} \leq U_0$), where \bar{U} is a working stress due to the internal force that is derived from the analysis of the structure and U_0 is the suitable stress and it's a function of the stress capacity of the material and the section properties of the member.

2.6.2 Serviceability Limit States

Serviceability limit states are those conditions that are not strength based but still may make the structure unsatisfactory for its planned use. The most widely recognized serviceability limit states in structural design are deflection, vibration, slenderness, and clearance. Serviceability limit states can be written in the general form:

Real Behavior \leq Allowable Behavior

A case of this is deflection and it tends to be communicated as: ($W_{\max} < W_a$) where W_{\max} is the maximum deflection (Real behavior) due to the design load and it must be kept less than the allowable deflection (allowable behavior) W_a .

Serviceability limit states tend to be less rigid requirements than strength based limit states since the safety of the structure is not in question. Serviceability limit states don't tend to put individuals' lives in danger nor do they risk property damage.

2.7 Survey of Elasticity Theory

The classical theory of plates is an important application of the theory of elasticity, which deals with relationships of forces, displacements, stresses, and strains in an elastic body. When a solid body is subjected to external forces, it deforms, producing internal strains and stresses. The deformation depends on the geometrical configuration of the body, on applied loading, and on the mechanical properties of its material. The classical theory of elasticity assumes the material is homogeneous and isotropic, i.e., its mechanical properties are the same in all directions and at all points (Ibearugbulem, 2017; Johnson, 2010; Beer et al., 2012; Ventsel and Krauthammer, 2001).

2.7.1 Stress at a Point: Stress Tensor

Consider plate subjected to external loads which are in equilibrium. At that point, consider a material point anywhere in the inside of the body. If we assign a Cartesian coordinate frame with axes x , y , and z , as shown in Figure (2.2), it is convenient to assign an infinitesimal element in the form of parallelepiped (d_x ; d_y ; d_z), with faces parallel to the coordinate planes. Stresses acting on the faces of this element describe the intensity of the internal forces at a point on a particular face. These stresses can be broken down into a normal component (normal stress) and tangent

component (shear stress) to the particular face. As a result, the three stress components denoted by σ_{xx} ; τ_{xy} ; τ_{xz} ; ...; will act on each face of the element. The subscript notation for the stress components is interpreted as follows: the first subscript indicates the direction of an outer normal to the face on which the stress component acts; the second subscript relates to the direction of the stress itself (Ibearugbulem, 2017; Johnson, 2010; Lancaster and Mitchell, 1980; Megson, 2007; Ventsel and Krauthammer, 2001; Ugural, 1999; Chennamsetti, 2012; Mansfield, 2005; Birman, 2011; Szilard, 2004; Jawad, 2004; Ezeh et al., 2018).

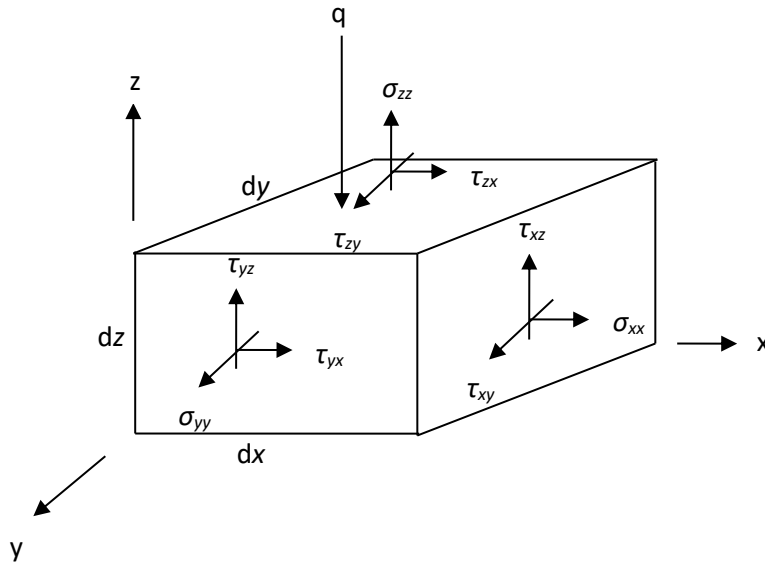


Figure 2.2: Normal stresses and Shear stresses on a loaded plate

On any one face these three stress components comprise a vector, called a surface traction. The above-mentioned set of the stress components acting on the faces of the element forms the stress tensor, T_S , i.e.,

$$T_S = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} \quad 2.9$$

Which is symmetric with respect to the principal diagonal because of the reciprocity law of shear stresses, i.e.,

$$\tau_{xy} = \tau_{yx} ; \tau_{xz} = \tau_{zx} ; \tau_{yz} = \tau_{zy} \quad 2.10$$

Thus, only the six stress components out of nine in the stress tensor T_S are independent. The stress tensor, T_S , completely characterizes the three-dimensional state of stress at a point of interest.

For elastic stress analysis of plates, the two-dimensional state of stress is of special importance. In this case,

$$\sigma_z = \tau_{yz} = \tau_{xz} = 0 \quad 2.11$$

Thus, the two-dimensional stress tensor has a form

$$T_S = \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{pmatrix} \quad 2.12$$

2.7.2 Strain Components

According to Beer et al., 2012, strain is the deformation to the original size of a material. This is:

$$\varepsilon = \frac{\Delta}{L} \quad 2.13$$

There are three planes in the cuboid and they are xy, xz and yz planes as shown in Figure (2.3). Strain can be broken down into a normal component (normal strains) and tangent component (shear strains) to the particular face (Ibearugbulem, 2017; Johnson, 2010; Lancaster and Mitchell, 1980; Megson, 2007; Ventsel and Krauthammer, 2001; Ugural, 1999; Chennamsetti, 2012; Mansfield, 2005; Birman, 2011; Szilard, 2004; Jawad, 2004; Ezeh et al., 2018)

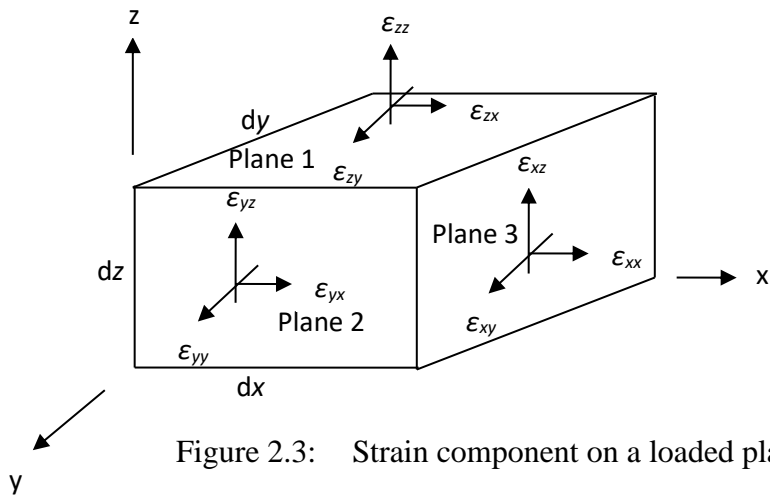


Figure 2.3: Strain component on a loaded plate.

After loading of structural material, it deforms normally by moving in the x, y and z directions. This movement is called displacement.

The displacement in x direction is u, displacement in y direction is v and the displacement in z direction is w.

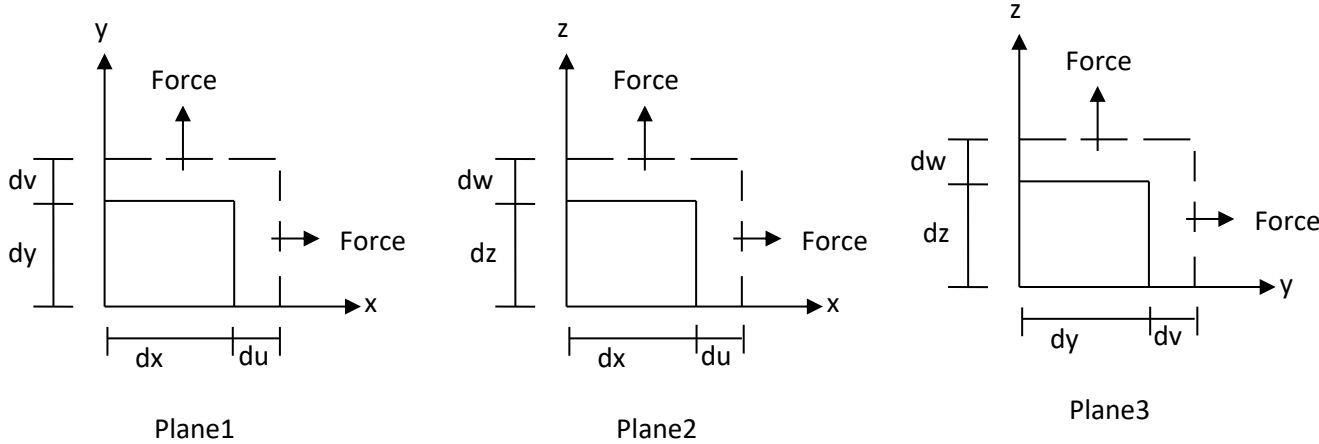


Figure 2.4: Displacement in the cuboid for the three planes; Plane 1 (xy), plane 2 (xz), plane 3 (yz).

This displacement shall make the cuboid to deform as well. The deformations of the cuboid are called elemental displacement; du, dv and dw in x, y, and z directions respectively. The normal

strain components shall be defined using Figure (2.4) and from this figure the normal strain of the plane cuboid can be written mathematically as:

$$\epsilon_{xx} = \frac{du}{dx} \quad 2.14$$

$$\epsilon_{yy} = \frac{dv}{dy} \quad 2.15$$

$$\epsilon_{zz} = \frac{dw}{dz} \quad 2.16$$

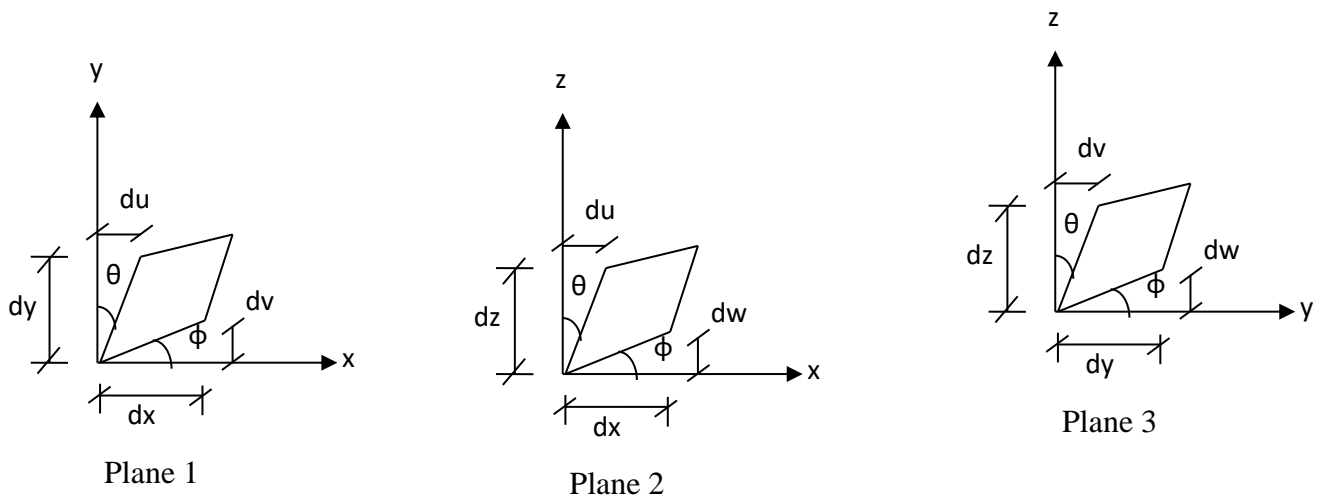


Figure 2.5: Twist in the cuboid for the three planes; Plane 1 (xy), plane 2 (xz), plane 3 (yz).

Shear strains

$$\epsilon_{xy} = \theta = \frac{dv}{dx} \quad 2.17$$

$$\epsilon_{yx} = \vartheta = \frac{du}{dy} \quad 2.18$$

$$\epsilon_{xz} = \theta = \frac{dw}{dx} \quad 2.19$$

$$\varepsilon_{zx} = \vartheta = \frac{du}{dz} \quad 2.20$$

$$\varepsilon_{yz} = \theta = \frac{dw}{dy} \quad 2.21$$

$$\varepsilon_{zy} = \vartheta = \frac{dv}{dz} \quad 2.22$$

The total angle of twist is obtained by summation of θ and ϑ together. That is:

$$\gamma_{ij} = \theta + \vartheta = \varepsilon_{ij} + \varepsilon_{ji} \quad 2.23$$

$$\gamma_{xy} = \varepsilon_{xy} + \varepsilon_{yx} = \frac{dv}{dx} + \frac{du}{dy} \quad 2.24$$

$$\gamma_{xz} = \varepsilon_{xz} + \varepsilon_{zx} = \frac{dw}{dx} + \frac{du}{dz} \quad 2.25$$

$$\gamma_{yz} = \varepsilon_{yz} + \varepsilon_{zy} = \frac{dw}{dy} + \frac{dv}{dz} \quad 2.26$$

$$\text{Note: } \gamma_{xy} = \gamma_{yx}, \gamma_{xz} = \gamma_{zx}, \gamma_{yz} = \gamma_{zy} \quad 2.27$$

2.7.3 Constitutive Equations

According to (Ibearugbulem, 2017; Lancaster and Mitchell, 1980; Chennamsetti, 2012; Mansfield, 2005; Beer et al., 2012; Birman, 2011; Jawad, 2004), the constitutive equations relate the stress components to strain components. For the linear elastic range, these equations represent the generalized Hooke's law. In the case of a three-dimensional isotropic body, the constitutive equations are given by:

$$\varepsilon_x = \frac{1}{E} \{ \sigma_x - \mu(\sigma_y + \sigma_z) \} \quad 2.28$$

$$\varepsilon_y = \frac{1}{E} \{ \sigma_y - \mu(\sigma_x + \sigma_z) \} \quad 2.29$$

$$\varepsilon_z = \frac{1}{E} \{ \sigma_z - \mu(\sigma_x + \sigma_y) \} \quad 2.30$$

$$\gamma_{xy} = \frac{1}{G} \tau_{xy} \quad 2.31$$

$$\gamma_{xz} = \frac{1}{G} \tau_{xz} \quad 2.32$$

$$\gamma_{yz} = \frac{1}{G} \tau_{yz} \quad 2.33$$

Where E, μ , G are the modulus of elasticity, Poisson's ratio, and the shear modulus, respectively.

The following relationship exists between E and G is:

$$G = \frac{E}{2(1 + \mu)} \quad 2.34$$

2.7.4 Equilibrium of Forces

According to Ibearugbulem (2017), Lancaster and Mitchell (1980), Chennamsetti (2012), Mansfield (2005), Birman (2011), Jawad (2004), the stress components introduced previously must satisfy the following differential equations of equilibrium:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + F_x = 0 \quad 2.35$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} + F_y = 0 \quad 2.36$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + F_z = 0 \quad 2.37$$

Where F_x , F_y and F_z are the body forces (e.g., gravitational, magnetic forces). In deriving these equations, the reciprocity of the shear stresses, has been used.

According to Mansfield (2005), relating Kirchhoff's theory into the stress component, strain component, constitutive relation and Equilibrium of forces equations, plate theories have been a success and many methods of plate analysis have been actualized, such as Ritz, Galerkin methods.

The Ritz method is a direct variational method in finding approximate solution for plate theory. The Ritz method has a function with "second derivative" as the highest derivative. The Ritz method of classical rectangular plate is given as:

$$\Pi = \left(\frac{D}{2} \iint \left[\left(\frac{\partial^2 w}{\partial x^2} \right)^2 + 2 \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 + \left(\frac{\partial^2 w}{\partial y^2} \right)^2 \right] - q \iint w \right) \partial x \partial y = 0 \quad 2.38$$

Galerkin method is an equilibrium method and it has a function with "four derivatives" as the highest derivative. The Galerkin method of classical rectangular plate is given as:

$$\Pi = \left(\frac{D}{2} \iint \left[\left(\frac{\partial^4 w}{\partial x^4} \right) w + 2 \left(\frac{\partial^4 w}{\partial x^2 \partial y^2} \right) w + \left(\frac{\partial^4 w}{\partial y^4} \right) w \right] - q \iint w \right) \partial x \partial y = 0 \quad 2.39$$

CHAPTER THREE

METHODOLOGY

3.1 Formulation of Third-Order Energy Functional

For a classical rectangular plate, it is expected that the thickness of the plate in the z-axis is moderately small compared with the other characteristic dimension in the x-axis and y-axis respectively. The elasticity theory and Kirchhoff's theory assumptions will be used to formulate the third-order energy functional. The sequence of the formulation includes: total potential energy, kinematic, strains-deflection relationships, stress-strain (constitutive) relationship.

3.1.1 Total Strain Energy

Strain energy is defined as average of indefinite summation of the product of stress and strain at every point in the plate within the plate domain.

From Hooke's Law,

$$P \propto \Delta \tag{3.1}$$

When,

P = Applied force (Load)

Δ = Deformation

Introducing proportional constant, Equation (3.1) becomes:

$$P = K\Delta \tag{3.2}$$

When,

K = Stiffness of the material

Recall strain and stress equation,

$$\text{Stress} = \sigma = \frac{P}{A} \quad 3.3$$

$$\text{Strain} = \varepsilon = \frac{\Delta}{L} \quad 3.4$$

When,

σ = Stress

A = Area where the force is applied

ε = Strain

L = Length of the material

Rearranging Equation (3.3) and Equation (3.4) gives:

$$P = \sigma A \quad 3.5$$

$$\Delta = \varepsilon L \quad 3.6$$

Substituting Equation (3.5) and Equation (3.6) into Equation (3.2) gives:

$$\sigma A = K \varepsilon L \quad 3.7$$

From Equation (3.7), Stress can be rewritten as:

$$\sigma = \varepsilon \left(\frac{K L}{A} \right) \quad 3.8$$

From Hooke's / Young's Law which states that: Stress is directly proportional to strain.

$$\sigma \propto \varepsilon \quad 3.9$$

Introducing the proportional constant,

$$\sigma = E \varepsilon \quad 3.10$$

When,

E = Young modulus of elasticity

From Equation (3.8) “ E ” can be said to be:

$$E = \frac{K L}{A} \quad 3.11$$

Recall,

From work energy principle, work is the result of force and distance travelled which can be mathematically represented as:

$$V = P \Delta \quad 3.12$$

When,

$$V = \text{Work}$$

Substitution of Equation (3.5) and Equation (3.6) into Equation (3.12) gives:

$$V = (\sigma A)(\varepsilon L) \quad 3.13$$

$$V = (\sigma \varepsilon)(A L) \quad 3.14$$

When,

$$(A L) = \text{Area} \cdot \text{Length} = \text{Volume}$$

Rewriting Equation (3.14):

$$V = \sigma \cdot \varepsilon(\text{Vol}) \quad 3.15$$

Note: work is done when the applied load causes the material to deform, and for this not to happen, the internal energy or strain energy of resistance which is the capacity for the material to oppose the deformation from the applied load, plays a major role.

The energy can be expressed mathematically as.

$$V_c \geq \sigma \cdot \varepsilon(\text{Vol}) \quad 3.16$$

The average strain energy can be defined as:

$$U = \frac{1}{2} V_c \quad 3.17$$

For a finite element isolated from a material in equilibrium, the volume will be:

$$\partial_{\text{vol}} = \partial_x \partial_y \partial_z \quad 3.18$$

The average strain energy of a finite element material in equilibrium can be expressed as:

$$\partial_u = \frac{1}{2} \sigma \cdot \varepsilon(\partial_{\text{vol}}) \quad 3.19$$

To get the total strain energy of the material, the strain energies of the entire finite element that make up the full material will be added, and this summation must be indefinite summation called integration.

That is:

$$\int \partial_u = \frac{1}{2} \sigma \varepsilon \left(\int \partial_{\text{vol}} \right) \quad 3.20$$

Integrating Equation (3.20) gives:

$$U = \frac{1}{2} \sigma \varepsilon \left(\int \partial_{\text{vol}} \right) \quad 3.21$$

Rewriting Equation (3.21) gives:

$$U = \iiint \frac{1}{2} \sigma \varepsilon \partial_x \partial_y \partial_z \quad 3.22$$

Equation (3.22) is called the Total Strain energy of a plate.

3.1.2 Kinematics

The displacement of a thin rectangular plate includes in-plane displacements u and v and out of plane displacement (deflection) w . The vertical normal strain of a plate is equal to zero.

Subsequently, upon these the vertical shear strains are negligible in classical plate analysis and assumed to be equal to zero. The six engineering strain components of a plate are defined in terms of displacements as:

$$\varepsilon_x = \frac{\partial u}{\partial x} \quad 3.23$$

$$\varepsilon_y = \frac{\partial v}{\partial y} \quad 3.24$$

$$\varepsilon_z = \frac{\partial w}{\partial z} = 0 \quad 3.25$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \quad 3.26$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = 0 \quad 3.27$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 0 \quad 3.28$$

3.1.3 Stress-Deflection Relationships

This is done to get the connections, which exist between the in-plane stresses and the out of plane displacement.

Solving Equation (3.27) gives:

$$\partial v = -\partial z \frac{\partial w}{\partial y} \quad 3.29$$

$$v = -z \frac{\partial w}{\partial y} \quad 3.30$$

Similar solution for Equation (3.28),

$$\partial u = -\partial z \frac{\partial w}{\partial x} \quad 3.31$$

$$u = -z \frac{\partial w}{\partial x} \quad 3.32$$

Substituting Equation (3.30) and Equation (3.32) into Equation (3.26) gives:

$$\gamma_{xy} = \frac{\partial}{\partial x} \left(-z \frac{\partial w}{\partial y} \right) + \frac{\partial}{\partial y} \left(-z \frac{\partial w}{\partial x} \right) \quad 3.33$$

Solving Equation (3.33) gives:

$$\gamma_{xy} = -2z \frac{\partial^2 w}{\partial y \partial x} \quad 3.34$$

Substituting Equation (3.32) into Equation (3.23) gives:

$$\begin{aligned} \epsilon_x &= \frac{\partial}{\partial x} \left(-z \frac{\partial w}{\partial x} \right) \\ \epsilon_x &= -z \frac{\partial^2 w}{\partial x^2} \end{aligned} \quad 3.35$$

Substituting Equation (3.30) into Equation (3.24) gives:

$$\begin{aligned} \epsilon_y &= \frac{\partial}{\partial y} \left(-z \frac{\partial w}{\partial y} \right) \\ \epsilon_y &= -z \frac{\partial^2 w}{\partial y^2} \end{aligned} \quad 3.36$$

3.1.4 Constitutive Relations

From the classical plate assumptions, the effects of normal stress and vertical shear stresses ($\sigma_z, \tau_{xz}, \tau_{yz}$) on the gross response of the plate are negligible when compared with the effects from the in-plane stresses ($\sigma_x, \sigma_y, \tau_{xy}$). This leaves us with only three stress components: ($\sigma_x, \sigma_y, \tau_{xy}$).

The resulting constitutive equations include:

$$E\epsilon_x = \sigma_x - \mu\sigma_y - \mu\sigma_z \quad 3.37$$

$$E\epsilon_y = \sigma_y - \mu\sigma_x - \mu\sigma_z \quad 3.38$$

$$E\epsilon_{xy} = \tau_{xy} + \mu\tau_{yx} \quad 3.39$$

For plane stress analysis:

$$\sigma_z = \sigma_{zx} = \sigma_{zy} = 0 \quad 3.40$$

Solving for σ_y and σ_x from Equation (3.37) and Equation (3.38) and Equation (3.40) gives:

$$\sigma_x = \frac{E}{1 - \mu^2} (\mu \varepsilon_y + \varepsilon_x) \quad 3.41$$

$$\sigma_y = \frac{E}{1 - \mu^2} (\mu \varepsilon_x + \varepsilon_y) \quad 3.42$$

Substituting Equation (3.35) and Equation (3.36) into Equation (3.41) gives:

$$\sigma_x = \frac{-zE}{1 - \mu^2} \left(\mu \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2} \right) \quad 3.43$$

Substituting Equation (3.35) and Equation (3.36) into Equation (3.42) gives:

$$\sigma_y = \frac{-zE}{1 - \mu^2} \left(\mu \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \quad 3.44$$

Solving for τ_{xy} from Equation (3.39) gives:

$$E \varepsilon_{xy} = (1 + \mu) \tau_{yx} \quad 3.45$$

Remember:

$$\gamma_{xy} = 2 \varepsilon_{xy} \quad 3.47$$

$$\varepsilon_{xy} = \frac{\gamma_{xy}}{2} \quad 3.48$$

Substituting Equation (3.48) into Equation (3.45) gives:

$$E \frac{\gamma_{xy}}{(1 + \mu)} = 2\tau_{yx} \quad 3.49$$

Substituting Equation (3.34) into Equation (3.49) gives:

$$\frac{-zE}{(1 + \mu)} \cdot \frac{\partial^2 w}{\partial y \partial x} = \tau_{yx} \quad 3.50$$

Rewriting Equation (3.50) gives:

$$\tau_{yx} = \frac{-zE(1 - \mu)}{(1 - \mu^2)} \cdot \frac{\partial^2 w}{\partial y \partial x} \quad 3.51$$

We now have solution of all the in-plane stresses and in-plane strains, which are:

$$\sigma_x = \frac{-zE}{1 - \mu^2} \left(\mu \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2} \right) \quad 3.52$$

$$\sigma_y = \frac{-zE}{1 - \mu^2} \left(\mu \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \quad 3.53$$

$$\tau_{yx} = \frac{-zE(1 - \mu)}{(1 - \mu^2)} \cdot \frac{\partial^2 w}{\partial y \partial x} \quad 3.54$$

$$\varepsilon_x = -z \frac{\partial^2 w}{\partial x^2} \quad 3.55$$

$$\varepsilon_y = -z \frac{\partial^2 w}{\partial y^2} \quad 3.56$$

$$\gamma_{xy} = -2z \frac{\partial^2 w}{\partial y \partial x} \quad 3.57$$

Recall for Total strain energy:

$$U = \iiint \frac{1}{2} (\sigma \epsilon) (\partial_x \partial_y \partial_z)$$

For thin continuum, only in-plane stresses and strains are considered.

So Total strain energy of a plate will be,

$$U = \iiint \frac{1}{2} (\sigma_x \epsilon_x + \tau_{yx} \gamma_{xy} + \sigma_y \epsilon_y) \partial_x \partial_y \partial_z \quad 3.58$$

Substituting the in-plane stresses and strains in Equation (3.52) to Equation (3.57) into Total strain energy of a plate in Equation (3.58) gives:

$$U = \iiint \frac{1}{2} \left(\frac{-zE}{1-\mu^2} \left(\mu \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2} \right) \cdot -z \frac{\partial^2 w}{\partial x^2} + \frac{-zE(1-\mu)}{(1-\mu^2)} \cdot \frac{\partial^2 w}{\partial y \partial x} \cdot -2z \frac{\partial^2 w}{\partial y \partial x} \right. \\ \left. + \frac{-zE}{1-\mu^2} \left(\mu \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \cdot -z \frac{\partial^2 w}{\partial y^2} \right) \partial_x \partial_y \partial_z$$

$$U = \iiint \frac{1}{2} \left(\frac{-zE}{1-\mu^2} \left(\mu \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2} \right) \cdot -z \frac{\partial^2 w}{\partial x^2} + \frac{-zE(1-\mu)}{(1-\mu^2)} \cdot \frac{\partial^2 w}{\partial y \partial x} \cdot -2z \frac{\partial^2 w}{\partial y \partial x} \right. \\ \left. + \frac{-zE}{1-\mu^2} \left(\mu \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \cdot -z \frac{\partial^2 w}{\partial y^2} \right) \partial_x \partial_y \partial_z$$

$$U = \frac{1}{2} \iiint \left(\frac{z^2 E}{1-\mu^2} \right) \left(\mu \frac{\partial^3 w}{\partial x \partial y^2} \cdot \frac{\partial w}{\partial x} + \frac{\partial^3 w}{\partial x^3} \cdot \frac{\partial w}{\partial x} + 2(1-\mu) \frac{\partial^3 w}{\partial x \partial y^2} \cdot \frac{\partial w}{\partial x} + \mu \frac{\partial^3 w}{\partial x \partial y^2} \cdot \frac{\partial w}{\partial x} \right. \\ \left. + \frac{\partial^3 w}{\partial y^3} \cdot \frac{\partial w}{\partial y} \right) \partial_x \partial_y \partial_z$$

$$U = \frac{1}{2} \iiint \left(\frac{z^2 E}{1 - \mu^2} \right) \left(2\mu \frac{\partial^3 w}{\partial x \partial y^2} \cdot \frac{\partial w}{\partial x} + \frac{\partial^3 w}{\partial x^3} \cdot \frac{\partial w}{\partial x} + 2 \frac{\partial^3 w}{\partial x \partial y^2} \cdot \frac{\partial w}{\partial x} - 2\mu \frac{\partial^3 w}{\partial x \partial y^2} \cdot \frac{\partial w}{\partial x} + \frac{\partial^3 w}{\partial y^3} \frac{\partial w}{\partial y} \right) \partial_x \partial_y \partial_z$$

$$U = \frac{1}{2} \iiint \left(\frac{z^2 E}{1 - \mu^2} \right) \left(\frac{\partial^3 w}{\partial x^3} \cdot \frac{\partial w}{\partial x} + 2 \frac{\partial^3 w}{\partial x \partial y^2} \cdot \frac{\partial w}{\partial x} + \frac{\partial^3 w}{\partial y^3} \cdot \frac{\partial w}{\partial y} \right) \partial_x \partial_y \partial_z \quad 3.59$$

Considering the mid-plane of the plate where the thickness is divided into two equal halves,

$$U = \frac{1}{2} \iint \left(\frac{E}{1 - \mu^2} \right) \left(\frac{\partial^3 w}{\partial x^3} \cdot \frac{\partial w}{\partial x} + 2 \frac{\partial^3 w}{\partial x \partial y^2} \cdot \frac{\partial w}{\partial x} + \frac{\partial^3 w}{\partial y^3} \cdot \frac{\partial w}{\partial y} \right) \partial_x \partial_y \left(\int_{-\frac{t}{2}}^{\frac{t}{2}} z^2 \right) \partial_z \quad 3.60$$

Solving a close integral from Equation (3.60) in respect to “z” gives:

$$\int_{-\frac{t}{2}}^{\frac{t}{2}} z^2 \partial_z = \frac{1}{3} \left(\frac{t^3}{2} - \left(-\frac{t^3}{2} \right) \right) \quad 3.61$$

Simplifying Equation (3.61) gives:

$$= \left(\frac{t^3}{6} + \frac{t^3}{6} \right) = \frac{t^3}{12} \quad 3.62$$

$$\int_{-\frac{t}{2}}^{\frac{t}{2}} z^2 \partial_z = \frac{t^3}{12} \quad 3.63$$

Substituting Equation (3.63) into Equation (3.60) gives:

$$U = \frac{1}{2} \iint \frac{t^3}{12} \left(\frac{E}{1 - \mu^2} \right) \left(\frac{\partial^3 w}{\partial x^3} \cdot \frac{\partial w}{\partial x} + 2 \frac{\partial^3 w}{\partial x \partial y^2} \cdot \frac{\partial w}{\partial x} + \frac{\partial^3 w}{\partial y^3} \cdot \frac{\partial w}{\partial y} \right) \partial_x \partial_y \quad 3.64$$

When,

$$D = \frac{t^3}{12} \left(\frac{E}{1 - \mu^2} \right) \quad 3.65$$

D = Flexural rigidity of the plate

Substituting Equation (3.65) into Equation (3.64) gives:

$$U = \frac{D}{2} \iint \left(\frac{\partial^3 w}{\partial x^3} \cdot \frac{\partial w}{\partial x} + 2 \frac{\partial^3 w}{\partial x \partial y^2} \cdot \frac{\partial w}{\partial x} + \frac{\partial^3 w}{\partial y^3} \cdot \frac{\partial w}{\partial y} \right) \partial_x \partial_y \quad 3.66$$

Equation (3.66) is now the general solution of Third Order Total Strain Energy Functional of a classical plate.

$$U = \frac{D}{2} \iint \left(\frac{\partial^3 w}{\partial x^3} \cdot \frac{\partial w}{\partial x} + 2 \frac{\partial^3 w}{\partial x \partial y^2} \cdot \frac{\partial w}{\partial x} + \frac{\partial^3 w}{\partial y^3} \cdot \frac{\partial w}{\partial y} \right) \partial_x \partial_y \quad 3.67$$

The external work due to lateral load on the plate, q is given as:

$$V = - \iint q w \partial_x \partial_y \quad 3.68$$

Summation of the external work from Equation (3.68) into the total energy functional in Equation (3.67) gives the general equation of a classical rectangular plate under pure bending and this is represented as:

$$\Pi = U + V \quad 3.69$$

That is,

$$\Pi = \frac{D}{2} \iint \left(\frac{\partial^3 w}{\partial x^3} \cdot \frac{\partial w}{\partial x} + 2 \frac{\partial^3 w}{\partial x \partial y^2} \cdot \frac{\partial w}{\partial x} + \frac{\partial^3 w}{\partial y^3} \cdot \frac{\partial w}{\partial y} \right) \partial_x \partial_y - \iint q w \partial_x \partial_y \quad 3.70$$

Equation (3.70) is the plate general equation under pure bending.

Now let us define the principal in-plane coordinates (x and y) in terms of non-dimension in-plane coordinates (R and Q) as:

$$x = aR \quad 3.71$$

$$y = bQ \quad 3.72$$

$$\alpha = \frac{b}{a} \quad 3.73$$

Inserting Equation (3.71, 3.72 and 3.73) into Equation (3.70) gives:

$$\Pi = \frac{D}{2} \int_0^1 \int_0^1 \left(\frac{\partial^3 w}{\partial R^3 a^3} \cdot \frac{\partial w}{\partial aR} + 2 \frac{\partial^3 w}{\partial aR \partial Q^2 b^2} \cdot \frac{\partial w}{\partial aR} + \frac{\partial^3 w}{\partial Q^3 b^3} \frac{\partial w}{\partial bQ} \right) \partial_x \partial_y - \int_0^1 \int_0^1 q w \partial_x \partial_y \quad 3.74$$

Multiply Equation (3.74) by a^4 gives:

$$\Pi = \frac{D}{2} \int_0^1 \int_0^1 \left(\frac{\partial^3 w}{\partial R^3} \cdot \frac{\partial w}{\partial R} + \frac{2}{\alpha^2} \cdot \frac{\partial^3 w}{\partial R \partial Q^2} \cdot \frac{\partial w}{\partial R} + \frac{1}{\alpha^4} \cdot \frac{\partial^3 w}{\partial Q^3} \cdot \frac{\partial w}{\partial Q} \right) \partial_x \partial_y - \int_0^1 \int_0^1 q w a^4 \partial_x \partial_y \quad 3.75$$

Rewriting Equation (3.75):

$$\Pi = \frac{D}{2} \int_0^1 \int_0^1 \left(\frac{\partial^3 w}{\partial R^3} \cdot \frac{\partial w}{\partial R} + \frac{2}{\alpha^2} \cdot \frac{\partial^3 w}{\partial R \partial Q^2} \cdot \frac{\partial w}{\partial R} + \frac{1}{\alpha^4} \cdot \frac{\partial^3 w}{\partial Q^3} \cdot \frac{\partial w}{\partial Q} - \frac{2a^4 q w}{D} \right) \partial_x \partial_y \quad 3.76$$

Equation (3.76) is the non-dimensional plate general equation under pure bending.

3.2 Determination of Deflection Function

The plate general equation is minimized with respect to deflection to obtain the equilibrium of forces governing equation of a thin rectangular plate. The solution of the resistant forces gives the general polynomial deflection equation. Satisfying the boundary conditions of the plate with respect to the general polynomial deflection equation, gives the peculiar deflection equation.

3.2.1 Governing Equation of a Classical Rectangular Plate

The governing equation of a plate is an equilibrium equation of all forces acting on the plate and since the load and the plate made the same displacement, the total work performed by them must be in equilibrium. That is, all the forces sum are equal to zero.

Therefore, differentiating Equation (3.70) with respect to deflection (w) gives resultant force, F as zero:

Recall Equation (3.70):

$$\Pi = \frac{D}{2} \iint \left(\frac{\partial^3 w}{\partial x^3} \cdot \frac{\partial w}{\partial x} + 2 \frac{\partial^3 w}{\partial x \partial y^2} \cdot \frac{\partial w}{\partial x} + \frac{\partial^3 w}{\partial y^3} \cdot \frac{\partial w}{\partial y} \right) \partial_x \partial_y - \iint q w \partial_x \partial_y$$

Its first derivative with respect to “ w ” is:

$$F = \frac{\partial \Pi}{\partial w} = \frac{D}{2} \iint \left(\frac{2 \partial^4 w}{\partial x^4} + \frac{4 \partial^4 w}{\partial x^2 \partial y^2} + \frac{2 \partial^4 w}{\partial y^4} \right) \partial_x \partial_y - \iint q \partial_x \partial_y$$

$$\int_0^1 \int_0^1 \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} - \frac{q}{D} = F = 0 \quad 3.77$$

The resultant force is zero because the plate is static (in a rest state).

3.2.2 General Deflection Equation

Solving the governing resultant force equation gives the general deflection equation.

3.2.2.1 Exact CPT Solution of Rectangular Plate

According to Ibearugbulem (2016), Split-deflection approach shall be used to obtain exact general orthogonal polynomial deflection equation of a rectangular plate. In doing this, the following assumptions were made:

$$\text{i) } w = w_x \cdot w_y \quad 3.78$$

$$\text{ii) } q = q_x \cdot q_y \quad 3.79$$

$$\text{iii) } D = D_x \cdot D_y \quad 3.80$$

iv) w_x is constant along y direction

v) w_y is constant along x direction

$$\text{vi) } \beta_1 + \beta_2 + \beta_3 = 1 \quad 3.81$$

vii) β_1 and β_3 are expressed as:

$$\beta_1 = \frac{q_x \cdot w_y}{D_x} \cdot \frac{D}{q} \quad 3.82$$

$$\beta_3 = \frac{q_y \cdot w_x}{D_y} \cdot \frac{D}{q} \quad 3.83$$

The assumptions above will be used to solve β_2 and Equation (3.77) can be rewritten as:

$$w_y \frac{\partial^4 w_x}{\partial x^4} + 2 \frac{\partial^2 w_x}{\partial x^2} \cdot \frac{\partial^2 w_y}{\partial y^2} + w_x \frac{\partial^4 w_y}{\partial y^4} = \frac{q}{D} (\beta_1 + \beta_2 + \beta_3) \quad 3.84$$

From Equation (3.84), using one possibility of solutions, the following equations can be obtained:

$$w_y \frac{\partial^4 w_x}{\partial x^4} = \frac{q}{D} \beta_1 \quad 3.85$$

$$2 \frac{\partial^2 w_x}{\partial x^2} \cdot \frac{\partial^2 w_y}{\partial y^2} = \frac{q}{D} \beta_2 \quad 3.86$$

$$w_x \frac{\partial^4 w_y}{\partial y^4} = \frac{q}{D} \beta_3 \quad 3.87$$

Substituting Equation (3.82) into Equation (3.85) gives:

$$w_y \frac{\partial^4 w_x}{\partial x^4} = \frac{q}{D} \cdot \frac{q_x \cdot w_y}{D_x} \cdot \frac{D}{q} \quad \text{That is:}$$

$$\frac{\partial^4 w_x}{\partial x^4} = \frac{q_x}{D_x} \quad 3.88$$

Similarly, substituting Equation (3.83) into Equation (3.87) gives:

$$w_x \frac{\partial^4 w_y}{\partial y^4} = \frac{q}{D} \cdot \frac{q_y \cdot w_x}{D_y} \cdot \frac{D}{q} \quad \text{That is:}$$

$$\frac{\partial^4 w_y}{\partial y^4} = \frac{q_y}{D_y} \quad 3.89$$

Integrating Equation (3.88) four times with respect to x gives:

$$w_x = a_0 + a_1 x + \frac{a_2 x^2}{2} + \frac{a_3 x^3}{6} + \frac{q_x x^4}{24 D_x} \quad 3.90$$

Integrating Equation (3.89) four times with respect to y gives:

$$w_y = b_0 + b_1 y + \frac{b_2 y^2}{2} + \frac{b_3 y^3}{6} + \frac{q_y y^4}{24 D_y} \quad 3.91$$

Let

$$a_0 = \frac{a_0 q_x}{D_x}; a_1 = \frac{a_1 q_x}{D_x}; a_2 = \frac{a_2 q_x}{D_x}; a_3 = \frac{a_3 q_x}{D_x} \quad 3.92$$

$$b_0 = \frac{b_0 q_y}{D_y}; b_1 = \frac{b_1 q_y}{D_y}; b_2 = \frac{b_2 q_y}{D_y}; b_3 = \frac{b_3 q_y}{D_y} \quad 3.93$$

Substituting Equations (3.92) into Equations (3.90) gives:

$$w_x = \frac{q_x}{D_x} \left(a_0 + a_1 x + \frac{a_2 x^2}{2} + \frac{a_3 x^3}{6} + \frac{x^4}{24} \right) \quad 3.94$$

Substituting Equations (3.93) into Equations (3.91) gives:

$$w_y = \frac{q_y}{D_y} \left(b_0 + b_1 y + \frac{b_2 y^2}{2} + \frac{b_3 y^3}{6} + \frac{y^4}{24} \right) \quad 3.95$$

Now Equation (3.94) shall multiply equation (3.95) to obtain:

$$\begin{aligned} w &= w_x w_y \\ &= \frac{q_x}{D_x} \cdot \frac{q_y}{D_y} \left(a_0 + a_1 x + \frac{a_2 x^2}{2} + \frac{a_3 x^3}{6} + \frac{x^4}{24} \right) \cdot \left(b_0 + b_1 y + \frac{b_2 y^2}{2} + \frac{b_3 y^3}{6} + \frac{y^4}{24} \right) \end{aligned} \quad 3.96$$

That is:

$$w = \frac{q}{D} \left(a_0 + a_1 x + \frac{a_2 x^2}{2} + \frac{a_3 x^3}{6} + \frac{x^4}{24} \right) \cdot \left(b_0 + b_1 y + \frac{b_2 y^2}{2} + \frac{b_3 y^3}{6} + \frac{y^4}{24} \right) \quad 3.97$$

Integrating Equation (3.86) two times with respect x, and two times with respect y gives:

$$w = w_x w_y = \frac{\beta_2 q}{2D} \left(a_5 + a_6 x + \frac{a_7 x^2}{2} \right) \cdot \left(b_5 + b_6 y + \frac{b_7 y^2}{2} \right) \quad 3.98$$

It is clear from the foregoing that Equation (3.97) and Equation (3.98) are equal. Thus, the equation for β_2 can be obtained by equating the two equations. That is:

$$\begin{aligned} \frac{q}{D} \left(a_0 + a_1 R + \frac{a_2 x^2}{2} + \frac{a_3 x^3}{6} + \frac{x^4}{24} \right) \cdot \left(b_0 + b_1 y + \frac{b_2 y^2}{2} + \frac{b_3 y^3}{6} + \frac{y^4}{24} \right) \\ = \frac{\beta_2 q}{2D} \left(a_5 + a_6 x + \frac{a_7 x^2}{2} \right) \cdot \left(b_5 + b_6 y + \frac{b_7 y^2}{2} \right) \end{aligned} \quad 3.99$$

That is:

$$\beta_2 = \frac{2 \left(a_0 + a_1 x + \frac{a_2 x^2}{2} + \frac{a_3 x^3}{6} + \frac{x^4}{24} \right) \left(b_0 + b_1 y + \frac{b_2 y^2}{2} + \frac{b_3 y^3}{6} + \frac{y^4}{24} \right)}{\left(a_5 + a_6 x + \frac{a_7 x^2}{2} \right) \left(b_5 + b_6 y + \frac{b_7 y^2}{2} \right)} \quad 3.100$$

That is:

$$\beta_2 \cong 2 \left(a_2 + a_3 x + \frac{x^2}{2} \right) \left(b_2 + b_3 y + \frac{y^2}{2} \right) \quad 3.101$$

It is pertinent now to validate the earlier assumptions. In doing this, Equation (3.97) shall be differentiated with respect to x and y.

Differentiating the Equation four times with respect to x gives:

$$w_y \left(\frac{\partial^4 w_x}{\partial x^4} \right) = \frac{q}{D} \left(b_0 + b_1 y + \frac{b_2 y^2}{2} + \frac{b_3 y^3}{6} + \frac{y^4}{24} \right) \quad 3.102$$

Differentiating Equation (3.97) four times with respect to y gives:

$$w_x \left(\frac{\partial^4 w_y}{\partial y^4} \right) = \frac{q}{D} \left(a_0 + a_1 x + \frac{a_2 x^2}{2} + \frac{a_3 x^3}{6} + \frac{x^4}{24} \right) \quad 3.103$$

Differentiating Equation (3.97) two times with respect to x and two times with respect to y gives:

$$2 \left(\frac{\partial^2 w_x}{\partial x^2} \right) \left(\frac{\partial^2 w_y}{\partial y^2} \right) = \frac{2q}{D} \left(a_2 + a_3 x + \frac{x^2}{2} \right) \left(b_2 + b_3 y + \frac{y^2}{2} \right) \quad 3.104$$

From Equation (3.85) and Equation (3.102), the value for β_1 is obtained as:

$$\beta_1 = \left(b_0 + b_1 y + \frac{b_2 y^2}{2} + \frac{b_3 y^3}{6} + \frac{y^4}{24} \right) \quad 3.105$$

Similarly, from Equation (3.86) and Equation (3.104), the value for β_2 is obtained as:

$$\beta_2 = 2 \left(a_2 + a_3 x + \frac{x^2}{2} \right) \left(b_2 + b_3 y + \frac{y^2}{2} \right) \quad 3.106$$

Furthermore, from Equation (3.87) and Equation (3.103), the value for β_3 is obtained as:

$$\beta_3 = \left(a_0 + a_1 x + \frac{a_2 x^2}{2} + \frac{a_3 x^3}{6} + \frac{x^4}{24} \right) \quad 3.107$$

Adding Equations (3.102, 3.103 and 3.104) gives:

$$\begin{aligned} w_y \left(\frac{\partial^4 w_x}{\partial x^4} \right) + 2 \left(\frac{\partial^2 w_x}{\partial x^2} \right) \left(\frac{\partial^2 w_y}{\partial y^2} \right) + w_x \left(\frac{\partial^4 w_y}{\partial y^4} \right) \\ = \frac{q}{D} \left(b_0 + b_1 y + \frac{b_2 y^2}{2} + \frac{b_3 y^3}{6} + \frac{y^4}{24} \right) + \frac{2q}{D} \left(a_2 + a_3 x + \frac{x^2}{2} \right) \left(b_2 + b_3 y + \frac{y^2}{2} \right) \\ + \frac{q}{D} \left(a_0 + a_1 x + \frac{a_2 x^2}{2} + \frac{a_3 x^3}{6} + \frac{x^4}{24} \right) \end{aligned} \quad 3.108$$

That is:

$$\begin{aligned}
w_y \left(\frac{\partial^4 w_x}{\partial x^4} \right) + 2 \left(\frac{\partial^2 w_x}{\partial x^2} \right) \left(\frac{\partial^2 w_y}{\partial y^2} \right) + w_x \left(\frac{\partial^4 w_y}{\partial y^4} \right) \\
= \frac{q}{D} \left[\left(b_0 + b_1 y + \frac{b_2 y^2}{2} + \frac{b_3 y^3}{6} + \frac{y^4}{24} \right) + 2 \left(a_2 + a_3 x + \frac{x^2}{2} \right) \left(b_2 + b_3 y + \frac{y^2}{2} \right) \right. \\
\left. + \left(a_0 + a_1 x + \frac{a_2 x^2}{2} + \frac{a_3 x^3}{6} + \frac{x^4}{24} \right) \right] \quad 3.109
\end{aligned}$$

Substituting Equations (3.105, 3.106 and 3.107) into Equation (3.109) gives:

$$w_y \left(\frac{\partial^4 w_x}{\partial x^4} \right) + 2 \left(\frac{\partial^2 w_x}{\partial x^2} \right) \left(\frac{\partial^2 w_y}{\partial y^2} \right) + w_x \left(\frac{\partial^4 w_y}{\partial y^4} \right) = \frac{q}{D} (\beta_1 + \beta_2 + \beta_3) = \frac{q}{D} \quad 3.110a$$

Substituting Equations (3.81) into Equation (3.110a) gives:

$$w_y \left(\frac{\partial^4 w_x}{\partial x^4} \right) + 2 \left(\frac{\partial^2 w_x}{\partial x^2} \right) \left(\frac{\partial^2 w_y}{\partial y^2} \right) + w_x \left(\frac{\partial^4 w_y}{\partial y^4} \right) = \frac{q}{D} (1) = \frac{q}{D} \quad 3.110b$$

Thus, the assumptions made are valid, and the exact general orthogonal polynomial deflection equation for a rectangular plate is Equation (3.110b).

From Equation (3.97), “w” can be written as:

$$w = [1 \ x \ x^2 \ x^3 \ x^4] \begin{bmatrix} \frac{a_0 q_x}{D_x} \\ \frac{a_1 q_x}{D_x} \\ \frac{a_2 q_x}{2D_x} \\ \frac{a_3 q_x}{6D_x} \\ \frac{q_x}{24D_x} \end{bmatrix} \times [1 \ y \ y^2 \ y^3 \ y^4] \begin{bmatrix} \frac{b_0 q_y}{D_y} \\ \frac{b_1 q_y}{D_y} \\ \frac{b_2 q_y}{2D_y} \\ \frac{b_3 q_y}{6D_y} \\ \frac{q_y}{24D_y} \end{bmatrix} \quad 3.111a$$

Where:

$$a_4 = \frac{q_x}{D_x}; b_4 = \frac{q_y}{D_y} \quad 3.111b$$

Now let us define the principal in-plane coordinates (x and y) in terms of non-dimensional in-plane coordinates (R and Q).

Substituting Equation (3.71 and 3.72) into Equation (3.111) gives:

$$w = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4) \cdot (b_0 + b_1Q + b_2Q^2 + b_3Q^3 + b_4Q^4) \quad 3.112$$

Equation (3.112) can be written in short form as:

$$w = [h_R][a_i] \times [h_Q][b_i] \quad 3.113$$

That is:

$$w = A h_R h_Q \quad 3.114$$

That is:

$$w = A h \quad 3.115$$

When:

$$A = [a_i][b_i] \quad 3.116a$$

$$h = h_R h_Q \quad 3.116b$$

From Equation (3.115), $w = Ah$ can conveniently be used to represent the deflection of a plate in which A and h are the coefficient of deflection and shape function respectively.

Maximum deflection will occur at h_{max} , where mid-span slope is zero. Hence,

$$Ah_{max} = w_{max} \quad 3.116c$$

Where w_{max} is the maximum deflection of the plate.

3.2.3 Peculiar Deflection Equation

The plate general polynomial deflection equation will be used to obtain the specific (peculiar) deflection of various plates boundary condition.

A thin rectangular plate has four edges as show in Figure (1.3) and the following cases of different edge conditions are considered

S S S S

C C C C

S S C C

C S C S

S S C S

C S C C

Note:

C = Clamped supported edge.

S = Simply supported edge.

Recall,

A plate has four edges

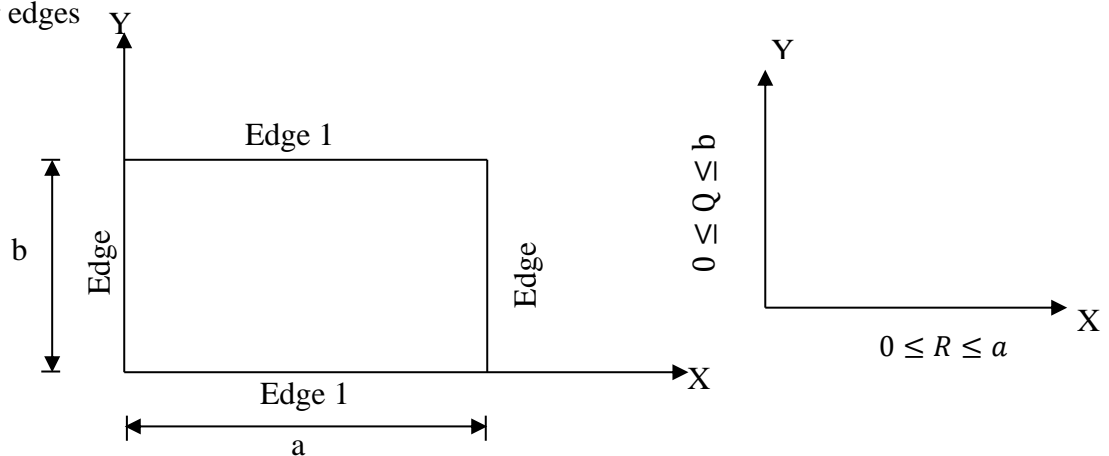


Figure 3.1: A classical rectangular plate with edge numbering and X and Y axes dimension of “a” and “b”.

3.2.3.1 Deflection Function for (SS) Edge Condition (Simply Supported Edge)

SS boundary conditions:

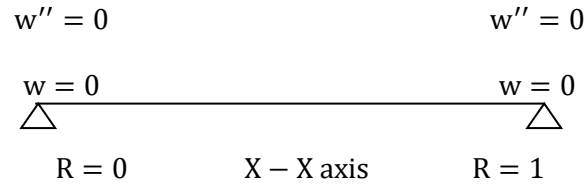


Figure 3.2: Simply supported edge (SS) plate strip along **x-axis**

General orthogonal polynomial deflection equation of a strip plate along x-axis gives:

$$w_R = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4) \quad 3.117$$

The second derivative of the deflection function of Equation (3.117) in the x-axis gives:

$$w_R'' = (0 + 0 + 2a_2 + 6a_3R + 12a_4R^2) \quad 3.118a$$

When

$$R = 0 \quad 3.118b$$

$$w_R = 0 \quad 3.118c$$

$$w_R'' = 0 \quad 3.118d$$

Substituting Equations (3.118b) and (3.118c) into Equation (3.117) gives:

$$w_R = 0 = a_0 + 0 + 0 + 0 + 0 \Rightarrow a_0 = 0 \quad 3.119$$

Substituting Equations (3.118b) and (3.118d) into Equation (3.118a) gives:

$$w_R'' = 0 = 2a_2 \Rightarrow 2a_2 = 0 \quad 3.120a$$

When

$$R = 1 \quad 3.120b$$

$$w_R = 0 \quad 3.120c$$

$$w_R'' = 0 \quad 3.120d$$

Substituting Equations (3.120b) and (3.120c) into Equation (3.117) gives:

$$w_R = 0 = (a_0 + a_1 + a_2 + a_3 + a_4) \quad 3.121$$

Substitution of Equation (3.119) and Equation (3.120a) into (3.121) gives:

$$w_R = 0 = (0 + a_1 + 0 + a_3 + a_4) \Rightarrow a_1 + a_3 = -a_4 \quad 3.122$$

Substituting Equation (3.120b) and Equation (3.120d) into Equation (3.118a) gives:

$$w_R'' = 0 = 2a_2 + 6a_3 + 12a_4 \quad 3.123$$

Substituting Equation (3.120a) into Equation (3.123) gives:

$$w_R'' = 0 = +6a_3 + 12a_4 \Rightarrow a_3 = -2a_4 \quad 3.124$$

Substituting Equation (3.124) into Equation (3.122) gives:

$$a_1 - 2a_4 = -a_4$$

$$a_1 = -a_4 + 2a_4$$

$$a_1 = a_4 \quad 3.125a$$

From Equation (3.119), (3.125a), (3.124) and (3.120a) gives:

$$a_0 = 0, a_1 = a_4, a_2 = 0, a_3 = -2a_4 \quad 3.125b$$

Substituting Equation (3.125b) into Equation (3.117) gives:

$$w_{RSS} = (a_4 R - 2a_4 R^3 + a_4 R^4)$$

$$w_{RSS} = a_4 (R - 2R^3 + R^4) \quad 3.126$$

$$w_{RSS} = w_{ySS}$$

3.2.3.2 Deflection Function for (CC) Edge Condition (Clamped Supported Edge)

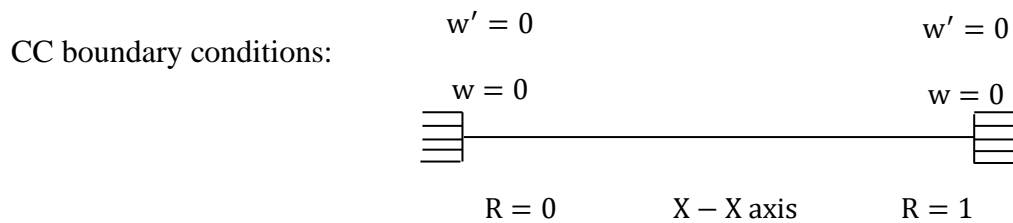


Figure 3.3: Clamped supported edge (CC) plate strip along **x-axis**

General orthogonal polynomial deflection equation of a strip plate along x-axis gives:

$$w_R = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4) \quad 3.127a$$

The first derivative of the deflection function of Equation (3.127a) in the x-axis gives:

$$w_R' = (0 + a_1 + 2a_2R + 3a_3R^2 + 4a_4R^3) \quad 3.127b$$

When

$$R = 0 \quad 3.127c$$

$$w_R = 0 \quad 3.127d$$

$$w_R' = 0 \quad 3.127e$$

Substituting Equations (3.127c) and Equation (3.127d) into Equation (3.127a) gives:

$$w_R = (a_0 + 0 + 0 + 0 + 0) = 0 \Rightarrow a_0 = 0 \quad 3.128a$$

Substituting Equations (3.127c) and Equation (3.127e) into Equation (3.127b) gives:

$$w_R' = (0 + a_1 + 0 + 0 + 0) = 0 \Rightarrow a_1 = 0 \quad 3.128b$$

When

$$R = 1 \quad 3.128c$$

$$w_R = 0 \quad 3.128d$$

$$w_R' = 0 \quad 3.128e$$

Substituting Equations (3.128c) and Equation (3.128d) into Equation (3.127a) gives:

$$w_R = (a_0 + a_1 + a_2 + a_3 + a_4) \quad 3.129$$

Substituting Equation (3.128a) and Equation (3.128b) into Equation (3.129) gives:

$$w_R = (0 + 0 + a_2 + a_3 + a_4) = 0 \Rightarrow a_2 + a_3 = -a_4 \quad 3.130$$

Substituting Equation (3.128c) and Equation (3.128e) into Equation (3.127b) gives:

$$w_R' = (0 + a_1 + 2a_2 + 3a_3 + 4a_4) = 0 \quad 3.131$$

Substituting Equation (3.128b) into Equation (3.131) gives:

$$w_R' = (0 + 0 + 2a_2 + 3a_3 + 4a_4) = 0 \Rightarrow 2a_2 + 3a_3 = -4a_4 \quad 3.132$$

Multiply Equation (3.130) by 2 gives:

$$2a_2 + 2a_3 = -2a_4 \quad 3.133$$

Subtract Equation (3.132) from Equation (3.133) gives

$$a_3 = -2a_4 \quad 3.134$$

Substitute Equation (3.134) into Equation (3.130) gives:

$$a_2 = a_4 \quad 3.135a$$

From Equation (3.128a), (3.128b), (3.134) and (3.135a) gives:

$$a_0 = 0; a_1 = 0; a_2 = a_4; a_3 = -2a_4 \quad 3.135b$$

Substitute Equation (3.135b) into Equation (3.127a) gives:

$$w_{RCC} = (a_4 R^2 - 2a_4 R^3 + a_4 R^4)$$

$$w_{RCC} = a_4 (R^2 - 2R^3 + R^4) \quad 3.136$$

$$w_{RCC} = w_{yCC}$$

3.2.3.3 Deflection Function for (CS) Edge Condition (Clamped and Simply Supported Edge)

CS boundary conditions:

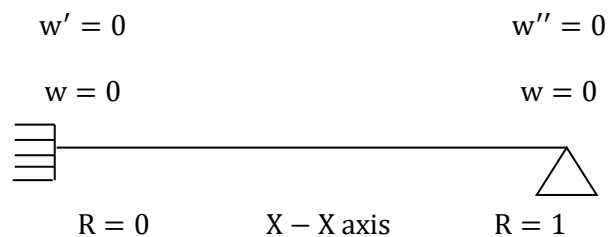


Figure 3.4: Clamped and Simply supported edge (CS) Plate strip along **x-axis**

General orthogonal polynomial deflection equation of a strip plate along x-axis gives:

$$w_R = (a_0 + a_1 R + a_2 R^2 + a_3 R^3 + a_4 R^4) \quad 3.137$$

The first derivative of the deflection function of Equation (3.137) in the x-axis gives:

$$w_R' = (0 + a_1 + 2a_2R + 3a_3R^2 + 4a_4R^3) \quad 3.138$$

The second derivative of the deflection function of Equation (3.137) in the x-axis gives:

$$w_R'' = (0 + 0 + 2a_2 + 6a_3R + 12a_4R^2) \quad 3.139a$$

When

$$R = 0 \quad 3.139b$$

$$w_R = 0 \quad 3.139c$$

$$w_R' = 0 \quad 3.139d$$

Substituting Equations (3.139b) and Equation (3.139c) into Equation (3.137) gives:

$$w_R = (a_0 + 0 + 0 + 0 + 0) = 0 \Rightarrow a_0 = 0 \quad 3.140$$

Substituting Equations (3.139b) and Equation (3.139d) into Equation (3.138) gives:

$$w_R' = (0 + a_1 + 0 + 0 + 0) = 0 \Rightarrow a_1 = 0 \quad 3.141$$

When

$$R = 1 \quad 3.142a$$

$$w_R = 0 \quad 3.142b$$

$$w_R'' = 0 \quad 3.142c$$

Substituting Equation (3.142a) and Equation (3.142b) into Equation (3.137) gives:

$$w_R = (a_0 + a_1 + a_2 + a_3 + a_4) = 0 \quad 3.142d$$

Substituting Equation (3.140) and Equation (3.141) into Equation (3.142d) gives:

$$w_R = (0 + 0 + a_2 + a_3 + a_4) = 0 \Rightarrow a_2 + a_3 = -a_4 \quad 3.143$$

Substituting Equation (3.142a) and Equation (3.142c) into Equation (3.139a) gives

$$w_R'' = (0 + 0 + 2a_2 + 6a_3 + 12a_4) = 0 \Rightarrow 2a_2 + 6a_3 = -12a_4 \quad 3.144$$

Multiplying Equation (3.143) by (2) gives

$$2a_2 + 2a_3 = -2a_4 \quad 3.145$$

Subtracting Equation (3.145) from Equation (3.144) gives:

$$a_3 = -2.5a_4 \quad 3.146a$$

Substituting Equation (3.146a) into Equation (3.143) gives:

$$a_2 - 2.5a_4 = -a_4$$

$$a_2 = 1.5a_4 \quad 3.146b$$

From Equation (3.140), (3.141), (3.146a) and (3.146b) gives:

$$a_0 = 0; a_1 = 0; a_2 = 1.5a_4; a_3 = -2.5a_4 \quad 3.146c$$

Substituting Equation (3.146c) into Equation (3.137) gives:

$$w_{RCS} = (1.5a_4R^2 - 2.5a_4R^3 + a_4R^4)$$

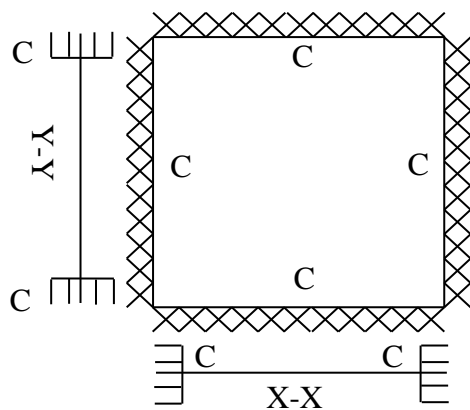
$$w_{RCS} = a_4(1.5R^2 - 2.5R^3 + R^4) \quad 3.147$$

$$w_{RCS} = w_{yCS}$$

3.2.3.4 Peculiar Deflection Function for Different Edge Condition of a Plate

3.2.3.4.1 Deflection Function for CCCC Plate

Substituting Equation (3.136) gives:



The diagram shows a rectangular plate with all four edges fixed, indicated by hatched lines and the letter 'C' at each corner. A vertical coordinate system is shown on the left with the origin at the top-left corner, and a horizontal coordinate system 'X-X' is shown at the bottom. The deflection function is given by:

$$w_{CCCC} = a_4(R^2 - 2R^3 + R^4) \cdot b_4(Q^2 - 2Q^3 + Q^4) \quad 3.148a$$

Figure 3.5: Deflection function for CCCC plate.

3.2.3.4.2 Deflection Function for SSSS Plate

Substituting Equation (3.126) gives:

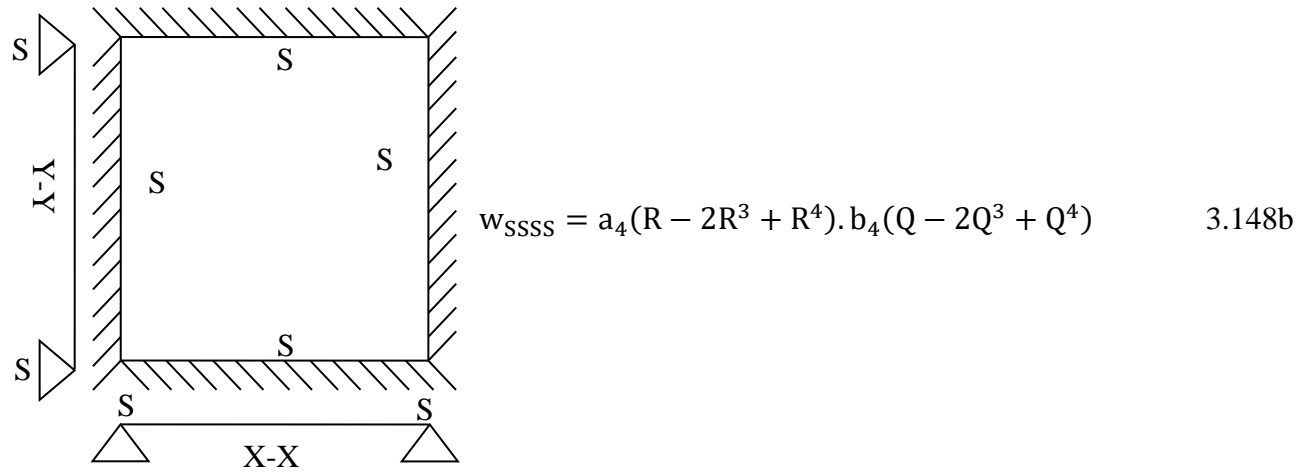


Figure 3.6: Deflection function for SSSS plate.

3.2.3.4.3 Deflection Function for CSCS Plate

Substituting Equation (3.126) and Equation (3.136) gives:

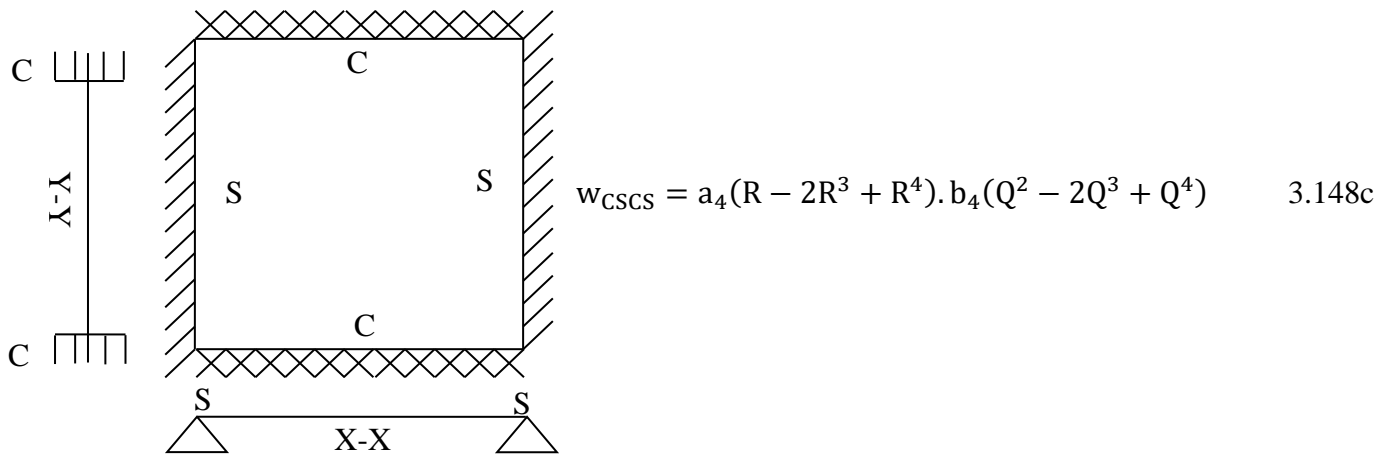


Figure 3.7: Deflection function for CSCS plate

3.2.3.4.4 Deflection Function for CCSS Plate

Substituting Equation (3.147) gives:

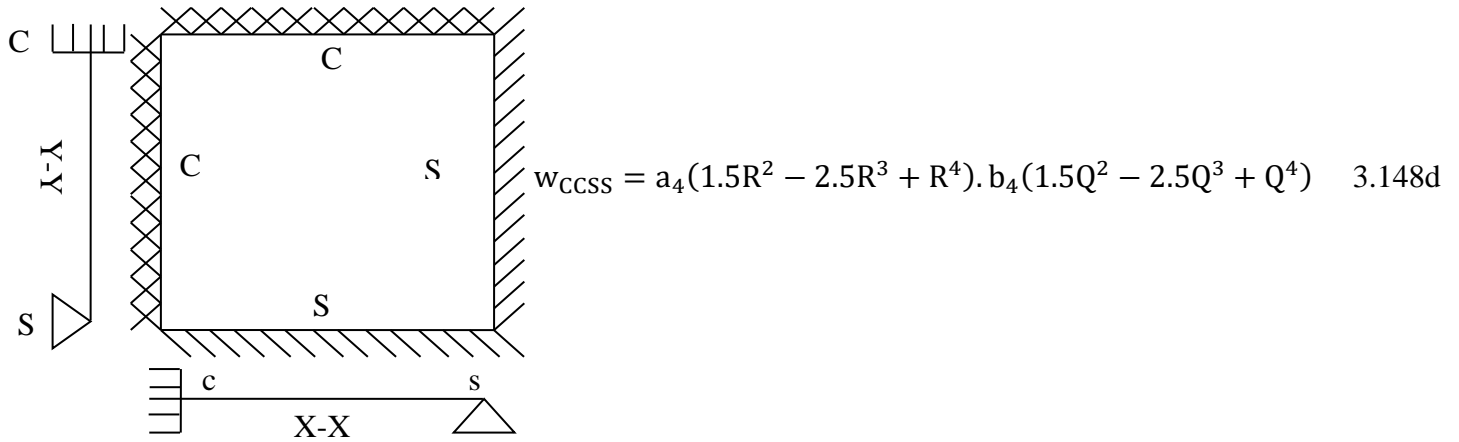


Figure 3.8: Deflection function for CCSS plate.

3.2.3.4.5 Deflection Function for CSSS Plate

Substituting Equation (3.126) and Equation (3.147) gives:

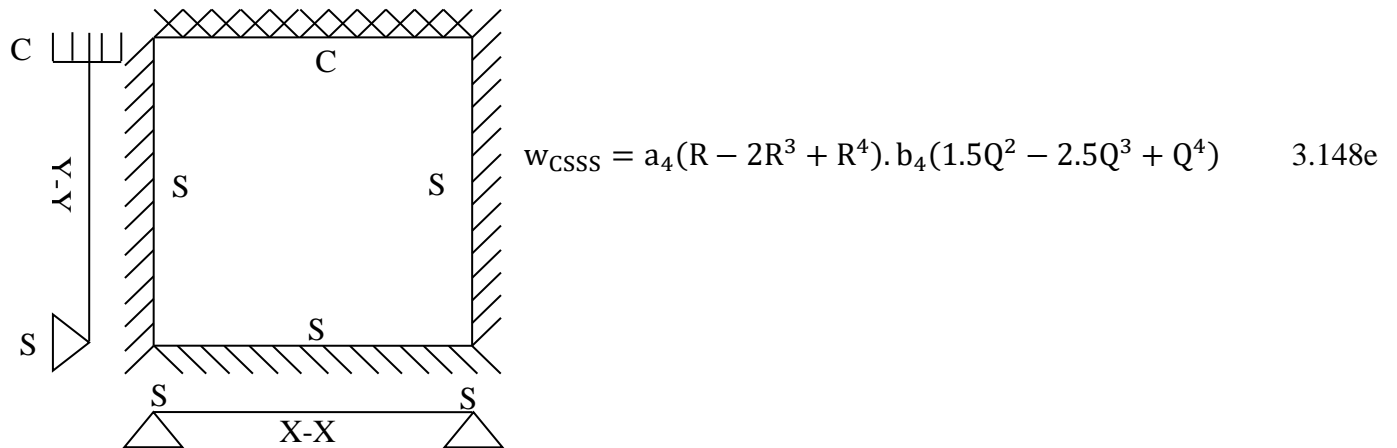


Figure 3.9: Deflection function for CSSS plate.

3.2.3.4.6 Deflection Function for CCCS Plate

Substituting Equation (3.147) and Equation (3.136) gives:

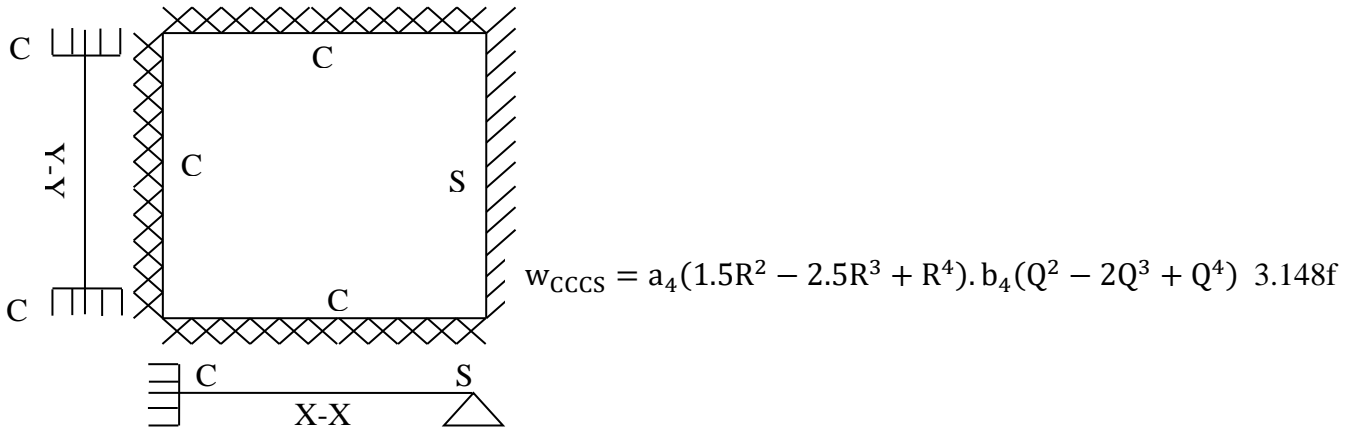


Figure 3.10: Deflection function for CCCS plate.

3.3 Determination of Stiffness Coefficients

From Equation (3.115) where:

$$w = Ah$$

When,

A = Amplitude of the deflection function (Coefficient of deflection)

h = Shape function

Substituting Equation (3.115) into Equation (3.75) gives:

$$\begin{aligned} \Pi = & \frac{D}{2} \int_0^1 \int_0^1 \left(\frac{\partial^3 Ah}{\partial R^3} \cdot \frac{\partial Ah}{\partial R} + \frac{2}{\alpha^2} \cdot \frac{\partial^3 Ah}{\partial R \partial Q^2} \cdot \frac{\partial Ah}{\partial R} + \frac{1}{\alpha^4} \cdot \frac{\partial^3 Ah}{\partial Q^3} \cdot \frac{\partial Ah}{\partial Q} \right) \partial_R \partial_Q \\ & - \int_0^1 \int_0^1 qa^4 Ah \partial_R \partial_Q \end{aligned} \quad 3.149$$

Rearranging Equation (3.149) gives:

$$\begin{aligned} \Pi = & \frac{DA^2}{2} \int_0^1 \int_0^1 \left(\frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} + \frac{2}{\alpha^2} \cdot \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} + \frac{1}{\alpha^4} \cdot \frac{\partial^3 h}{\partial Q^3} \cdot \frac{\partial h}{\partial Q} \right) \partial_R \partial_Q \\ & - qa^4 A \int_0^1 \int_0^1 h \partial_R \partial_Q \end{aligned} \quad 3.150$$

Differentiating Equation (3.150) with the respect of A gives:

$$DA \int_0^1 \int_0^1 \left(\frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} + \frac{2}{\alpha^2} \cdot \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} + \frac{1}{\alpha^4} \cdot \frac{\partial^3 h}{\partial Q^3} \cdot \frac{\partial h}{\partial Q} \right) \partial_R \partial_Q - qa^4 \int_0^1 \int_0^1 h \partial_R \partial_Q = 0 \quad 3.151$$

Rearranging Equation (3.151) in respect of (A) gives:

$$DA \int_0^1 \int_0^1 \left(\frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} + \frac{2}{\alpha^2} \cdot \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} + \frac{1}{\alpha^4} \cdot \frac{\partial^3 h}{\partial Q^3} \cdot \frac{\partial h}{\partial Q} \right) \partial_R \partial_Q = qa^4 \int_0^1 \int_0^1 h \partial_R \partial_Q \quad 3.152$$

$$A = \frac{qa^4 \left(\int_0^1 \int_0^1 h \right) \partial_R \partial_Q}{D \int_0^1 \int_0^1 \left(\frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} + \frac{2}{\alpha^2} \cdot \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} + \frac{1}{\alpha^4} \cdot \frac{\partial^3 h}{\partial Q^3} \cdot \frac{\partial h}{\partial Q} \right) \partial_R \partial_Q} \quad 3.153$$

Let the stiffness coefficient be:

$$k_q = \int_0^a \int_0^a h \partial_R \partial_Q \text{ (Load stiffness)} \quad 3.154$$

$$k_x = \int_0^a \int_0^a \frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} \partial_R \partial_Q \text{ (Material stiffness in X direction)} \quad 3.155$$

$$k_{xy} = \int_0^a \int_0^a \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} \partial_R \partial_Q \text{ (Material stiffness in x - y direction)} \quad 3.156$$

$$k_y = \int_0^a \int_0^a \frac{\partial^3 h}{\partial Q^3} \cdot \frac{\partial h}{\partial Q} \partial_R \partial_Q \text{ (Material stiffness in y direction)} \quad 3.157$$

Substituting Equation (3.154), (3.155), (3.156), (3.157) into Equation (3.153) gives:

$$A = \frac{qa^4}{D} \left(\frac{k_q}{k_x + \frac{2}{\alpha^2} k_{xy} + \frac{1}{\alpha^4} k_y} \right) \quad 3.158$$

From Equation (3.158) the stiffness can be written as:

$$K = \frac{k_q}{k_x + \frac{2}{\alpha^2} k_{xy} + \frac{1}{\alpha^4} k_y} \quad 3.159$$

Let the material stiffness be k_T and substituting it into Equation (3.159) gives:

$$K = \frac{k_q}{k_T} \quad 3.160$$

Equation (3.158) can be written as:

$$A = \frac{qa^4}{D} \cdot \frac{k_q}{k_T} \quad 3.161$$

$$A = \frac{qa^4}{D} \cdot K \quad 3.162$$

3.3.1 Solving for Stiffness Coefficient for the Different Plate Conditions

From Equation (3.154) to Equation (3.157) where the stiffness coefficient was stated and this equation can be solved by definite integration of the shape functions deflection (h)

3.3.1.1 Stiffness Coefficient for SSSS Classical Rectangular Plate

From Equation (3.148b);

$$w_{SSSS} = a_4(R - 2R^3 + R^4) \cdot b_4(Q - 2Q^3 + Q^4)$$

$$w_{SSSS} = Ah$$

$$w_{SSSS} = A((R - 2R^3 + R^4). (Q - 2Q^3 + Q^4)) \quad 3.163a$$

Separation of the shape function for the deflection function (Equation 3.148b) of SSSS classical rectangular plate gives:

$$h = ((R - 2R^3 + R^4). (Q - 2Q^3 + Q^4)) \quad 3.163b$$

Solving a close integral for load stiffness gives:

$$k_q = \int_0^1 \int_0^1 h \partial_R \partial_Q (\text{Load stiffness})$$

Substituting Equation (3.163b) into Equation (3.154) gives:

$$\int_0^1 \int_0^1 ((R - 2R^3 + R^4). (Q - 2Q^3 + Q^4)) (\partial_R \partial_Q) \quad 3.163c$$

$$= \left(\frac{1}{2} - \frac{2}{4} + \frac{1}{5}\right) \left(\frac{1}{2} - \frac{2}{4} + \frac{1}{5}\right) \quad 3.164$$

$$k_q = 0.04 \quad 3.165$$

Solving a close integral for Material stiffness along x plane of SSSS classical rectangular plate gives:

$$k_x = \int_0^a \int_0^a \frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} \partial_R \partial_Q (\text{Material stiffness})$$

Substituting Equation (3.163b) into Equation (3.155) gives:

$$\frac{\partial h}{\partial R} = (1 - 6R^2 + 4R^3)(Q - 2Q^3 + Q^4) \quad 3.166$$

$$\frac{\partial^2 h}{\partial R^2} = 12(R^2 - R)(Q - 2Q^3 + Q^4) \quad 3.167$$

$$\left(\frac{\partial^2 h}{\partial R^2}\right)^2 = 144(R^4 - 2R^3 + R^2)(Q^2 - 4Q^4 + 2Q^5 + 4Q^6 - 4Q^7 + Q^8) \quad 3.168$$

$$\int_0^1 \int_0^1 \frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} = 144 \left(\frac{1}{5} - \frac{2}{4} + \frac{1}{3}\right) \left(\frac{1}{3} - \frac{4}{5} + \frac{2}{6} + \frac{4}{7} - \frac{4}{8} + \frac{1}{9}\right) \quad 3.169$$

$$k_x = 0.2361904762 \quad 3.170$$

Solving a close integral for Material stiffness along x-y plane of SSSS classical rectangular plate gives:

$$k_{xy} = \int_0^a \int_0^a \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} \partial_R \partial_Q \text{ (Material stiffness)}$$

Substituting Equation (3.163b) into Equation (3.156) gives:

$$\frac{\partial^2 h}{\partial R \partial Q} = (1 - 6R^2 + 4R^3)(1 - 6Q^2 + 4Q^3) \quad 3.171$$

$$\begin{aligned} & \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} \\ &= (1 - 12R^2 + 8R^3 + 36R^4 - 48R^5 + 16R^6)(1 - 12Q^2 + 8Q^3 + 36Q^4 - 48Q^5 \\ &+ 16Q^6) \end{aligned} \quad 3.172$$

$$\int_0^1 \int_0^1 \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} = \left(1 - \frac{12}{3} + \frac{8}{4} + \frac{36}{5} - \frac{48}{6} + \frac{16}{7}\right) \left(1 - \frac{12}{3} + \frac{8}{4} + \frac{36}{5} - \frac{48}{6} + \frac{16}{7}\right) \quad 3.173$$

$$k_{xy} = 0.235918 \quad 3.174$$

Solving a close integral for Material stiffness along y plane of SSSS classical rectangular plate gives:

$$k_y = \int_0^a \int_0^a \frac{\partial^3 h}{\partial Q^3} \cdot \frac{\partial h}{\partial Q} \partial_R \partial_Q \text{ (Material stiffness)}$$

Substituting Equation (3.163b) into Equation (3.157) gives:

$$\frac{\partial h}{\partial Q} = (R - 2R^3 + R^4)(1 - 6Q^2 + 4Q^3) \quad 3.175$$

$$\frac{\partial^2 h}{\partial Q^2} = 12(Q^2 - Q)(R - 2R^3 + R^4) \quad 3.176$$

$$\left(\frac{\partial^2 h}{\partial Q^2}\right)^2 = 144(Q^4 - 2Q^3 + Q^2)(R^2 - 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8) \quad 3.177$$

$$\int_0^1 \int_0^1 \frac{\partial^3 h}{\partial Q^3} \cdot \frac{\partial h}{\partial Q} = 144 \left(\frac{1}{5} - \frac{2}{4} + \frac{1}{3}\right) \left(\frac{1}{3} - \frac{4}{5} + \frac{2}{6} + \frac{4}{7} - \frac{4}{8} + \frac{1}{9}\right) \quad 3.178$$

$$k_y = 0.2361904762 \quad 3.179$$

3.3.1.2 Stiffness Coefficient for CCCC Classical Rectangular Plate

From Equation (3.148a);

$$w_{CCCC} = a_4(R^2 - 2R^3 + R^4) \cdot b_4(Q^2 - 2Q^3 + Q^4)$$

$$w_{CCCC} = Ah$$

$$w_{CCCC} = A((R^2 - 2R^3 + R^4) \cdot (Q^2 - 2Q^3 + Q^4))$$

Separation of the shape function for the deflection function (Equation 3.148a) of CCCC classical rectangular plate gives:

$$h = (R^2 - 2R^3 + R^4) \cdot (Q^2 - 2Q^3 + Q^4) \quad 3.180$$

Solving a close integral for load stiffness gives:

$$k_q = \int_0^1 \int_0^1 h \partial_R \partial_Q (\text{Load stiffness})$$

Substituting Equation (3.180) into Equation (3.154) gives:

$$\int_0^1 \int_0^1 (R^2 - 2R^3 + R^4) \cdot (Q^2 - 2Q^3 + Q^4) (\partial_R \partial_Q) \quad 3.181$$

$$= \left(\frac{1}{3} - \frac{2}{4} + \frac{1}{5} \right) \left(\frac{1}{3} - \frac{2}{4} + \frac{1}{5} \right) \quad 3.182$$

$$k_q = 0.001111 \quad 3.183$$

Solving a close integral for Material stiffness along x plane of CCCC classical rectangular plate gives:

$$k_x = \int_0^a \int_0^a \frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} \partial_R \partial_Q (\text{Material stiffness})$$

Substituting Equation (3.180) into Equation (3.155) gives:

$$\frac{\partial h}{\partial R} = (2R - 6R^2 + 4R^3)(Q^2 - 2Q^3 + Q^4) \quad 3.184$$

$$\frac{\partial^2 h}{\partial R^2} = (2 - 12R + 12R^2)(Q^2 - 2Q^3 + Q^4) \quad 3.185$$

$$\left(\frac{\partial^2 h}{\partial R^2}\right)^2 = (4 - 48R + 192R^2 - 288R^3 + 144R^4)(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8) \quad 3.186$$

$$\int_0^1 \int_0^1 \frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} = \left(4 - \frac{48}{2} + \frac{192}{3} - \frac{288}{4} + \frac{144}{5}\right) \left(\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9}\right) \quad 3.187$$

$$k_x = 0.00127 \quad 3.188$$

Solving a close integral for Material stiffness along x-y plane of CCCC classical rectangular plate gives:

$$k_{xy} = \int_0^a \int_0^a \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} \partial_R \partial_Q \text{ (Material stiffness)}$$

Substituting Equation (3.180) into Equation (3.156) gives:

$$\frac{\partial^2 h}{\partial R \partial Q} = (2R - 6R^2 + 4R^3)(2Q - 6Q^2 + 4Q^3) \quad 3.189$$

$$\begin{aligned} \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} &= (4R^2 - 24R^3 + 52R^4 - 48R^5 + 16R^6)(4Q^2 - 24Q^3 + 52Q^4 - 48Q^5 \\ &\quad + 16Q^6) \end{aligned} \quad 3.190$$

$$\int_0^1 \int_0^1 \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} = \left(\frac{4}{3} - \frac{24}{4} + \frac{52}{5} - \frac{48}{6} + \frac{16}{7}\right) \left(\frac{4}{3} - \frac{24}{4} + \frac{52}{5} - \frac{48}{6} + \frac{16}{7}\right) \quad 3.191$$

$$k_{xy} = 0.000363 \quad 3.192$$

Solving a close integral for Material stiffness along y plane of CCCC classical rectangular plate gives:

$$k_y = \int_0^a \int_0^a \frac{\partial^3 h}{\partial Q^3} \cdot \frac{\partial h}{\partial Q} \partial_R \partial_Q \text{ (Material stiffness)}$$

Substituting Equation (3.180) into Equation (3.157) gives:

$$\frac{\partial h}{\partial Q} = (2Q - 6Q^2 + 4Q^3)(R^2 - 2R^3 + R^4) \quad 3.193$$

$$\frac{\partial^2 h}{\partial Q^2} = (2 - 12Q + 12Q^2)(R^2 - 2R^3 + R^4) \quad 3.194$$

$$\left(\frac{\partial^2 h}{\partial Q^2}\right)^2 = (4 - 48Q + 192Q^2 - 288Q^3 + 144Q^4)(R^4 - 4R^5 + 6R^6 - 4R^7 + R^8) \quad 3.195$$

$$\int_0^1 \int_0^1 \frac{\partial^3 h}{\partial Q^3} \cdot \frac{\partial h}{\partial Q} = \left(4 - \frac{48}{2} + \frac{192}{3} - \frac{288}{4} + \frac{144}{5}\right) \left(\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9}\right) \quad 3.196$$

$$k_y = 0.00127 \quad 3.197$$

3.3.1.3 Stiffness Coefficient for CSCS Classical Rectangular Plate

From Equation (3.148c);

$$w_{\text{CSCS}} = a_4(R - 2R^3 + R^4) \cdot b_4(Q^2 - 2Q^3 + Q^4)$$

$$w_{\text{CSCS}} = Ah$$

$$w_{\text{CSCS}} = A((R - 2R^3 + R^4) \cdot (Q^2 - 2Q^3 + Q^4))$$

Separation of the shape function for the deflection function (Equation 3.148c) of CSCS classical rectangular plate gives:

$$h = (R - 2R^3 + R^4) \cdot (Q^2 - 2Q^3 + Q^4) \quad 3.198$$

Solving a close integral for load stiffness gives:

$$k_q = \int_0^1 \int_0^1 h \partial_R \partial_Q (\text{Load stiffness})$$

Substituting Equation (3.198) into Equation (3.154) gives:

$$\int_0^1 \int_0^1 (R - 2R^3 + R^4) \cdot (Q^2 - 2Q^3 + Q^4) (\partial_R \partial_Q) \quad 3.199$$

$$= \left(\frac{1}{2} - \frac{2}{4} + \frac{1}{5}\right) \left(\frac{1}{3} - \frac{2}{4} + \frac{1}{5}\right) \quad 3.200$$

$$k_q = 0.006667 \quad 3.201$$

Solving a close integral for Material stiffness along x plane of CSCS classical rectangular plate gives:

$$k_x = \int_0^a \int_0^a \frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} \partial_R \partial_Q (\text{Material stiffness})$$

Substituting Equation (3.198) into Equation (3.155) gives:

$$\frac{\partial h}{\partial R} = (1 - 6R^2 + 4R^3)(Q^2 - 2Q^3 + Q^4) \quad 3.202$$

$$\frac{\partial^2 h}{\partial R^2} = 12(R^2 - R)(Q^2 - 2Q^3 + Q^4) \quad 3.203$$

$$\left(\frac{\partial^2 h}{\partial R^2}\right)^2 = 144(R^4 - 2R^3 + R^2)(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8) \quad 3.204$$

$$\int_0^1 \int_0^1 \frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} = 144 \left(\frac{1}{5} - \frac{2}{4} + \frac{1}{3} \right) \left(\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9} \right) \quad 3.205$$

$$k_x = 0.007619 \quad 3.206$$

Solving a close integral for Material stiffness along x-y plane of CSCS classical rectangular plate gives:

$$k_{xy} = \int_0^a \int_0^a \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} \partial_R \partial_Q \text{ (Material stiffness)}$$

Substituting Equation (3.198) into Equation (3.156) gives:

$$\frac{\partial^2 h}{\partial R \partial Q} = (1 - 6R^2 + 4R^3)(2Q - 6Q^2 + 4Q^3) \quad 3.207$$

$$\begin{aligned} \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} = & (1 - 12R^2 + 8R^3 + 36R^4 - 48R^5 + 16R^6)(4Q^2 - 24Q^3 + 52Q^4 - 48Q^5 \\ & + 16Q^6) \end{aligned} \quad 3.208$$

$$\int_0^1 \int_0^1 \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} = \left(1 - \frac{12}{3} + \frac{8}{4} + \frac{36}{5} - \frac{48}{6} + \frac{16}{7} \right) \left(\frac{4}{3} - \frac{24}{4} + \frac{52}{5} - \frac{48}{6} + \frac{16}{7} \right) \quad 3.209$$

$$k_{xy} = 0.009252 \quad 3.210$$

Solving a close integral for Material stiffness along y plane of CSCS classical rectangular plate gives:

$$k_y = \int_0^a \int_0^a \frac{\partial^3 h}{\partial Q^3} \cdot \frac{\partial h}{\partial Q} \partial_R \partial_Q \text{ (Material stiffness)}$$

Substituting Equation (3.198) into Equation (3.157) gives:

$$\frac{\partial h}{\partial Q} = (2Q - 6Q^2 + 4Q^3)(R - 2R^3 + R^4) \quad 3.211$$

$$\frac{\partial^2 h}{\partial Q^2} = (2 - 12Q + 12Q^2)(R - 2R^3 + R^4) \quad 3.212$$

$$\left(\frac{\partial^2 h}{\partial Q^2}\right)^2 = (4 - 48Q + 192Q^2 - 288Q^3 + 144Q^4)(R^2 - 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8) \quad 3.213$$

$$\int_0^1 \int_0^1 \frac{\partial^3 h}{\partial Q^3} \cdot \frac{\partial h}{\partial Q} = \left(4 - \frac{48}{2} + \frac{192}{3} - \frac{288}{4} + \frac{144}{5}\right) \left(\frac{1}{3} - \frac{4}{5} + \frac{2}{6} + \frac{4}{7} - \frac{4}{8} + \frac{1}{9}\right) \quad 3.214$$

$$k_y = 0.039365 \quad 3.215$$

3.3.1.4 Stiffness Coefficient for CCSS Classical Rectangular Plate

From Equation (3.148d);

$$w_{CCSS} = a_4(1.5R^2 - 2.5R^3 + R^4) \cdot b_4(1.5Q^2 - 2.5Q^3 + Q^4)$$

$$w_{CCSS} = Ah$$

$$w_{CCSS} = A((1.5R^2 - 2.5R^3 + R^4) \cdot (1.5Q^2 - 2.5Q^3 + Q^4))$$

Separation of the shape function for the deflection function (Equation 3.148d) of CCSS classical rectangular plate gives:

$$h = (1.5R^2 - 2.5R^3 + R^4) \cdot (1.5Q^2 - 2.5Q^3 + Q^4) \quad 3.216$$

Solving a close integral for load stiffness gives:

$$k_q = \int_0^1 \int_0^1 h \partial_R \partial_Q (\text{Load stiffness})$$

Substituting Equation (3.216) into Equation (3.154) gives:

$$\int_0^1 \int_0^1 (1.5R^2 - 2.5R^3 + R^4) \cdot (1.5Q^2 - 2.5Q^3 + Q^4) (\partial_R \partial_Q) \quad 3.217$$

$$= \left(\frac{1.5}{3} - \frac{2.5}{4} + \frac{1}{5} \right) \left(\frac{1.5}{3} - \frac{2.5}{4} + \frac{1}{5} \right) \quad 3.218$$

$$k_q = 0.005625 \quad 3.219$$

Solving a close integral for Material stiffness along x plane of CCSS classical rectangular plate gives:

$$k_x = \int_0^a \int_0^a \frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} \partial_R \partial_Q (\text{Material stiffness})$$

Substituting Equation (3.216) into Equation (3.155) gives:

$$\frac{\partial h}{\partial R} = (3R - 7.5R^2 + 4R^3)(1.5Q^2 - 2.5Q^3 + Q^4) \quad 3.220$$

$$\frac{\partial^2 h}{\partial R^2} = (3 - 15R + 12R^2)(1.5Q^2 - 2.5Q^3 + Q^4) \quad 3.221$$

$$\left(\frac{\partial^2 h}{\partial R^2} \right)^2 = (9 - 90R + 297R^2 - 360R^3 + 144R^4)(2.25Q^4 - 7.5Q^5 + 9.25Q^6 - 5Q^7 + Q^8) \quad 3.222$$

$$\int_0^1 \int_0^1 \frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} = \left(9 - \frac{90}{2} + \frac{297}{3} - \frac{360}{4} + \frac{144}{5} \right) \left(\frac{2.25}{5} - \frac{7.5}{6} + \frac{9.25}{7} - \frac{5}{8} + \frac{1}{9} \right) \quad 3.223$$

$$k_x = 0.013571 \quad 3.224$$

Solving a close integral for Material stiffness along x-y plane of CCSS classical rectangular plate gives:

$$k_{xy} = \int_0^a \int_0^a \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} \partial_R \partial_Q \text{ (Material stiffness)}$$

Substituting Equation (3.216) into Equation (3.156) gives:

$$\frac{\partial^2 h}{\partial R \partial Q} = (3R - 7.5R^2 + 4R^3)(3Q - 7.5Q^2 + 4Q^3) \quad 3.225$$

$$\begin{aligned} \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} = & (9R^2 - 45R^3 + 80.25R^4 - 60R^5 + 16R^6)(9Q^2 - 45Q^3 + 80.25Q^4 - 60Q^5 \\ & + 16Q^6) \end{aligned} \quad 3.226$$

$$\int_0^1 \int_0^1 \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} = \left(\frac{9}{3} - \frac{45}{4} + \frac{80.25}{5} - \frac{60}{6} + \frac{16}{7} \right) \left(\frac{9}{3} - \frac{45}{4} + \frac{80.25}{5} - \frac{60}{6} + \frac{16}{7} \right) \quad 3.227$$

$$k_{xy} = 0.007347 \quad 3.228$$

Solving a close integral for Material stiffness along y plane of CCSS classical rectangular plate gives:

$$k_y = \int_0^a \int_0^a \frac{\partial^3 h}{\partial Q^3} \cdot \frac{\partial h}{\partial Q} \partial_R \partial_Q \text{ (Material stiffness)}$$

Substituting Equation (3.216) into Equation (3.157) gives:

$$\frac{\partial h}{\partial Q} = (3Q - 7.5Q^2 + 4Q^3)(1.5R^2 - 2.5R^3 + R^4) \quad 3.229$$

$$\frac{\partial^2 h}{\partial Q^2} = (3 - 15Q + 12Q^2)(1.5R^2 - 2.5R^3 + R^4) \quad 3.230$$

$$\left(\frac{\partial^2 h}{\partial Q^2}\right)^2 = (9 - 90Q + 297Q^2 - 360Q^3 + 144Q^4)(2.25R^4 - 7.5R^5 + 9.25R^6 - 5R^7 + R^8) \quad 3.231$$

$$\int_0^1 \int_0^1 \left(\frac{\partial^3 h}{\partial R^3}\right) \left(\frac{\partial h}{\partial R}\right) = \left(9 - \frac{90}{2} + \frac{297}{3} - \frac{360}{4} + \frac{144}{5}\right) \left(\frac{2.25}{5} - \frac{7.5}{6} + \frac{9.25}{7} - \frac{5}{8} + \frac{1}{9}\right) \quad 3.232$$

$$k_y = 0.013571 \quad 3.233$$

3.3.1.5 Stiffness Coefficient for CSSS Classical Rectangular Plate

From Equation (3.148e);

$$w_{\text{CSSS}} = a_4(R - 2R^3 + R^4) \cdot b_4(1.5Q^2 - 2.5Q^3 + Q^4)$$

$$w_{\text{CSSS}} = Ah$$

$$w_{\text{CSSS}} = A((R - 2R^3 + R^4) \cdot (1.5Q^2 - 2.5Q^3 + Q^4))$$

Separation of the shape function for the deflection function (Equation 3.148e) of CSSS classical rectangular plate gives:

$$h = (R - 2R^3 + R^4) \cdot (1.5Q^2 - 2.5Q^3 + Q^4) \quad 3.234$$

Solving a close integral for load stiffness gives:

$$k_q = \int_0^1 \int_0^1 h \partial_R \partial_Q (\text{Load stiffness})$$

Substituting Equation (3.234) into Equation (3.154) gives:

$$\int_0^1 \int_0^1 (R - 2R^3 + R^4) \cdot (1.5Q^2 - 2.5Q^3 + Q^4) (\partial_R \partial_Q) \quad 3.235$$

$$= \left(\frac{1}{2} - \frac{2}{4} + \frac{1}{5}\right) \left(\frac{1.5}{3} - \frac{2.5}{4} + \frac{1}{5}\right) \quad 3.236$$

$$k_q = 0.015 \quad 3.237$$

Solving a close integral for Material stiffness along x plane of CSSS classical rectangular plate gives:

$$k_x = \int_0^a \int_0^a \frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} \partial_R \partial_Q \text{ (Material stiffness)}$$

Substituting Equation (3.234) into Equation (3.155) gives:

$$\frac{\partial h}{\partial R} = (1 - 6R^2 + 4R^3) \quad 3.238$$

$$\frac{\partial^2 h}{\partial R^2} = 12(R^2 - R)(1.5Q^2 - 2.5Q^3 + Q^4) \quad 3.239$$

$$\left(\frac{\partial^2 h}{\partial R^2}\right)^2 = 144(R^4 - 2R^3 + R^2)(2.25Q^4 - 7.5Q^5 + 9.25Q^6 - 5Q^7 + Q^8) \quad 3.240$$

$$\int_0^1 \int_0^1 \frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} = 144 \left(\frac{1}{5} - \frac{2}{4} + \frac{1}{3}\right) \left(\frac{2.25}{5} - \frac{7.5}{6} + \frac{9.25}{7} - \frac{5}{8} + \frac{1}{9}\right) \quad 3.241$$

$$k_x = 0.03619 \quad 3.242$$

Solving a close integral for Material stiffness along x-y plane of CSSS classical rectangular plate gives:

$$k_{xy} = \int_0^a \int_0^a \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} \partial_R \partial_Q \text{ (Material stiffness)}$$

Substituting Equation (3.234) into Equation (3.156) gives:

$$\frac{\partial^2 h}{\partial R \partial Q} = (1 - 6R^2 + 4R^3)(3Q - 7.5Q^2 + 4Q^3) \quad 3.243$$

$$\begin{aligned} \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} = & (1 - 12R^2 + 8R^3 + 36R^4 - 48R^5 + 16R^6)(9Q^2 - 45Q^3 + 80.25Q^4 - 60Q^5 \\ & + 16Q^6) \end{aligned} \quad 3.244$$

$$\int_0^1 \int_0^1 \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} = \left(1 - \frac{12}{3} + \frac{8}{4} + \frac{36}{5} - \frac{48}{6} + \frac{16}{7}\right) \left(\frac{9}{3} - \frac{45}{4} + \frac{80.25}{5} - \frac{60}{6} + \frac{16}{7}\right) \quad 3.245$$

$$k_{xy} = 0.041633 \quad 3.246$$

Solving a close integral for Material stiffness along y plane of CSSS classical rectangular plate gives:

$$k_y = \int_0^a \int_0^a \frac{\partial^3 h}{\partial Q^3} \cdot \frac{\partial h}{\partial Q} \partial_R \partial_Q \text{ (Material stiffness)}$$

$$\frac{\partial h}{\partial Q} = (3Q - 7.5Q^2 + 4Q^3)(R - 2R^3 + R^4) \quad 3.247$$

$$\frac{\partial^2 h}{\partial Q^2} = (3 - 15Q + 12Q^2)(R - 2R^3 + R^4) \quad 3.248$$

$$\left(\frac{\partial^2 h}{\partial Q^2}\right)^2 = (9 - 90Q + 297Q^2 - 360Q^3 + 144Q^4)(R^2 - 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8) \quad 3.249$$

$$\int_0^1 \int_0^1 \frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} = \left(9 - \frac{90}{2} + \frac{297}{3} - \frac{360}{4} + \frac{144}{5}\right) \left(\frac{1}{3} - \frac{4}{5} + \frac{2}{6} + \frac{4}{7} - \frac{4}{8} + \frac{1}{9}\right) \quad 3.250$$

$$k_y = 0.088571 \quad 3.251$$

3.3.1.6 Stiffness Coefficient for CCCS Classical Rectangular Plate

From Equation (3.148f);

$$w_{SCCC} = a_4(1.5R^2 - 2.5R^3 + R^4) \cdot b_4(Q^2 - 2Q^3 + Q^4)$$

$$w_{SCCC} = Ah$$

$$w_{SCCC} = A((1.5R^2 - 2.5R^3 + R^4) \cdot (Q^2 - 2Q^3 + Q^4))$$

Separation of the shape function for the deflection function (Equation 3.148f) of CCCS classical rectangular plate gives:

$$h = (1.5R^2 - 2.5R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \quad 3.252$$

Solving a close integral for load stiffness gives:

$$k_q = \int_0^1 \int_0^1 h \partial_R \partial_Q (\text{Load stiffness})$$

Substituting Equation (3.252) into Equation (3.154) gives:

$$\int_0^1 \int_0^1 (1.5R^2 - 2.5R^3 + R^4)(\partial_R \partial_Q) \cdot (Q^2 - 2Q^3 + Q^4) \quad 3.253$$

$$= \left(\frac{1.5}{3} - \frac{2.5}{4} + \frac{1}{5}\right) \left(\frac{1}{3} - \frac{2}{4} + \frac{1}{5}\right) \quad 3.254$$

$$k_q = 0.0025 \quad 3.255$$

Solving a close integral for Material stiffness along x plane of CCCS classical rectangular plate gives:

$$k_x = \int_0^a \int_0^a \frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} \partial_R \partial_Q \text{ (Material stiffness)}$$

Substituting Equation (3.252) into Equation (3.155) gives:

$$\frac{\partial h}{\partial R} = (3R - 7.5R^2 + 4R^3)(Q^2 - 2Q^3 + Q^4) \quad 3.256$$

$$\frac{\partial^2 h}{\partial R^2} = (3 - 15R + 12R^2)(Q^2 - 2Q^3 + Q^4) \quad 3.257$$

$$\left(\frac{\partial^2 h}{\partial R^2}\right)^2 = (9 - 90R + 297R^2 - 360R^3 + 144R^4)(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8) \quad 3.258$$

$$\int_0^1 \int_0^1 \frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} = \left(9 - \frac{90}{2} + \frac{297}{3} - \frac{360}{4} + \frac{144}{5}\right) \left(\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9}\right) \quad 3.259$$

$$k_x = 0.002857 \quad 3.260$$

Solving a close integral for Material stiffness along x-y plane of CCCS classical rectangular plate gives:

$$k_{xy} = \int_0^a \int_0^a \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} \partial_R \partial_Q \text{ (Material stiffness)}$$

Substituting Equation (3.252) into Equation (3.156) gives:

$$\frac{\partial^2 h}{\partial R \partial Q} = (2R - 6R^2 + 4R^3)(3Q - 7.5Q^2 + 4Q^3) \quad 3.261$$

$$\begin{aligned} \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} &= (4R^2 - 24R^3 + 52R^4 - 48R^5 + 16R^6)(9Q^2 - 45Q^3 + 80.25Q^4 - 60Q^5 \\ &\quad + 16Q^6) \end{aligned} \quad 3.262$$

$$\int_0^1 \int_0^1 \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} = \left(\frac{4}{3} - \frac{24}{4} + \frac{52}{5} - \frac{48}{6} + \frac{16}{7} \right) \left(\frac{9}{3} - \frac{45}{4} + \frac{80.25}{5} - \frac{60}{6} + \frac{16}{7} \right) \quad 3.263$$

$$k_{xy} = 0.001633 \quad 3.264$$

Solving a close integral for Material stiffness along y plane of CCCS classical rectangular plate gives:

$$k_y = \int_0^a \int_0^a \frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} \partial_R \partial_Q \text{ (Material stiffness)}$$

$$\frac{\partial h}{\partial Q} = (1.5R - 2.5R^3 + R^4)(2Q - 6Q^2 + 4Q^3) \quad 3.265$$

$$\frac{\partial^2 h}{\partial Q^2} = (1.5R^2 - 2.5R^3 + R^4)(2 - 12Q + 12Q^2) \quad 3.266$$

$$\begin{aligned} \frac{\partial^3 h}{\partial Q^3} \cdot \frac{\partial h}{\partial Q} &= (2.25R^4 - 7.5R^5 + 9.25R^6 - 5R^7 + R^8)(4 - 48Q + 192Q^2 - 288Q^3 \\ &\quad + 144Q^4) \end{aligned} \quad 3.267$$

$$\int_0^1 \int_0^1 \frac{\partial^3 h}{\partial Q^3} \cdot \frac{\partial h}{\partial Q} = \left(\frac{2.25}{5} - \frac{7.5}{6} + \frac{9.25}{7} - \frac{5}{8} + \frac{1}{9} \right) \left(4 - \frac{48}{2} + \frac{192}{3} - \frac{288}{4} + \frac{144}{5} \right) \quad 3.268$$

$$k_y = 0.006032 \quad 3.269$$

With this solution of stiffness coefficient of different plate edge conditions, the Coefficient of deflection (A) of several boundary conditions of a plate can be written as:

From Equation (3.158) Coefficient of deflection of different plate edge conditions gives:

Coefficient of deflection for **SSSS** plate gives:

$$A = \frac{qa^4}{D} \left(\frac{0.04}{0.2361904762 + \frac{2}{\alpha^2} 0.235918 + \frac{1}{\alpha^4} 0.2361904762} \right) \quad 3.270$$

Coefficient of deflection for **CCCC** plate gives:

$$A = \frac{qa^4}{D} \left(\frac{0.001111}{0.00127 + \frac{2}{\alpha^2} 0.000363 + \frac{1}{\alpha^4} 0.00127} \right) \quad 3.271$$

Coefficient of deflection for **CCSS** plate gives:

$$A = \frac{qa^4}{D} \left(\frac{0.005625}{0.013571 + \frac{2}{\alpha^2} 0.007347 + \frac{1}{\alpha^4} 0.013571} \right) \quad 3.272$$

Coefficient of deflection for **CSCS** plate gives:

$$A = \frac{qa^4}{D} \left(\frac{0.006667}{0.007619 + \frac{2}{\alpha^2} 0.009252 + \frac{1}{\alpha^4} 0.039365} \right) \quad 3.273$$

Coefficient of deflection for **CSSS** plate gives:

$$A = \frac{qa^4}{D} \left(\frac{0.015}{0.03619 + \frac{2}{\alpha^2} 0.041633 + \frac{1}{\alpha^4} 0.088571} \right) \quad 3.274$$

Coefficient of deflection for CCCS plate gives:

$$A = \frac{qa^4}{D} \left(\frac{0.0025}{0.002857 + \frac{2}{\alpha^2} 0.001633 + \frac{1}{\alpha^4} 0.006032} \right) \quad 3.275$$

3.4 Determination of Critical Design Parameters of Classical Rectangular Plates under Uniformly Distributed Lateral Load

3.4.1 Serviceability Limit State of Deflection Pure Bending Analysis of Classical Rectangular Plate

From Deflection Limit State which states that the maximum deflection is less than allowable deflection and this can be mathematically written as:

$$W_{\max} < W_a \quad 3.276$$

When,

W_{\max} = Maximum deflection

W_a = Allowable deflection

3.4.1.1 Determination of Maximum Deflection Pure Bending Analysis of a Classical Rectangular Plate

From Equation (3.115) where:

$$w = Ah$$

Substitution of Equation (3.162) into Equation (3.115) gives:

$$w = \frac{qa^4}{D} \cdot K \cdot h \quad 3.277$$

Equation (3.277) is the deflection equation for pure bending analysis of a classical plate and the maximum deflection gives:

$$W_{\max} = \frac{qa^4}{D} \cdot K \cdot h_{\max} \quad 3.278$$

Equation (3.278) gives the maximum deflection for pure bending analysis of a classical plate and it's denoted as W_{\max}

When,

$h_{\max} = h$ near the center of the plate.

Substitution of W_{\max} from Equation (3.278) into the deflection Limit state condition in Equation (3.276) gives:

$$\frac{qa^4}{D} \cdot K \cdot h_{\max} < W_a \quad 3.279$$

Rewriting Equation (3.279), gives

$$\frac{qa^4}{D} \cdot K_c < W_a \quad 3.27$$

When

$K_c =$ Maximum deflection Coefficient

3.4.1.2 Determination of Critical Imposed Load for Deflection Limit State Pure Bending Analysis of Classical Rectangular Plate

Solving for the critical lateral loading from Equation (3.279) gives:

$$q < \frac{W_a \cdot D}{K \cdot h_{\max} \cdot a^4} \quad 3.280$$

Substituting Equation (3.65) into Equation (3.280) gives:

$$q < \frac{W_a \cdot t^3 \cdot E}{K \cdot h_{\max} \cdot a^4 \cdot 12 (1 - \mu^2)} \quad 3.281$$

From Euro code 1 EN 1991-1-1 and McCormac et al, (2012, 2014),

$$\text{If } q = q_s + q_i \quad 3.282$$

$$q_s = \varphi \cdot t \quad 3.283$$

When,

q = Applied load

q_s = Dead load

q_i = Imposed Load

φ = Unit weight of material

t = Thickness of the plate

Substituting Equation (3.282) into Equation (3.281) gives:

$$q_s + q_i < \frac{W_a \cdot t^3 \cdot E}{K \cdot h_{\max} \cdot a^4 \cdot 12 (1 - \mu^2)} \quad 3.284$$

Substituting Equation (3.283) into Equation (3.284) gives:

$$q_i < \frac{W_a \cdot t^3 \cdot E}{K \cdot h_{\max} \cdot a^4 \cdot 12 (1 - \mu^2)} - \varphi \cdot t \quad 3.285$$

Let

$$\phi_1 = \frac{1}{12 \cdot K \cdot h_{\max}} \quad 3.286$$

Rewriting Equation (3.285) gives:

$$q_{icD} < \phi_1 \frac{W_a \cdot t^3 \cdot E}{a^4 \cdot (1 - \mu^2)} - \varphi \cdot t \quad 3.287$$

Equation (3.287) is the critical imposed lateral load equation of classical rectangular plate with a known thickness at specified deflection.

3.4.1.3 Determination of Critical Thickness for Deflection Limit State Pure Bending Analysis of Classical Rectangular Plate

From Equation (3.281) the critical thickness (t_{cD}) equation can be derived when the load is known and it's given as:

$$\frac{q \cdot K \cdot h_{\max} \cdot a^4 \cdot 12 (1 - \mu^2)}{W_a \cdot E} < t^3 \quad 3.288$$

Rewriting Equation (3.288) gives:

$$t^3 > \frac{q \cdot K \cdot h_{\max} \cdot a^4 \cdot 12 (1 - \mu^2)}{W_a \cdot E} \quad 3.289$$

Solving for the critical thickness from Equation (3.289) gives:

$$t > \sqrt[3]{\frac{q \cdot K \cdot h_{\max} \cdot a^4 \cdot 12 (1 - \mu^2)}{W_a \cdot E}} \quad 3.290$$

Let

$$\phi_2 = \sqrt[3]{12 \cdot K \cdot h_{\max}} \quad 3.291$$

Rewriting Equation (3.290) gives:

$$t_{cD} > \phi_2 \left(\frac{q \cdot a^4 (1 - \mu^2)}{W_a \cdot E} \right)^{\frac{1}{3}} \quad 3.292$$

Equation (3.292) is the critical thickness equation of classical rectangular plate at specified lateral load and deflection.

Equation (3.287) and Equation (3.292) are the critical imposed lateral load and critical thickness of classical rectangular plate using third order energy functional.

Equation (3.287) is the critical imposed load equation (q_{iCD}), a classical rectangular plate thickness can withstand at specified thickness and deflection.

Equation (3.292) is the critical thickness equation (t_{cD}) of the rectangular plate, such that it can carry a specified lateral load at a specified deflection.

3.4.2 Ultimate Limit State of Stress Pure Bending Analysis of Classical Rectangular Plate

From Elasticity theory according to Ibearugbulem (2017), the strain energy limit state states as:

$$(\bar{U} \leq U_0) \quad 3.293$$

When,

\bar{U} = Total strain energy per volume.

U_0 = Allowable total strain energy per volume.

3.4.2.1 Determination of Strain Energy per Volume of a Classical Rectangular Plate

3.4.2.1.1 Determination of Total Strain Energy per Volume of a Classical Rectangular Plate

From Equation (3.58) total strain energy of a thin rectangular plate is:

$$U = \iiint \frac{1}{2} (\sigma_x \varepsilon_x + \tau_{yx} \cdot \gamma_{xy} + \sigma_y \varepsilon_y) \partial_x \partial_y \partial_z$$

The total strain energy of a plate can be rewritten per volume as:

$$\bar{U} = \frac{1}{2} (\sigma_x \varepsilon_x + \tau_{yx} \cdot \gamma_{xy} + \sigma_y \varepsilon_y) \quad 3.294a$$

When

\bar{U} = Total strain energy per volume.

From constitutive relations in Equation (3.37) to Equation (3.40), the stress normal to the middle plane, σ_z , is Zero, for a thin plate with small deflection and this gives:

$$\varepsilon_x = \frac{1}{E} (\sigma_x - \mu \sigma_y) \quad 3.294b$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \mu\sigma_x) \quad 3.295$$

$$\gamma_{xy} = \frac{1}{E}(2\tau_{xy} + 2\mu\tau_{yx}) \quad 3.296$$

Substituting Equation (3.294b), (3.295) and (3.296) into Equation (3.294a) gives:

$$\bar{U} = \frac{1}{2} \left(\sigma_x \left(\frac{1}{E}(\sigma_x - \mu\sigma_y) \right) + \tau_{yx} \left(\frac{1}{E}(2\tau_{xy} + 2\mu\tau_{yx}) \right) + \sigma_y \left(\frac{1}{E}(\sigma_y - \mu\sigma_x) \right) \right) \quad 3.297$$

Solving further Equation (3.297) gives:

$$\bar{U} = \frac{1}{2E} \left(\sigma_x(\sigma_x - \mu\sigma_y) + \tau_{yx}(2\tau_{xy} + 2\mu\tau_{yx}) + \sigma_y(\sigma_y - \mu\sigma_x) \right) \quad 3.298$$

Simplifying further Equation (3.298) gives:

$$\bar{U} = \frac{1}{2E} \left(\sigma_x^2 - \mu\sigma_y\sigma_x + 2\tau_{xy}^2 + 2\mu\tau_{yx}^2 + \sigma_y^2 - \mu\sigma_x\sigma_y \right) \quad 3.299$$

Simplifying further Equation (3.299) gives:

$$\bar{U} = \frac{1}{2E} \left(\sigma_x^2 + \sigma_y^2 + 2\tau_{xy}^2 + 2\mu\tau_{yx}^2 - 2\mu\sigma_y\sigma_x \right) \quad 3.300$$

Simplifying further Equation (3.300) gives:

$$\bar{U} = \frac{1}{2E} \left(\sigma_x^2 + \sigma_y^2 + 2\tau_{xy}^2 + 2\mu(\tau_{yx}^2 - \sigma_y\sigma_x) \right) \quad 3.301$$

Equation (3.301) is the Total strain energy per volume of a classical plate and it's denoted as \bar{U} .

3.4.2.1.2 Determination of Allowable Strain Energy per Volume of a Classical Rectangular Plate

Substituting Equation (3.301) into Equation (3.293) gives:

$$\frac{1}{2E} \left(\sigma_x^2 + \sigma_y^2 + 2\tau_{xy}^2 + 2\mu(\tau_{yx}^2 - \sigma_y\sigma_x) \right) \leq U_0 \quad 3.302$$

Let the stress on x axis be the material stress and this gives:

$$\sigma_x = f_y \quad 3.303$$

Let

$$\sigma_y = \tau_{yx} = 0 \quad 3.304$$

Substituting Equation (3.303) and Equation (3.304) into Equation (3.302) gives:

$$\frac{1}{2E} (f_y^2 + 0 + 0 + 0) \leq U_0 \quad 3.305$$

Simplifying further Equation (3.305) gives:

$$\frac{f_y^2}{2E} \leq U_0 \quad 3.306$$

Rewriting Equation (3.306) gives:

$$U_0 \geq \frac{f_y^2}{2E} \quad 3.307$$

Equation (3.307) is the allowable total strain energy of a classical plate and it's denoted as U_0 .

3.4.2.2 Determination of Critical Imposed Load for Elastic Limit State Pure Bending Analysis of Classical Rectangular Plate

This is done in line with the work of Ibearugbulem (2017).

Substituting allowable total strain energy in Equation (3.307) into equation (3.302) gives:

$$\frac{1}{2E} \left(\sigma_x^2 + \sigma_y^2 + 2\tau_{xy}^2 + 2\mu(\tau_{yx}^2 - \sigma_y\sigma_x) \right) \leq \frac{f_y^2}{2E} \quad 3.308$$

Simplifying Equation (3.308) gives:

$$\left(\sigma_x^2 + \sigma_y^2 + 2\tau_{xy}^2 + 2\mu(\tau_{yx}^2 - \sigma_y\sigma_x) \right) \leq f_y^2 \quad 3.309$$

Let the ratios relating σ_x , σ_y and τ_{xy} be given as:

$$\sigma_y = n_1\sigma_x \quad 3.310$$

$$\tau_{xy} = n_2\sigma_x \quad 3.311$$

Substituting Equation (3.310) and Equation (3.311) into Equation (3.309) gives:

$$\sigma_x^2 + (n_1\sigma_x)^2 + 2(n_2\sigma_x)^2 + 2\mu((n_2\sigma_x)^2 - (n_1\sigma_x)\sigma_x) \leq f_y^2 \quad 3.312$$

Simplifying Equation (3.312) gives:

$$\left(\sigma_x^2 + (n_1\sigma_x)^2 + 2(n_2\sigma_x)^2 + 2\mu(n_2\sigma_x)^2 - (2\mu n_1\sigma_x^2) \right) \leq f_y^2 \quad 3.313$$

Simplifying Equation (3.313) gives:

$$\sigma_x^2(1 + n_1^2 + 2n_2^2 + 2\mu n_2^2 - 2\mu n_1) \leq f_y^2 \quad 3.314$$

Rearranging Equation (3.314) gives:

$$\sigma_x^2 \leq \frac{f_y^2}{\left(1 + n_1^2 + 2n_2^2 + (2\mu n_2^2 - (2\mu n_1))\right)} \quad 3.315$$

Taking Square root of Equation (3.315) gives:

$$\sigma_x \leq \frac{f_y}{\sqrt{(1 + (n_1)^2 + 2n_2^2 + 2\mu n_2^2 - 2\mu n_1)}} \quad 3.316$$

Equation (3.316) can be called critical stress.

From Equation (3.310) and Equation (3.311) can rewritten as:

$$n_1 = \frac{\sigma_y}{\sigma_x} \quad 3.317$$

$$n_2 = \frac{\tau_{xy}}{\sigma_x} \quad 3.318$$

Substitution of Equation (3.52) to Equation (3.54) into Equation (3.317) and Equation (3.318)

gives:

$$n_1 = \frac{\frac{-zE}{1 - \mu^2} \left(\mu \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)}{\frac{-zE}{1 - \mu^2} \left(\mu \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2} \right)} \quad 3.319$$

$$n_2 = \frac{\frac{-zE(1 - \mu)}{(1 - \mu^2)} \cdot \frac{\partial^2 w}{\partial y \partial x}}{\frac{-zE}{1 - \mu^2} \left(\mu \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2} \right)} \quad 3.320$$

From Equation (3.115) When:

$$w = Ah$$

So, Equation (3.319) and Equation (3.320) gives:

$$n_1 = \frac{\frac{-zEA}{1-\mu^2} \left(\mu \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right)}{\frac{-zEA}{1-\mu^2} \left(\mu \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial x^2} \right)} \quad 3.321$$

$$n_2 = \frac{\frac{-zEA(1-\mu)}{(1-\mu^2)} \cdot \frac{\partial^2 h}{\partial y \partial x}}{\frac{-zEA}{1-\mu^2} \left(\mu \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial x^2} \right)} \quad 3.322$$

Substituting Equation (3.71) to Equation (3.73) into Equation (3.321) and Equation (3.322) gives:

$$n_1 = \frac{\left(\mu \frac{\partial^2 h}{\partial R^2} + \frac{\partial^2 h}{\alpha^2 \partial Q^2} \right)}{\left(\mu \frac{\partial^2 h}{\alpha^2 \partial Q^2} + \frac{\partial^2 h}{\partial R^2} \right)} \quad 3.323$$

$$n_2 = \frac{(1-\mu) \cdot \frac{\partial^2 h}{\alpha \partial R \partial Q}}{\left(\mu \frac{\partial^2 h}{\alpha^2 \partial Q^2} + \left(\frac{\partial^2 h}{\partial R^2} \right) \right)} \quad 3.324$$

Let the second derivative of the shape function stresses be denoted as:

$$\Phi_{y_1} = \frac{\partial^2 h}{\partial Q^2} \quad 3.325$$

$$\Phi_{x_1} = \frac{\partial^2 h}{\partial R^2} \quad 3.326$$

$$\Phi_{xy_1} = \frac{\partial^2 h}{\partial R \partial Q} \quad 3.327$$

Substituting Equation (3.325), (3.326), and Equation (3.327) into Equation (3.323) and Equation (3.324) gives:

$$n_1 = \frac{\left(\mu\Phi_{x_1} + \frac{1}{\alpha^2}\Phi_{y_1}\right)}{\left(\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}\right)} \quad 3.328$$

$$n_2 = \frac{(1 - \mu) \cdot \frac{1}{\alpha}\Phi_{xy_1}}{\left(\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}\right)} \quad 3.329$$

Substituting Equation (3.328) and Equation (3.329) into Equation (3.316), so the critical stress can be rewritten as:

$$\sigma_x \leq \frac{f_y}{\sqrt{1 + \left(\frac{\mu\Phi_{x_1} + \frac{1}{\alpha^2}\Phi_{y_1}}{\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}}\right)^2 + 2\left(\frac{(1 - \mu) \cdot \frac{1}{\alpha}\Phi_{xy_1}}{\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}}\right)^2 + 2\mu\left(\frac{(1 - \mu) \cdot \frac{1}{\alpha}\Phi_{xy_1}}{\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}}\right) - 2\mu\left(\frac{\mu\Phi_{x_1} + \frac{1}{\alpha^2}\Phi_{y_1}}{\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}}\right)}} \quad 3.330$$

Let

$$n = \sqrt{1 + \left(\frac{\mu\Phi_{x_1} + \frac{1}{\alpha^2}\Phi_{y_1}}{\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}}\right)^2 + 2\left(\frac{(1 - \mu) \cdot \frac{1}{\alpha}\Phi_{xy_1}}{\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}}\right)^2 + 2\mu\left(\frac{(1 - \mu) \cdot \frac{1}{\alpha}\Phi_{xy_1}}{\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}}\right) - 2\mu\left(\frac{\mu\Phi_{x_1} + \frac{1}{\alpha^2}\Phi_{y_1}}{\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}}\right)} \quad 3.331$$

Substituting Equation (3.331) into Equation (3.330) gives:

$$\sigma_x \leq \frac{f_y}{n} \quad 3.332$$

Substituting Equation (3.115) and the non-dimension in-plane coordinate (R and Q) from Equation (3.71), (3.72) and (3.73) into Equation (3.52) gives:

$$\sigma_x = \frac{-zEA}{1 - \mu^2} \left(\mu \frac{\partial^2 h}{\alpha^2 \partial Q^2} + \frac{\partial^2 h}{\partial R^2} \right) \quad 3.333$$

Substituting Equation (3.325) and Equation (3.326) into Equation (3.333) gives:

$$\sigma_x = \frac{-zEA}{1 - \mu^2} \left(\mu \frac{\Phi_{y_1}}{\alpha^2} + \Phi_{x_1} \right) \quad 3.334$$

Substituting Equation (3.334) into Equation (3.332) gives:

$$\frac{-zEA}{1 - \mu^2} \left(\mu \frac{\Phi_{y_1}}{\alpha^2} + \Phi_{x_1} \right) \leq \frac{f_y}{n} \quad 3.335$$

From Equation (3.162):

$$A = \frac{qa^4 \cdot K}{D}$$

Where K is from Equation (3.159):

$$K = \frac{k_q}{k_x + \frac{2}{\alpha^2} k_{xy} + \frac{1}{\alpha^4} k_y}$$

Considering the Mid plane, where by the thickness is divided into two equal halves gives:

$$Z = \frac{t}{2} \quad 3.336$$

Substituting Equation (3.162) and Equation (3.336) (Z and A) into Equation (3.335) gives:

$$\left(\frac{-t \cdot E \cdot q \cdot a^4 \cdot K \cdot \left(\mu \frac{1}{\alpha^2} \Phi_{y_1} + \Phi_{x_1} \right)}{2 \cdot D \cdot (1 - \mu^2)} \right) \leq \frac{f_y}{n} \quad 3.337$$

Solving for critical loading gives:

$$q \leq \frac{f_y \cdot 2 \cdot t^3 \cdot (1 - \mu^2)}{-t \cdot n \cdot 12 \cdot (1 - \mu^2) \cdot a^4 \cdot K \cdot \left(\mu \frac{1}{\alpha^2} \Phi_{y_1} + \Phi_{x_1} \right)} \quad 3.338$$

Substituting Equation (3.282) into Equation (3.338) gives:

$$q_i + q_s \leq \frac{f_y \cdot 2 \cdot t^2 \cdot (1 - \mu^2)}{-1 \cdot n \cdot 12 \cdot (1 - \mu^2) \cdot a^4 \cdot K \cdot \left(\mu \frac{1}{\alpha^2} \Phi_{y_1} + \Phi_{x_1} \right)} \quad 3.339$$

Substituting Equation (3.283) into Equation (3.339) gives:

$$q_i + (\varphi \cdot t) \leq \frac{f_y \cdot t^2 \cdot (1 - \mu^2)}{-6 \cdot K \cdot n \cdot (1 - \mu^2) \cdot a^4 \cdot \left(\mu \frac{1}{\alpha^2} \Phi_{y_1} + \Phi_{x_1} \right)} \quad 3.340$$

Let

$$\phi_3 = \frac{1}{-6 \cdot K} \quad 3.341$$

Substituting Equation (3.341) into Equation (3.340) to get the critical imposed lateral load and this gives:

$$q_{icE} \leq \frac{\phi_3 \cdot f_y \cdot t^2}{n \cdot a^4 \cdot \left(\mu \frac{1}{\alpha^2} \Phi_{y_1} + \Phi_{x_1} \right)} - (\varphi \cdot t) \quad 3.342$$

Equation (3.342) is the critical imposed lateral load equation of classical rectangular plate with a known thickness at specified material strength.

3.4.2.3 Determination of Critical Thickness for Elastic Limit State Pure Bending Analysis of Classical Rectangular Plate

From Equation (3.338) the critical thickness (t_{cE}) can be derived when the load is known and it's given as:

$$t_c^2 \geq \frac{-1 \cdot q \cdot n \cdot a^4 \cdot \left(\mu \frac{1}{\alpha^2} \Phi_{y_1} + \Phi_{x_1} \right) \cdot 12 (1 - \mu^2) \cdot k}{f_y \cdot 2 \cdot (1 - \mu^2)} \quad 3.343$$

Let,

$$\phi_4 = (6 \cdot k)^{\frac{1}{2}} \quad 3.344$$

Substituting Equation (3.344) into Equation (3.343) to get the critical thickness and this gives:

$$t_{cE} \geq \phi_4 \left(\frac{-1 \cdot q \cdot n \cdot a^4 \cdot \left(\mu \frac{1}{\alpha^2} \Phi_{y_1} + \Phi_{x_1} \right)}{f_y} \right)^{\frac{1}{2}} \quad 3.345$$

Equation (3.345) is the critical thickness equation of classical rectangular plate with a specified load and material strength.

Equation (3.342) and Equation (3.345) are the critical imposed load and critical thickness equations of a classical rectangular plates using third order energy functional.

Equation (3.342) is the critical imposed load equation (q_{icE}), a classical rectangular plate thickness can withstand at specified thickness and material strength.

Equation (3.345) is the critical thickness equation (t_{cE}) of classical rectangular plate such that it can carry a specified lateral load at a specified material strength.

Equations of critical design parameters of classical rectangular plates under uniformly distributed lateral load are:

$$q_{icD} < \phi_1 \frac{W_a \cdot t^3 \cdot E}{a^4 \cdot (1 - \mu^2)} - \phi \cdot t \quad 3.346$$

$$t_{cD} > \phi_2 \left(\frac{q \cdot a^4 (1 - \mu^2)}{W_a \cdot E} \right)^{\frac{1}{3}} \quad 3.347$$

$$q_{icE} \leq \frac{\phi_3 \cdot f_y \cdot t^2}{n \cdot a^4 \cdot \left(\mu \frac{1}{\alpha^2} \Phi_{y_1} + \Phi_{x_1} \right)} - (\phi \cdot t) \quad 3.348$$

$$t_{cE} \geq \phi_4 \left(\frac{-1 \cdot q \cdot n \cdot a^4 \cdot \left(\mu \frac{1}{\alpha^2} \Phi_{y_1} + \Phi_{x_1} \right)}{f_y} \right)^{\frac{1}{2}} \quad 3.349$$

3.4.3 Determination of Maximum Stress Coefficient for Various Boundary Conditions.

The point of maximum stress for a classical rectangular plate under uniformly distributed lateral load occurs at the center of the plate and this can be mathematically represented as:

h_{max} occurs at ($R = Q = 0.5$).

3.4.3.1 Maximum Stress Coefficient for SSSS Classical Rectangular Plate

From Equation (3.163b), the shape function of SSSS plate is given as

$$h = ((R - 2R^3 + R^4) \cdot (Q - 2Q^3 + Q^4))$$

$$\frac{\partial^2 h}{\partial R^2} = 12(R^2 - R)(Q - 2Q^3 + Q^4) \quad 3.350$$

$$\frac{\partial^2 h}{\partial R \partial Q} = (1 - 6R^2 + 4R^3)(1 - 6Q^2 + 4Q^3) \quad 3.351$$

$$\frac{\partial^2 h}{\partial Q^2} = 12(Q^2 - Q)(R - 2R^3 + R^4) \quad 3.352$$

At $(R = Q = 0.5)$ (point of maximum deflection)

$$\frac{\partial^2 h}{\partial R^2} = 12(0.5^2 - 0.5)(0.5 - [2 * 0.5^3] + 0.5^4) \quad 3.353$$

$$\Phi_{x_1} = -0.9375 \quad 3.354$$

$$\frac{\partial^2 h}{\partial R \partial Q} = (1 - [6 * 0.5^2] + [4 * 0.5^3])(1 - [6 * 0.5^2] + [4 * 0.5^3]) \quad 3.355$$

$$\Phi_{xy_1} = 0.0000 \quad 3.356$$

$$\frac{\partial^2 h}{\partial Q^2} = 12(0.5^2 - 0.5)(0.5 - [2 * 0.5^3] + 0.5^4) \quad 3.357$$

$$\Phi_{y_1} = -0.9375 \quad 3.358$$

3.4.3.2 Maximum Stress Coefficient for CCCC Classical Rectangular Plate

From Equation (3.180), the shape function of CCCC plate is given as

$$h = (R^2 - 2R^3 + R^4) \cdot (Q^2 - 2Q^3 + Q^4)$$

$$\frac{\partial^2 h}{\partial R^2} = (2 - 12R + 12R^2)(Q^2 - 2Q^3 + Q^4) \quad 3.359$$

$$\frac{\partial^2 h}{\partial R \partial Q} = (2R - 6R^2 + 4R^3)(2Q - 6Q^2 + 4Q^3) \quad 3.360$$

$$\frac{\partial^2 h}{\partial Q^2} = (2 - 12Q + 12Q^2)(R^2 - 2R^3 + R^4) \quad 3.361$$

At (R = Q = 0.5) (point of maximum deflection)

$$\frac{\partial^2 h}{\partial R^2} = (2 - [12 * 0.5] + [12 * 0.5^2])(0.5^2 - [2 * 0.5^3] + 0.5^4) \quad 3.362$$

$$\Phi_{x_1} = -0.0625 \quad 3.363$$

$$\frac{\partial^2 h}{\partial R \partial Q} = ([2 * 0.5] - [6 * 0.5^2] + [4 * 0.5^3])([2 * 0.5] - [6 * 0.5^2] + [4 * 0.5^3]) \quad 3.364$$

$$\Phi_{xy_1} = 0.0000 \quad 3.365$$

$$\frac{\partial^2 h}{\partial Q^2} = (2 - [12 * 0.5] + [12 * 0.5^2])(0.5^2 - [2 * 0.5^3] + 0.5^4) \quad 3.366$$

$$\Phi_{y_1} = -0.0625 \quad 3.367$$

3.4.3.3 Maximum Stress Coefficient for CSCS Classical Rectangular Plate

From Equation (3.198), the shape function of CSCS plate is given as

$$h = (R - 2R^3 + R^4) \cdot (Q^2 - 2Q^3 + Q^4)$$

$$\frac{\partial^2 h}{\partial R^2} = 12(R^2 - R)(Q^2 - 2Q^3 + Q^4) \quad 3.368$$

$$\frac{\partial^2 h}{\partial R \partial Q} = (1 - 6R^2 + 4R^3)(2Q - 6Q^2 + 4Q^3) \quad 3.369$$

$$\frac{\partial^2 h}{\partial Q^2} = (2 - 12Q + 12Q^2)(R - 2R^3 + R^4) \quad 3.370$$

At (R = Q = 0.5) (point of maximum deflection)

$$\frac{\partial^2 h}{\partial R^2} = 12(0.5^2 - 0.5)(0.5^2 - [2 * 0.5^3] + 0.5^4) \quad 3.371$$

$$\Phi_{x_1} = -0.1875 \quad 3.372$$

$$\frac{\partial^2 h}{\partial R \partial Q} = (1 - [6 * 0.5^2] + [4 * 0.5^3])([2 * 0.5] - [6 * 0.5^2] + [4 * 0.5^3]) \quad 3.373$$

$$\Phi_{xy_1} = 0.0000 \quad 3.374$$

$$\frac{\partial^2 h}{\partial Q^2} = (2 - [12 * 0.5] + [12 * 0.5^2])(0.5 - [2 * 0.5^3] + 0.5^4) \quad 3.375$$

$$\Phi_{y_1} = -0.3125 \quad 3.376$$

3.4.3.4 Maximum Stress Coefficient for CCSS Classical Rectangular Plate

From Equation (3.216), the shape function of CCSS plate is given as

$$h = (1.5R^2 - 2.5R^3 + R^4) \cdot (1.5Q^2 - 2.5Q^3 + Q^4)$$

$$\frac{\partial^2 h}{\partial R^2} = (3 - 15R + 12R^2)(1.5Q^2 - 2.5Q^3 + Q^4) \quad 3.377$$

$$\frac{\partial^2 h}{\partial R \partial Q} = (3R - 7.5R^2 + 4R^3)(3Q - 7.5Q^2 + 4Q^3) \quad 3.378$$

$$\frac{\partial^2 h}{\partial Q^2} = (3 - 15Q + 12Q^2)(1.5R^2 - 2.5R^3 + R^4) \quad 3.379$$

At (R = Q = 0.5)(point of maximum deflection)

$$\frac{\partial^2 h}{\partial R^2} = (3 - [15 * 0.5] + [12 * 0.5^2])([1.5 * 0.5^2] - [2.5 * 0.5^3] + 0.5^4) \quad 3.380$$

$$\Phi_{x_1} = -0.1875 \quad 3.381$$

$$\frac{\partial^2 h}{\partial R \partial Q} = ([3 * 0.5] - [7.5 * 0.5^2] + [4 * 0.5^3])([3 * 0.5] - [7.5 * 0.5^2] + [4 * 0.5^3]) \quad 3.382$$

$$\Phi_{xy_1} = 0.015625 \quad 3.383$$

$$\frac{\partial^2 h}{\partial Q^2} = (3 - [15 * 0.5] + [12 * 0.5^2])([1.5 * 0.5^2] - [2.5 * 0.5^3] + 0.5^4) \quad 3.384$$

$$\Phi_{y_1} = -0.1875 \quad 3.385$$

3.4.3.5 Maximum Stress Coefficient for CSSS Classical Rectangular Plate

From Equation (3.234), the shape function of CSSS plate is given as

$$h = (R - 2R^3 + R^4). (1.5Q^2 - 2.5Q^3 + Q^4)$$

$$\frac{\partial^2 h}{\partial R^2} = 12(R^2 - R)(1.5Q^2 - 2.5Q^3 + Q^4) \quad 3.386$$

$$\frac{\partial^2 h}{\partial R \partial Q} = (1 - 6R^2 + 4R^3)(3Q - 7.5Q^2 + 4Q^3) \quad 3.387$$

$$\frac{\partial^2 h}{\partial Q^2} = (3 - 15Q + 12Q^2)(R - 2R^3 + R^4) \quad 3.388$$

At (R = Q = 0.5)(point of maximum deflection)

$$\frac{\partial^2 h}{\partial R^2} = 12(0.5^2 - 0.5)([1.5 * 0.5^2] - [2.5 * 0.5^3] + 0.5^4) \quad 3.389$$

$$\Phi_{x_1} = -0.375 \quad 3.390$$

$$\frac{\partial^2 h}{\partial R \partial Q} = (1 - [6 * 0.5^2] + [4 * 0.5^3])([3 * 0.5] - [7.5 * 0.5^2] + [4 * 0.5^3]) \quad 3.391$$

$$\Phi_{xy_1} = 0.0000 \quad 3.392$$

$$\frac{\partial^2 h}{\partial Q^2} = (3 - [15 * 0.5] + [12 * 0.5^2])(0.5 - [2 * 0.5^3] + 0.5^4) \quad 3.393$$

$$\Phi_{y_1} = -0.46875 \quad 3.394$$

3.4.3.6 Maximum Stress Coefficient for CCCS Classical Rectangular Plate

From Equation (3.252), the shape function of CCCS plate is given as

$$h = (1.5R^2 - 2.5R^3 + R^4)(Q^2 - 2Q^3 + Q^4)$$

$$\frac{\partial^2 h}{\partial R^2} = (3 - 15R + 12R^2)(Q^2 - 2Q^3 + Q^4) \quad 3.395$$

$$\frac{\partial^2 h}{\partial R \partial Q} = (2R - 6R^2 + 4R^3)(3Q - 7.5Q^2 + 4Q^3) \quad 3.396$$

$$\frac{\partial^2 h}{\partial Q^2} = (1.5R^2 - 2.5R^3 + R^4)(2 - 12Q + 12Q^2) \quad 3.397$$

At $(R = Q = 0.5)$ (point of maximum deflection)

$$\frac{\partial^2 h}{\partial R^2} = (3 - [15 * 0.5] + [12 * 0.5^2])(0.5^2 - [2 * 0.5^3] + 0.5^4) \quad 3.398$$

$$\Phi_{x_1} = -0.09375 \quad 3.399$$

$$\frac{\partial^2 h}{\partial R \partial Q} = ([2 * 0.5] - [6 * 0.5^2] + [4 * 0.5^3])([3 * 0.5] - [7.5 * 0.5^2] + [4 * 0.5^3]) \quad 3.400$$

$$\Phi_{xy_1} = 0.0000 \quad 3.401$$

$$\frac{\partial^2 h}{\partial Q^2} = ([1.5 * 0.5^2] - [2.5 * 0.5^3] + 0.5^4)(2 - [12 * 0.5] + [12 * 0.5^2]) \quad 3.402$$

$$\Phi_{y_1} = -0.125 \quad 3.403$$

3.5 Numerical Examples

Numerical examples were performed using the critical design (limit) parameters listed in Equation (3.346), Equation (3.347), Equation (3.348) and Equation (3.349), and the parameter used for this example are as follows in Table 3.1:

Table 3.1: Parameters for Numerical Examples

SYMBOLS	VALUES
E	$207 \times 10^9 \text{ N/m}^2$
μ	0.3
φ	77 kN/m^3
a	1m
f_y	$250\text{N/mm}^2, 415\text{N/mm}^2, 500\text{N/mm}^2$
W_a	5mm, 10mm, 15mm, 20mm
t	5mm, 10mm, 15mm, 20mm, 25mm, 30mm, 35mm, 40mm
$\alpha = \frac{b}{a}$	1, 1.5, 2
q	50kN, 100kN, 150kN, 200kN, 250kN, 300kN, 350kN, 400kN

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.1 Presentation of Results

4.1.1 Third-Order Total Energy Functional of a Classical Rectangular Plate

The equation of third-order energy functional of a classical rectangular plate is presented in Equation (4.1).

$$\Pi = \frac{D}{2} \int_0^1 \int_0^1 \left(\frac{\partial^3 w}{\partial R^3} \cdot \frac{\partial w}{\partial R} + \frac{2}{\alpha^2} \cdot \frac{\partial^3 w}{\partial R \partial Q^2} \cdot \frac{\partial w}{\partial R} + \frac{1}{\alpha^4} \cdot \frac{\partial^3 w}{\partial Q^3} \cdot \frac{\partial w}{\partial Q} - \frac{2a^4 q w}{D} \right) \partial_x \partial_y \quad 4.1$$

4.1.2 Deflection Function

4.1.2.1 General Deflection Equation

The general deflection equation of classical rectangular plate obtained here is presented in Equation (4.2).

$$w = (a_0 + a_1 R + a_2 R^2 + a_3 R^3 + a_4 R^4) \cdot (b_0 + b_1 Q + b_2 Q^2 + b_3 Q^3 + b_4 Q^4) \quad 4.2$$

4.1.2.2 The Peculiar Deflection Equations for the Six Plates

The peculiar deflection equations for the six plates treated here are presented on Table 4.1.

Table 4.1: Peculiar deflection equations for the six plates

BOUNDARY CONDITION	DEFLECTED FUNCTIONS ($w = Ah$)
w_{SSSS}	$A(R - 2R^3 + R^4) \cdot (Q - 2Q^3 + Q^4)$
w_{CCCC}	$A(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)$
w_{CSCS}	$A(R - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)$
w_{CCSS}	$A(1.5R^2 - 2.5R^3 + R^4) \cdot (1.5Q^2 - 2.5Q^3 + Q^4)$
w_{CSSS}	$A(R - 2R^3 + R^4)(1.5Q^2 - 2.5Q^3 + Q^4)$

w_{cccs}	$A(1.5R^2 - 2.5R^3 + R^4) \cdot (Q^2 - 2Q^3 + Q^4)$
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4.1.3 Stiffness Coefficients

4.1.3.1 Stiffness Coefficients Equations

The equations for stiffness coefficient as derived herein are presented on Equation (4.3) to Equation (4.7).

$$K = \left(\frac{k_q}{k_x + \frac{2}{\alpha^2} k_{xy} + \frac{1}{\alpha^4} k_y} \right) \partial_R \partial_Q \quad 4.3$$

When,

$$k_q = \int_0^a \int_0^a h \partial_R \partial_Q \text{ (Load stiffness)} \quad 4.4$$

$$k_x = \int_0^a \int_0^a \frac{\partial^3 h}{\partial R^3} \cdot \frac{\partial h}{\partial R} \partial_R \partial_Q \text{ (Material stiffness in X direction)} \quad 4.5$$

$$k_{xy} = \int_0^a \int_0^a \frac{\partial^3 h}{\partial R \partial Q^2} \cdot \frac{\partial h}{\partial R} \partial_R \partial_Q \text{ (Material stiffness in x - y direction)} \quad 4.6$$

$$k_y = \int_0^a \int_0^a \frac{\partial^3 h}{\partial Q^3} \cdot \frac{\partial h}{\partial Q} \partial_R \partial_Q \text{ (Material stiffness in y direction)} \quad 4.7$$

4.1.3.2 Stiffness Coefficient Values for the Six Plates

The values of stiffness coefficients for the six plates are presented on Table 4.2.

Table 4.2: Values of stiffness coefficients for the six plates

STRESS COEFFICIENT	EDGE CONDITION					
	SSSS	CCCC	CSCS	CCSS	CSSS	CCCS
k_q	0.04	0.001111	0.006667	0.005625	0.015	0.0025
k_x	0.2361905	0.00127	0.007619	0.013571	0.03619	0.002857
k_{xy}	0.235918	0.000363	0.009252	0.007347	0.041633	0.001633
k_y	0.2361905	0.00127	0.039365	0.013571	0.088571	0.006032

The values of stress coefficients for the six plates are presented on Table 4.3.

Table 4.3: Stress coefficients for the six plates

STRESS COEFFICIENT	EDGE CONDITION					
	SSSS	CCCC	CSCS	CCSS	CSSS	CCCS
Φ_{x_1}	-0.9375	-0.0625	-0.1875	-0.1875	-0.375	-0.09375
Φ_{xy_1}	0	0	0	0.015625	0	0
Φ_{y_1}	-0.9375	-0.0625	-0.3125	-0.1875	-0.46875	-0.125

The non-dimensional deflection coefficients, K at various aspect ratios for the six plates are presented on Table 4.4.

Table 4.4: The values of non-dimensional deflection coefficients for the six plates

$\alpha = \frac{b}{a}$	Stiffness Coefficients $\mathbf{K} = \left(\frac{k_q}{k_x + \frac{2}{\alpha^2} k_{xy} + \frac{1}{\alpha^4} k_y} \right)$					
	SSSS	CCCC	CSCS	CCSS	CSSS	CCCS
0.1	1.66E-05	8.7E-05	1.69E-05	4.1E-05	1.68E-05	4.12E-05
0.2	0.000251	0.001366	0.000266	0.000635	0.000261	0.000649
0.3	0.001155	0.006688	0.001314	0.003037	0.001261	0.003189
0.4	0.003223	0.020048	0.004014	0.008851	0.003735	0.009656
0.5	0.006777	0.045358	0.009371	0.019431	0.008397	0.022235
0.6	0.011872	0.0849	0.018378	0.035355	0.015774	0.042755
0.7	0.018325	0.138165	0.031849	0.056204	0.026086	0.07216
0.8	0.025805	0.201818	0.050265	0.080746	0.039213	0.110197
0.9	0.033936	0.270845	0.0737	0.107355	0.054748	0.155445
1.0	0.042363	0.340171	0.101805	0.134454	0.072106	0.205677
1.1	0.050796	0.405856	0.13388	0.160788	0.090634	0.258368
1.2	0.059018	0.46551	0.168986	0.185522	0.109707	0.311177
1.3	0.066881	0.51813	0.206084	0.2082	0.128787	0.362239
1.4	0.074293	0.563673	0.244151	0.228653	0.147451	0.410273
1.5	0.08121	0.602648	0.282275	0.246902	0.165394	0.45454
1.6	0.087615	0.635809	0.319703	0.263076	0.182414	0.494737
1.7	0.093516	0.66397	0.355859	0.277363	0.198396	0.530863
1.8	0.098932	0.687903	0.390339	0.289964	0.21329	0.56311
1.9	0.103892	0.708293	0.422889	0.30108	0.227095	0.591777
2.0	0.108427	0.725729	0.453374	0.310899	0.239838	0.617208

4.1.4 Critical Design Parameters of Classical Rectangular Plates under Uniformly Distributed Lateral Load

The equation for maximum deflection obtained herein is presented on Equation (4.8).

$$W_{\max} = \frac{qa^4}{D} \cdot K \cdot h_{\max} \quad 4.8$$

The point of maximum deflection for a classical rectangular plate under uniformly distributed lateral load occurs at the center of the plate and this can be mathematically represented as:

$$h_{\max} \text{ occurs at } (R = Q = 0.5) \quad 4.8b$$

When h_{\max} is the value of shape function at the center of the plate.

Substituting Equation (4.8b) into Table 4.1 gives Table 4.5.

The values of h_{\max} for the six plates are presented on Table 4.5.

Table 4.5: Values of Center shape function h_{\max} for the six plates

PLATES	Center shape function (h_{\max})
SSSS	0.0977
CCCC	0.00391
CSCS	0.0195
CCSS	0.0156
CSSS	0.0391
CCCS	0.00781

The values of non-dimensional deflections at the centers of the six plates are presented on Table 4.6.

Table 4.6: The values of non-dimensional center deflection of six rectangular classical plates at various aspect ratios.

$\alpha = \frac{b}{a}$	Non-Dimensional Center Deflection ($K \cdot h_{\max}$)					
	SSSS	CCCC	CSCS	CCSS	CSSS	CCCS
0.1	1.62E-06	3.4E-07	3.29E-07	6.4E-07	6.56E-07	3.22E-07
0.2	2.45E-05	5.34E-06	5.19E-06	9.9E-06	1.02E-05	5.07E-06
0.3	0.000113	2.61E-05	2.56E-05	4.74E-05	4.93E-05	2.49E-05
0.4	0.000315	7.84E-05	7.83E-05	0.000138	0.000146	7.54E-05
0.5	0.000662	0.000177	0.000183	0.000303	0.000328	0.000174
0.6	0.00116	0.000332	0.000358	0.000552	0.000617	0.000334
0.7	0.00179	0.00054	0.000621	0.000877	0.00102	0.000564
0.8	0.002521	0.000789	0.00098	0.00126	0.001533	0.000861
0.9	0.003316	0.001059	0.001437	0.001675	0.002141	0.001214
1.0	0.004139	0.00133	0.001985	0.002097	0.002819	0.001606
1.1	0.004963	0.001587	0.002611	0.002508	0.003544	0.002018
1.2	0.005766	0.00182	0.003295	0.002894	0.00429	0.00243
1.3	0.006534	0.002026	0.004019	0.003248	0.005036	0.002829
1.4	0.007258	0.002204	0.004761	0.003567	0.005765	0.003204
1.5	0.007934	0.002356	0.005504	0.003852	0.006467	0.00355
1.6	0.00856	0.002486	0.006234	0.004104	0.007132	0.003864
1.7	0.009137	0.002596	0.006939	0.004327	0.007757	0.004146
1.8	0.009666	0.00269	0.007612	0.004523	0.00834	0.004398
1.9	0.01015	0.002769	0.008246	0.004697	0.008879	0.004622
2.0	0.010593	0.002838	0.008841	0.00485	0.009378	0.00482

4.1.4.1 Critical Imposed Load for Serviceability Limit State of Deflection

The critical imposed load equation for a specified deflection obtained herein is presented on Equation (4.9).

$$q_{icD} < \phi_1 \frac{W_a \cdot t^3 \cdot E}{a^4 \cdot (1 - \mu^2)} - \phi \cdot t \quad 4.9$$

When,

$$\phi_1 = \frac{1}{12 \cdot K \cdot h_{max}} \quad 4.10$$

The value of ϕ_1 for the six plates at various aspect ratios are present on Table 4.7.

Table 4.7: Value of ϕ_1 for the six plates at various aspect ratios

$\phi_1 = \frac{1}{12 \cdot K \cdot h_{max}}$						
$\alpha = \frac{b}{a}$	ϕ_1					
	SSSS	CCCC	CSCS	CCSS	CSSS	CCCS
0.1	51375.91	245047.6	253518.1	130287.7	127035.1	258853.8
0.2	3404.366	15599.45	16071.85	8416.71	8166.342	16451.16
0.3	738.6159	3186.894	3251.822	1759.041	1690.261	3345.443
0.4	264.657	1063.09	1064.667	603.5382	570.6745	1104.97
0.5	125.8654	469.8807	456.0509	274.9126	253.8207	479.8677
0.6	71.84629	251.0363	232.5277	151.0942	135.1098	249.5622
0.7	46.54636	154.2564	134.1823	95.04372	81.70116	147.8667
0.8	33.05338	105.6044	85.01976	66.15641	54.35222	96.82752
0.9	25.13425	78.69029	57.98551	49.75887	38.9292	68.64199
1.0	20.13427	62.65332	41.97739	39.73029	29.55769	51.87793
1.1	16.7916	52.51344	31.92045	33.22315	23.51521	41.29795

1.2	14.45236	45.78389	25.28905	28.79375	19.427	34.2894
1.3	12.75333	41.13419	20.73672	25.65742	16.54888	29.45589
1.4	11.48084	37.81068	17.50352	23.36238	14.45416	26.00728
1.5	10.50303	35.36538	15.1395	21.63567	12.88613	23.47444
1.6	9.735185	33.52088	13.36712	20.30543	11.68381	21.56718
1.7	9.120921	32.09916	12.00899	19.25953	10.7426	20.09952
1.8	8.621596	30.9824	10.94818	18.42256	9.992419	18.94849
1.9	8.210012	30.09047	10.10551	17.74237	9.385016	18.03059
2.0	7.866585	29.36755	9.426011	17.18204	8.886358	17.28767

4.1.4.2 Critical Thickness for Serviceability Limit State of Deflection

The equation for critical thickness of the plate which can withstand a specified lateral load without deflecting more than the specified deflection is presented on Equation (4.11).

$$t_{cD} > \phi_2 \left(\frac{q \cdot a^4 (1 - \mu^2)}{W_a \cdot E} \right)^{\frac{1}{3}} \quad 4.11$$

When,

$$\phi_2 = \sqrt[3]{12 \cdot K \cdot h_{\max}} \quad 4.12$$

The value of ϕ_2 for the six plates at various aspect ratios are present on Table 4.8.

Table 4.8: Value of ϕ_2 for the six plates at various aspect ratios

$\phi_2 = \sqrt[3]{12 \cdot K \cdot h_{\max}}$						
$\alpha = \frac{b}{a}$	ϕ_2					
	SSSS	CCCC	CSCS	CCSS	CSSS	CCCS
0.1	0.0269	0.01598	0.0158	0.019726	0.019893	0.015691

0.2	0.066474	0.040022	0.039626	0.049161	0.049658	0.039319
0.3	0.110627	0.067953	0.067498	0.08284	0.083949	0.066862
0.4	0.155754	0.097981	0.097933	0.118331	0.12056	0.096727
0.5	0.199541	0.128629	0.129916	0.153792	0.15794	0.12773
0.6	0.240546	0.158521	0.16262	0.187752	0.194882	0.158833
0.7	0.277996	0.18646	0.19533	0.219125	0.230457	0.189108
0.8	0.311598	0.211563	0.227419	0.247254	0.263994	0.217771
0.9	0.341385	0.23336	0.258361	0.27188	0.295059	0.244232
1.0	0.367582	0.251778	0.287736	0.293062	0.323427	0.268126
1.1	0.390513	0.26704	0.315242	0.311066	0.349047	0.289306
1.2	0.410538	0.27953	0.340687	0.326263	0.37199	0.307808
1.3	0.428015	0.289689	0.363988	0.339049	0.392413	0.323799
1.4	0.443277	0.29794	0.385146	0.349806	0.410521	0.337522
1.5	0.456627	0.304654	0.404231	0.358875	0.426539	0.349249
1.6	0.46833	0.310143	0.421361	0.366547	0.440695	0.359254
1.7	0.478616	0.314656	0.436681	0.373065	0.453207	0.367794
1.8	0.487683	0.318392	0.450353	0.378631	0.464276	0.375095
1.9	0.4957	0.321507	0.462538	0.383409	0.474084	0.381356
2.0	0.502811	0.324124	0.473395	0.387533	0.482791	0.386742

4.1.4.3 Critical Imposed Load for Ultimate Limit State of Stress

The equation of critical imposed load of a rectangular classical plate at specified material strength and thickness obtained here is presented on Equation (4.13).

$$q_{icE} \leq \frac{\phi_3 \cdot f_y \cdot t^2}{n \cdot a^4 \cdot \left(\mu \frac{1}{\alpha^2} \Phi_{y_1} + \Phi_{x_1} \right)} - (\varphi \cdot t) \quad 4.13$$

When,

$$\phi_3 = \frac{1}{-6 \cdot K} \quad 4.14$$

n

$$= \sqrt{1 + \left(\frac{\mu \Phi_{x_1} + \frac{1}{\alpha^2} \Phi_{y_1}}{\mu \frac{1}{\alpha^2} \Phi_{y_1} + \Phi_{x_1}} \right)^2 + 2 \left(\frac{(1-\mu) \cdot \frac{1}{\alpha} \Phi_{xy_1}}{\mu \frac{1}{\alpha^2} \Phi_{y_1} + \Phi_{x_1}} \right)^2 + 2\mu \left(\frac{(1-\mu) \cdot \frac{1}{\alpha} \Phi_{xy_1}}{\mu \frac{1}{\alpha^2} \Phi_{y_1} + \Phi_{x_1}} \right)^2 - 2\mu \left(\frac{\mu \Phi_{x_1} + \frac{1}{\alpha^2} \Phi_{y_1}}{\mu \frac{1}{\alpha^2} \Phi_{y_1} + \Phi_{x_1}} \right)} \quad 4.15$$

The value of ϕ_3 for the six plates at various aspect ratios are present on Table 4.9.

Table 4.9: Value of ϕ_3 for the six plates at various aspect ratios

$\phi_3 = \frac{1}{-6 \cdot K}$						
$\alpha = \frac{b}{a}$	ϕ_3					
	SSSS	CCCC	CSCS	CCSS	CSSS	CCCS
0.1	-10039	-1916.3	-9887.2	-4065	-9934.1	-4043.3
0.2	-665.21	-121.99	-626.8	-262.6	-638.61	-256.97
0.3	-144.33	-24.922	-126.82	-54.882	-132.18	-52.256
0.4	-51.714	-8.3134	-41.522	-18.83	-44.627	-17.26
0.5	-24.594	-3.6745	-17.786	-8.5773	-19.849	-7.4955
0.6	-14.039	-1.9631	-9.0686	-4.7141	-10.566	-3.8982
0.7	-9.0952	-1.2063	-5.2331	-2.9654	-6.389	-2.3097
0.8	-6.4586	-0.8258	-3.3158	-2.0641	-4.2503	-1.5125
0.9	-4.9112	-0.6154	-2.2614	-1.5525	-3.0443	-1.0722
1.0	-3.9342	-0.49	-1.6371	-1.2396	-2.3114	-0.8103

1.1	-3.2811	-0.4107	-1.2449	-1.0366	-1.8389	-0.6451
1.2	-2.824	-0.358	-0.9863	-0.8984	-1.5192	-0.5356
1.3	-2.492	-0.3217	-0.8087	-0.8005	-1.2941	-0.4601
1.4	-2.2434	-0.2957	-0.6826	-0.7289	-1.1303	-0.4062
1.5	-2.0523	-0.2766	-0.5904	-0.675	-1.0077	-0.3667
1.6	-1.9023	-0.2621	-0.5213	-0.6335	-0.9137	-0.3369
1.7	-1.7822	-0.251	-0.4684	-0.6009	-0.8401	-0.314
1.8	-1.6847	-0.2423	-0.427	-0.5748	-0.7814	-0.296
1.9	-1.6042	-0.2353	-0.3941	-0.5536	-0.7339	-0.2816
2.0	-1.5371	-0.2297	-0.3676	-0.5361	-0.6949	-0.27

4.1.4.4 Critical Thickness for Ultimate Limit State of Stress

The equation of critical thickness of rectangular classical plate for a specified imposed lateral load and material strength obtained here is presented on Equation (4.16).

$$t_{cE} \geq \phi_4 \left(\frac{-1. q. n . a^4 . \left(\mu \frac{1}{\alpha^2} \Phi_{y_1} + \Phi_{x_1} \right)}{f_y} \right)^{\frac{1}{2}} \quad 4.16$$

When,

$$\phi_4 = (6. k)^{\frac{1}{2}} \quad 4.17$$

The value of ϕ_4 for the six plates at various aspect ratios are present on Table 4.10.

Table 4.10: Value of ϕ_4 for the six plates at various aspect ratios

$\alpha = \frac{b}{a}$	$\phi_4 = (\mathbf{6.k})^{\frac{1}{2}}$					
	ϕ_4					
	SSSS	CCCC	CSCS	CCSS	CSSS	CCCS
0.1	0.00998	0.02284	0.01006	0.01569	0.01003	0.01573
0.2	0.03877	0.09054	0.03994	0.06171	0.03957	0.06238
0.3	0.08324	0.20032	0.0888	0.13499	0.08698	0.13834
0.4	0.13906	0.34683	0.15519	0.23045	0.14969	0.24071
0.5	0.20164	0.52168	0.23712	0.34145	0.22446	0.36526
0.6	0.26689	0.71372	0.33207	0.46057	0.30765	0.50649
0.7	0.33159	0.91049	0.43714	0.58071	0.39562	0.658
0.8	0.39349	1.10041	0.54917	0.69604	0.48505	0.81313
0.9	0.45124	1.27478	0.66498	0.80258	0.57314	0.96575
1.0	0.50416	1.42865	0.78156	0.89818	0.65775	1.11088
1.1	0.55207	1.56049	0.89626	0.98221	0.73743	1.24507
1.2	0.59507	1.67125	1.00694	1.05505	0.81132	1.36641
1.3	0.63347	1.76317	1.11198	1.11768	0.87905	1.47426
1.4	0.66765	1.83903	1.21033	1.17129	0.94059	1.56896
1.5	0.69804	1.90155	1.3014	1.21713	0.99617	1.65144
1.6	0.72505	1.95317	1.385	1.25637	1.04618	1.72291
1.7	0.74906	1.99595	1.46122	1.29003	1.09104	1.78471
1.8	0.77045	2.0316	1.53037	1.31901	1.13126	1.83811
1.9	0.78953	2.06149	1.5929	1.34406	1.16729	1.88432
2.0	0.80658	2.08671	1.64932	1.36579	1.1996	1.92438

The coefficient values of Material stress Φ_{y_1} on y plane of classical rectangular plate on pure bending analysis are presented on Table 4.11.

Table 4.11: Coefficient of Material stress of Φ_{y_1} for the six plates at various aspect ratios

Coefficient Values of Material Stress on y Plane $\frac{1}{\alpha^2} \Phi_{y_1}$						
$\alpha = \frac{b}{a}$	SSSS	CCCC	CSCS	CCSS	CSSS	CCCS
0.1	-93.75	-6.25	-31.25	-18.75	-46.875	-12.5
0.2	-23.438	-1.5625	-7.8125	-4.6875	-11.7188	-3.125
0.3	-10.417	-0.6944	-3.4722	-2.0833	-5.20833	-1.3889
0.4	-5.8594	-0.3906	-1.9531	-1.1719	-2.92969	-0.7813
0.5	-3.75	-0.25	-1.25	-0.75	-1.875	-0.5
0.6	-2.6042	-0.1736	-0.8681	-0.5208	-1.30208	-0.3472
0.7	-1.9133	-0.1276	-0.6378	-0.3827	-0.95663	-0.2551
0.8	-1.4648	-0.0977	-0.4883	-0.293	-0.73242	-0.1953
0.9	-1.1574	-0.0772	-0.3858	-0.2315	-0.5787	-0.1543
1.0	-0.9375	-0.0625	-0.3125	-0.1875	-0.46875	-0.125
1.1	-0.7748	-0.0517	-0.2583	-0.155	-0.3874	-0.1033
1.2	-0.651	-0.0434	-0.217	-0.1302	-0.32552	-0.0868
1.3	-0.5547	-0.037	-0.1849	-0.111	-0.27737	-0.074
1.4	-0.4783	-0.0319	-0.1594	-0.0957	-0.23916	-0.0638
1.5	-0.4167	-0.0278	-0.1389	-0.0833	-0.20833	-0.0556
1.6	-0.3662	-0.0244	-0.1221	-0.0732	-0.18311	-0.0488
1.7	-0.3244	-0.0216	-0.1081	-0.0649	-0.1622	-0.0433
1.8	-0.2894	-0.0193	-0.0965	-0.0579	-0.14468	-0.0386

1.9	-0.2597	-0.0173	-0.0866	-0.0519	-0.12985	-0.0346
2.0	-0.2344	-0.0156	-0.0781	-0.0469	-0.11719	-0.0313

The coefficient values of Material stress Φ_{xy_1} on x – y plane of classical rectangular plate on pure bending analysis are presented on Table 4.12.

Table 4.12: Coefficients of Material stresses Φ_{xy_1} on x – y plane of classical rectangular plate on pure bending analysis.

Coefficient Values of Material Stress On x – y Plane $\left(\frac{1}{\alpha} \Phi_{xy_1}\right)$						
$\alpha = \frac{b}{a}$	SSSS	CCCC	CSCS	CCSS	CSSS	CCCS
0.1	0	0	0	0.15625	0	0
0.2	0	0	0	0.07813	0	0
0.3	0	0	0	0.05208	0	0
0.4	0	0	0	0.03906	0	0
0.5	0	0	0	0.03125	0	0
0.6	0	0	0	0.02604	0	0
0.7	0	0	0	0.02232	0	0
0.8	0	0	0	0.01953	0	0
0.9	0	0	0	0.01736	0	0
1.0	0	0	0	0.01563	0	0
1.1	0	0	0	0.01421	0	0
1.2	0	0	0	0.01302	0	0
1.3	0	0	0	0.01202	0	0
1.4	0	0	0	0.01116	0	0

1.5	0	0	0	0.01042	0	0
1.6	0	0	0	0.00977	0	0
1.7	0	0	0	0.00919	0	0
1.8	0	0	0	0.00868	0	0
1.9	0	0	0	0.00822	0	0
2.0	0	0	0	0.00781	0	0

4.1.5 Numerical Example Results for Classical Rectangular Plates

4.1.5.1 Lateral Imposed Loads for Classical Rectangular Plates

Table 4.13: Value on **n** with Poisson's ratio, $\mu = 0.3$

n						
$\alpha = \frac{b}{a}$	SSSS	CCCC	CSCS	CCSS	CSSS	CCCS
0.1	13.41	7.786	12.183	11.151	12.769	10.071
0.2	11.645	3.5057	8.5644	6.7678	9.8784	5.3903
0.3	9.5144	2.236	5.7346	4.1996	7.1392	3.2611
0.4	7.5574	1.788	4.0311	2.9483	5.1856	2.366
0.5	5.993	1.5901	3.0611	2.3201	3.9289	1.9471
0.6	4.8256	1.4871	2.4961	1.9785	3.1365	1.726
0.7	3.9791	1.4269	2.1513	1.7772	2.6286	1.5971
0.8	3.3689	1.3888	1.9299	1.6501	2.2929	1.516
0.9	2.9259	1.3631	1.7811	1.5652	2.0634	1.4617
1	2.6	1.3449	1.677	1.5058	1.9013	1.4236
1.1	2.3564	1.3316	1.6015	1.4628	1.7834	1.3959
1.2	2.1711	1.3216	1.5452	1.4305	1.6953	1.3751
1.3	2.0279	1.3138	1.5021	1.4058	1.6279	1.3591

1.4	1.9153	1.3077	1.4684	1.3864	1.5753	1.3465
1.5	1.8256	1.3028	1.4416	1.3709	1.5334	1.3364
1.6	1.7531	1.2988	1.4198	1.3583	1.4997	1.3282
1.7	1.6937	1.2955	1.402	1.348	1.472	1.3214
1.8	1.6446	1.2927	1.3872	1.3393	1.4491	1.3158
1.9	1.6035	1.2903	1.3748	1.3321	1.4299	1.311
2	1.5687	1.2883	1.3643	1.3259	1.4136	1.3069

Table 4.14: Lateral imposed loads for SSSS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for SSSS							
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1$						
	q_{icD} (kN)				q_{icE} (kN)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
5	2.4775	5.33999	8.20249	11.065	7.37484	12.4963	15.1347
10	22.13	45.0299	67.9299	90.8299	30.2694	50.7553	61.3087
15	76.1324	153.42	230.707	307.995	68.6835	114.777	138.522
20	181.66	364.86	548.059	731.259	122.617	204.561	246.775
25	355.887	713.699	1071.51	1429.32	192.071	320.108	386.067
30	615.989	1234.29	1852.59	2470.89	277.044	461.418	556.398
35	979.141	1960.98	2942.81	3924.65	377.537	628.49	757.769
40	1462.52	2928.12	4393.72	5859.31	493.55	821.325	990.179

Table 4.15: Lateral imposed loads for CCCC classical rectangular plate on aspect ratio of 1, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for CCCC							
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1$						
	q_{icD} (kN)				q_{icE} (kN)		

	W _a 5mm	W _a 10 mm	W _a 15 mm	W _a 20 mm	f _y 250N/mm ²	f _y 415N/mm ²	f _y 500N/mm ²
5	8.52244	17.4299	26.3373	35.2448	27.638	46.1331	55.661
10	70.4896	141.749	213.009	284.268	111.322	185.303	223.414
15	239.346	479.847	720.348	960.849	251.052	417.508	503.259
20	568.536	1138.61	1708.69	2278.77	446.828	742.75	895.195
25	1111.51	2224.94	3338.37	4451.8	698.649	1161.03	1399.22
30	1921.7	3845.71	5769.71	7693.72	1006.52	1672.34	2015.34
35	3052.56	6107.81	9163.07	12218.3	1370.43	2276.69	2743.56
40	4557.53	9118.14	13678.8	18239.4	1790.39	2974.08	3583.86

Table 4.16: Lateral imposed loads for CSCS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for CSCS							
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1$						
	q _{icD} (kN)				q _{icE} (kN)		
	W _a 5mm	W _a 10 mm	W _a 15 mm	W _a 20 mm	f _y 250N/mm ²	f _y 415N/mm ²	f _y 500N/mm ²
5	5.58294	11.5509	17.5188	23.4868	21.3094	35.6277	43.0037
10	46.9735	94.717	142.461	190.204	86.0075	143.281	172.785
15	159.979	321.114	482.248	643.382	194.094	322.959	389.344
20	380.408	762.356	1144.3	1526.25	345.57	574.663	692.68
25	744.067	1490.06	2236.05	2982.05	540.434	898.391	1082.79
30	1286.77	2575.84	3864.92	5153.99	778.687	1294.15	1559.69
35	2044.31	4091.31	6138.31	8185.32	1060.33	1761.93	2123.35
40	3052.51	6108.09	9163.68	12219.3	1385.36	2301.73	2773.8

Table 4.17: Lateral imposed loads for CCSS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for CCSS							
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1$						
	q _{icD} (kN)				q _{icE} (kN)		

	W _a 5mm	W _a 10 mm	W _a 15 mm	W _a 20 mm	f _y 250N/mm ²	f _y 415N/mm ²	f _y 500N/mm ²
5	5.26347	10.9119	16.5604	22.2089	20.7224	34.6533	41.8298
10	44.4178	89.6055	134.793	179.981	83.6597	139.383	168.089
15	151.354	303.862	456.371	608.88	188.812	314.19	378.778
20	359.962	721.464	1082.97	1444.47	336.179	559.073	673.897
25	704.134	1410.19	2116.25	2822.31	525.76	874.033	1053.45
30	1217.76	2437.83	3657.9	4877.97	757.557	1259.07	1517.42
35	1934.73	3872.16	5809.58	7747.01	1031.57	1714.18	2065.83
40	2888.94	5780.95	8672.97	11565	1347.79	2239.37	2698.67

Table 4.18: Lateral imposed loads for CSSS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for CSSS							
t(mm)	Aspect ration $\alpha = \frac{b}{a} = 1$						
	q _{icD} (kN)				q _{icE} (kN)		
	W _a 5mm	W _a 10 mm	W _a 15 mm	W _a 20 mm	f _y 250N/mm ²	f _y 415N/mm ²	f _y 500N/mm ²
5	3.81723	8.01945	12.2217	16.4239	14.3506	24.0761	29.0862
10	32.8478	66.4656	100.083	133.701	58.1723	97.0743	117.115
15	112.305	225.765	339.225	452.685	131.465	218.995	264.086
20	267.403	536.345	805.287	1074.23	234.229	389.837	469.999
25	523.353	1048.63	1573.91	2099.19	366.465	609.602	734.854
30	905.371	1813.05	2720.73	3628.41	528.171	878.288	1058.65
35	1438.67	2880.03	4321.4	5762.76	719.349	1195.9	1441.39
40	1438.67	4300	6451.54	8603.08	939.997	1562.43	1883.07

Table 4.19: Lateral imposed loads for CCCS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for CCCS							
t(mm)	Aspect ration $\alpha = \frac{b}{a} = 1$						
	q _{icD} (kN)				q _{icE} (kN)		

	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
5	6.9905	14.366	21.7415	29.117	26.7202	44.6097	53.8255
10	58.234	117.238	176.242	235.246	107.651	179.209	216.072
15	197.984	397.122	596.261	795.399	242.792	403.797	486.739
20	470.492	942.524	1414.56	1886.59	432.144	718.375	865.827
25	920.013	1841.95	2763.89	3685.83	675.706	1122.94	1353.34
30	1590.8	3183.91	4777.02	6370.13	973.478	1617.5	1949.27
35	2527.1	5056.9	7586.7	10116.5	1325.46	2202.04	2653.62
40	3773.18	7549.44	11325.7	15102	1731.65	2876.58	3466.39

Table 4.20: Lateral imposed loads for SSSS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for SSSS							
t(mm)	Aspect ration $\alpha = \frac{b}{a} = 1.5$						
	q_{icD} (kN)				q_{icE} (kN)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
5	1.10822	2.60144	4.09466	5.58788	6.22784	10.5923	12.8407
10	11.1758	23.1215	35.0673	47.013	25.6814	43.1393	52.1327
15	39.1619	79.4789	119.796	160.113	58.3606	97.6408	117.876
20	94.0261	189.592	285.158	380.724	104.265	174.097	210.071
25	184.728	371.38	558.032	744.685	163.396	272.508	328.717
30	320.226	642.761	965.296	1287.83	235.752	392.873	473.815
35	509.479	1021.65	1533.83	2046	321.334	535.193	645.363
40	761.449	1525.98	2290.51	3055.03	420.142	699.468	843.364

Table 4.21: Lateral imposed loads for CCCC classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for CCCC							
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$						
	q_{icD} (kN)				q_{icE} (kN)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
5	4.64291	9.67082	14.6987	19.7266	18.3457	30.708	37.0765
10	39.4533	79.6765	119.9	160.123	74.1529	123.602	149.076
15	134.599	270.352	406.106	541.859	167.422	278.682	335.998
20	320.246	642.032	963.818	1285.6	298.152	495.948	597.844
25	626.564	1255.05	1883.54	2512.03	466.343	775.4	934.612
30	1083.72	2169.75	3255.77	4341.8	671.996	1117.04	1346.3
35	1721.88	3446.45	5171.02	6895.6	915.111	1520.86	1832.92
40	2571.21	5145.5	7719.79	10294.1	1195.69	1986.87	2394.45

Table 4.22: Lateral imposed loads for CSCS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for CSCS							
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$						
	q_{icD} (kN)				q_{icE} (kN)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
5	1.76739	3.91978	6.07216	8.22455	10.7855	18.1581	21.9561
10	16.4491	33.6682	50.8873	68.1064	43.9121	73.4023	88.5942
15	56.9595	115.074	173.188	231.303	99.3797	165.733	199.915
20	136.213	273.966	411.719	549.471	177.188	295.149	355.917
25	267.124	536.172	805.22	1074.27	277.338	461.652	556.601
30	462.606	927.522	1392.44	1857.35	399.829	665.241	801.968
35	735.574	1473.84	2212.11	2950.38	544.661	905.916	1092.02
40	1098.94	2200.97	3302.99	4405.01	711.834	1183.68	1426.75

Table 4.23: Lateral imposed loads for CCSS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for CCSS							
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$						
	q_{icD} (kN)				q_{icE} (kN)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
5	2.69095	5.7669	8.84285	11.9188	14.0974	23.6557	28.5797
10	23.8376	48.4452	73.0528	97.6604	57.1595	95.3929	115.089
15	81.8957	164.946	247.997	331.048	129.186	215.212	259.528
20	195.321	392.182	589.042	785.903	230.178	383.112	461.896
25	382.569	767.063	1151.56	1536.05	360.134	599.093	722.193
30	662.095	1326.5	1990.91	2655.31	519.055	863.156	1040.42
35	1052.36	2107.41	3162.46	4217.51	706.941	1175.3	1416.58
40	1571.81	3146.69	4721.58	6296.47	923.791	1535.53	1850.66

Table 4.24: Lateral imposed loads for CSSS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for CSSS							
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$						
	q_{icD} (kN)				q_{icE} (kN)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
5	1.44703	3.27905	5.11108	6.9431	9.00281	15.1988	18.3906
10	13.8862	28.5424	43.1986	57.8548	36.7812	61.565	74.3324
15	48.3097	97.7744	147.239	196.704	83.3353	139.099	167.826
20	115.71	232.959	350.209	467.459	148.665	247.8	298.87
25	227.078	456.082	685.085	914.088	232.77	387.669	467.465
30	393.408	789.125	1184.84	1580.56	335.651	558.705	673.612
35	625.69	1254.08	1882.46	2510.85	457.308	760.909	917.31
40	625.69	1872.92	2810.91	3748.91	597.74	994.28	1198.56

Table 4.25: Lateral imposed loads for CCCS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for CCCS							
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$						
	q _{icD} (kN)				q _{icE} (kN)		
	W _a 5mm	W _a 10 mm	W _a 15 mm	W _a 20 mm	f _y 250N/mm ²	f _y 415N/mm ²	f _y 500N/mm ²
5	2.95237	6.28974	9.62711	12.9645	15.1456	25.3959	30.6763
10	25.929	52.6279	79.3269	106.026	61.3525	102.353	123.475
15	88.954	179.063	269.172	359.281	138.621	230.873	278.396
20	212.052	425.643	639.235	852.827	246.95	410.954	495.44
25	415.246	832.417	1249.59	1666.76	386.341	642.596	774.607
30	718.562	1439.43	2160.31	2881.18	556.793	925.801	1115.9
35	1142.02	2286.74	3431.46	4576.18	758.306	1260.57	1519.31
40	1705.65	3414.39	5123.12	6831.85	990.88	1646.89	1984.84

Table 4.26: Lateral imposed loads for SSSS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for SSSS							
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 2$						
	q _{icD} (kN)				q _{icE} (kN)		
	W _a 5mm	W _a 10 mm	W _a 15 mm	W _a 20 mm	f _y 250N/mm ²	f _y 415N/mm ²	f _y 500N/mm ²
5	0.7334	1.85179	2.97019	4.08858	5.69159	9.70215	11.7682
10	8.17716	17.1243	26.0715	35.0186	23.5364	39.5786	47.8427
15	29.0417	59.2383	89.435	119.632	53.5343	89.6293	108.224
20	70.0373	141.615	213.192	284.769	95.6855	159.854	192.911
25	137.874	277.674	417.473	557.273	149.99	250.254	301.905
30	239.263	480.837	722.41	963.983	216.447	360.827	435.205
35	380.915	764.524	1148.13	1531.74	295.058	491.575	592.811
40	569.538	1142.16	1714.78	2287.39	385.822	642.497	774.724

Table 4.27 Lateral imposed loads for CCCC classical rectangular plate on aspect ratio of 2, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for CCCC							
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 2$						
	q _{icD} (kN)				q _{icE} (kN)		
	W _a 5mm	W _a 10 mm	W _a 15 mm	W _a 20 mm	f _y 250N/mm ²	f _y 415N/mm ²	f _y 500N/mm ²
5	3.79019	7.96539	12.1406	16.3158	16.197	27.1411	32.779
10	32.6316	66.0331	99.4347	132.836	65.5579	109.334	131.886
15	111.575	224.306	337.036	449.766	148.083	246.58	297.321
20	265.672	532.885	800.097	1067.31	263.772	438.877	529.083
25	519.974	1041.87	1563.77	2085.67	412.625	686.227	827.174
30	899.532	1801.37	2703.22	3605.06	594.641	988.629	1191.59
35	1429.4	2861.49	4293.58	5725.67	809.822	1346.08	1622.34
40	2134.62	4272.32	6410.02	8547.72	1058.17	1758.59	2119.41

Table 4.28: Lateral imposed loads for CSCS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for CSCS							
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 2$						
	q _{icD} (kN)				q _{icE} (kN)		
	W _a 5mm	W _a 10 mm	W _a 15 mm	W _a 20 mm	f _y 250N/mm ²	f _y 415N/mm ²	f _y 500N/mm ²
5	0.9551	2.2952	3.6353	4.9754	7.59906	12.8685	15.5831
10	9.95079	20.6716	31.3924	42.1132	31.1662	52.2441	63.1024
15	35.0277	71.2104	107.393	143.576	70.7015	118.127	142.558
20	84.2263	169.993	255.759	341.525	126.205	210.517	253.95
25	165.587	333.1	500.612	668.125	197.676	329.413	397.278
30	287.151	576.613	866.074	1155.54	285.116	474.817	572.542
35	456.959	916.613	1376.27	1835.92	388.524	646.728	779.742
40	683.051	1369.18	2055.31	2741.44	507.9	845.146	1018.88

Table 4.29: Lateral imposed loads for CCSS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for CCSS							
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 2$						
	q_{icD} (kN)				q_{icE} (kN)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
5	2.05778	4.50055	6.94333	9.38611	12.152	20.4264	24.6889
10	18.7722	38.3144	57.8566	77.3988	49.3779	82.4755	99.5258
15	64.8	130.755	196.71	262.665	111.678	186.147	224.511
20	154.798	311.135	467.473	623.811	199.052	331.442	399.643
25	303.422	608.769	914.116	1219.46	311.499	518.359	624.924
30	525.33	1052.97	1580.61	2108.25	449.021	746.899	900.352
35	835.177	1673.05	2510.92	3348.79	611.617	1017.06	1225.93
40	1247.62	2498.32	3749.02	4999.73	799.286	1328.85	1601.65

Table 4.30: Lateral imposed loads for CSSS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for CSSS							
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 2$						
	q_{icD} (kN)				q_{icE} (kN)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
5	0.87838	2.14175	3.40513	4.66851	7.10598	12.05	14.597
10	9.33701	19.444	29.551	39.6581	29.1939	48.9701	59.1579
15	32.9562	67.0673	101.179	135.29	66.2638	110.76	133.683
20	79.3161	160.172	241.028	321.884	118.316	197.421	238.171
25	155.997	313.919	471.841	629.763	185.35	308.951	372.624
30	270.579	543.469	816.358	1089.25	267.365	445.351	537.041
35	430.643	863.981	1297.32	1730.66	364.363	606.622	731.421
40	430.643	1290.62	1937.47	2584.32	476.343	792.762	955.766

Table 4.31: Lateral imposed loads for CCCS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, thicknesses and material strengths.

Lateral imposed loads for CCCS							
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 2$						
	q _{icD} (kN)				q _{icE} (kN)		
	W _a 5mm	W _a 10 mm	W _a 15 mm	W _a 20 mm	f _y 250N/mm ²	f _y 415N/mm ²	f _y 500N/mm ²
5	2.07279	4.53059	6.98838	9.44617	12.1371	20.4017	24.6592
10	18.8923	38.5547	58.217	77.8794	49.3184	82.3768	99.4068
15	65.2054	131.566	197.926	264.287	111.544	185.925	224.243
20	65.2054	313.058	470.356	627.655	198.814	331.047	399.167
25	305.299	612.523	919.747	1226.97	311.128	517.742	624.18
30	528.573	1059.46	1590.34	2121.22	448.486	746.011	899.281
35	840.328	1683.35	2526.37	3369.4	610.888	1015.85	1224.47
40	1255.31	2513.7	3772.09	5030.48	798.335	1327.27	1599.75

4.1.5.2 Thicknesses for Classical Rectangular Plates

Table 4.32: Thicknesses for SSSS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for SSSS							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1$						
	t _{cD} (m)				t _{cE} (m)		
	W _a 5mm	W _a 10 mm	W _a 15 mm	W _a 20 mm	f _y 250N/mm ²	f _y 415N/mm ²	f _y 500N/mm ²
50	0.01297	0.0103	0.009	0.00817	0.01269	0.00985	0.00898
100	0.01635	0.01297	0.01133	0.0103	0.01795	0.01393	0.01269
150	0.01871	0.01485	0.01297	0.01179	0.02198	0.01706	0.01554
200	0.02059	0.01635	0.01428	0.01297	0.02538	0.0197	0.01795
250	0.02218	0.01761	0.01538	0.01398	0.02838	0.02203	0.02007
300	0.02357	0.01871	0.01635	0.01485	0.03109	0.02413	0.02198
350	0.02482	0.0197	0.01721	0.01563	0.03358	0.02606	0.02374

400	0.02595	0.02059	0.01799	0.01635	0.0359	0.02786	0.02538
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Table 4.33: Thicknesses for CCCC classical rectangular plate on aspect ratio of 1, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for CCCC							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1$						
	t_{cD} (m)				t_{cE} (m)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
50	0.00889	0.00705	0.00616	0.0056	0.00668	0.00518	0.00472
100	0.0112	0.00889	0.00776	0.00705	0.00945	0.00733	0.00668
150	0.01282	0.01017	0.00889	0.00807	0.01157	0.00898	0.00818
200	0.01411	0.0112	0.00978	0.00889	0.01336	0.01037	0.00945
250	0.0152	0.01206	0.01054	0.00957	0.01493	0.01159	0.01056
300	0.01615	0.01282	0.0112	0.01017	0.01636	0.0127	0.01157
350	0.017	0.01349	0.01179	0.01071	0.01767	0.01372	0.0125
400	0.01777	0.01411	0.01232	0.0112	0.01889	0.01466	0.01336

Table 4.34: Thicknesses for CSCS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for CSCS							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1$						
	t_{cD} (m)				t_{cE} (m)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
50	0.01016	0.00806	0.00704	0.0064	0.00759	0.00589	0.00537
100	0.0128	0.01016	0.00887	0.00806	0.01074	0.00833	0.00759
150	0.01465	0.01163	0.01016	0.00923	0.01315	0.0102	0.0093
200	0.01612	0.0128	0.01118	0.01016	0.01518	0.01178	0.01074
250	0.01737	0.01378	0.01204	0.01094	0.01697	0.01317	0.012
300	0.01845	0.01465	0.0128	0.01163	0.01859	0.01443	0.01315
350	0.01943	0.01542	0.01347	0.01224	0.02008	0.01559	0.0142

400	0.02031	0.01612	0.01408	0.0128	0.02147	0.01666	0.01518
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Table 4.35: Thicknesses for CCSS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for CCSS							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1$						
	t_{cD} (m)				t_{cE} (m)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
50	0.01034	0.00821	0.00717	0.00652	0.0077	0.00597	0.00544
100	0.01303	0.01034	0.00904	0.00821	0.01088	0.00845	0.0077
150	0.01492	0.01184	0.01034	0.0094	0.01333	0.01035	0.00943
200	0.01642	0.01303	0.01138	0.01034	0.01539	0.01195	0.01088
250	0.01769	0.01404	0.01226	0.01114	0.01721	0.01336	0.01217
300	0.0188	0.01492	0.01303	0.01184	0.01885	0.01463	0.01333
350	0.01979	0.0157	0.01372	0.01246	0.02036	0.0158	0.0144
400	0.02069	0.01642	0.01434	0.01303	0.02177	0.01689	0.01539

Table 4.36: Thicknesses for CSSS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for CSSS							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1$						
	t_{cD} (m)				t_{cE} (m)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
50	0.01142	0.00906	0.00792	0.00719	0.01154	0.00715	0.00651
100	0.01438	0.01142	0.00997	0.00906	0.01632	0.01011	0.00921
150	0.01646	0.01307	0.01142	0.01037	0.01999	0.01238	0.01128
200	0.01812	0.01438	0.01256	0.01142	0.02308	0.0143	0.01303
250	0.01952	0.01549	0.01353	0.0123	0.0258	0.01599	0.01456
300	0.02074	0.01646	0.01438	0.01307	0.02827	0.01751	0.01595
350	0.02184	0.01733	0.01514	0.01376	0.03053	0.01891	0.01723

400	0.02283	0.01812	0.01583	0.01438	0.03264	0.02022	0.01842
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Table 4.37: Thicknesses for CCCS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for CCCS							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1$						
	t_{cD} (m)				t_{cE} (m)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
50	0.00946	0.01365	0.00656	0.00596	0.00679	0.00527	0.0048
100	0.01192	0.01437	0.00827	0.00751	0.0096	0.00745	0.00679
150	0.01365	0.01502	0.00946	0.0086	0.01176	0.00913	0.00832
200	0.01502	0.01365	0.01042	0.00946	0.01358	0.01054	0.0096
250	0.01618	0.01437	0.01122	0.01019	0.01519	0.01179	0.01074
300	0.0172	0.01502	0.01192	0.01083	0.01663	0.01291	0.01176
350	0.0181	0.01365	0.01255	0.0114	0.01797	0.01395	0.01271
400	0.01893	0.01437	0.01312	0.01192	0.01921	0.01491	0.01358

Table 4.38: Thicknesses for SSSS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for SSSS							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$						
	t_{cD} (m)				t_{cE} (m)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
50	0.01612	0.01279	0.01117	0.01015	0.01375	0.01067	0.00972
100	0.02031	0.01612	0.01408	0.01279	0.01944	0.01509	0.01375
150	0.02324	0.01845	0.01612	0.01464	0.02381	0.01848	0.01684
200	0.02558	0.02031	0.01774	0.01612	0.0275	0.02134	0.01944
250	0.02756	0.02187	0.01911	0.01736	0.03074	0.02386	0.02174
300	0.02928	0.02324	0.02031	0.01845	0.03368	0.02614	0.02381
350	0.03083	0.02447	0.02138	0.01942	0.03638	0.02823	0.02572

400	0.03223	0.02558	0.02235	0.02031	0.03889	0.03018	0.0275
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Table 4.39: Thicknesses for CCCC classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for CCCC							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$						
	t_{cD} (m)				t_{cE} (m)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
50	0.01075	0.00853	0.00746	0.00677	0.00817	0.00634	0.00578
100	0.01355	0.01075	0.00939	0.00853	0.01155	0.00897	0.00817
150	0.01551	0.01231	0.01075	0.00977	0.01415	0.01098	0.01001
200	0.01707	0.01355	0.01183	0.01075	0.01634	0.01268	0.01155
250	0.01839	0.01459	0.01275	0.01158	0.01827	0.01418	0.01292
300	0.01954	0.01551	0.01355	0.01231	0.02001	0.01553	0.01415
350	0.02057	0.01633	0.01426	0.01296	0.02161	0.01678	0.01528
400	0.0215	0.01707	0.01491	0.01355	0.02311	0.01793	0.01634

Table 4.40: Thicknesses for CSCS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for CSCS							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$						
	t_{cD} (m)				t_{cE} (m)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
50	0.01427	0.01132	0.00989	0.00899	0.01058	0.00821	0.00748
100	0.01798	0.01427	0.01246	0.01132	0.01496	0.01161	0.01058
150	0.02058	0.01633	0.01427	0.01296	0.01832	0.01422	0.01296
200	0.02265	0.01798	0.0157	0.01427	0.02116	0.01642	0.01496
250	0.0244	0.01936	0.01692	0.01537	0.02365	0.01836	0.01673
300	0.02592	0.02058	0.01798	0.01633	0.02591	0.02011	0.01832
350	0.02729	0.02166	0.01892	0.01719	0.02799	0.02172	0.01979

400	0.02853	0.02265	0.01978	0.01798	0.02992	0.02322	0.02116
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Table 4.41: Thicknesses for CCSS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for CCSS							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$						
	t_{cD} (m)				t_{cE} (m)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
50	0.01267	0.01005	0.00878	0.00798	0.00929	0.00721	0.00657
100	0.01596	0.01267	0.01107	0.01005	0.01314	0.0102	0.00929
150	0.01827	0.0145	0.01267	0.01151	0.01609	0.01249	0.01138
200	0.02011	0.01596	0.01394	0.01267	0.01858	0.01442	0.01314
250	0.02166	0.01719	0.01502	0.01364	0.02077	0.01612	0.01469
300	0.02302	0.01827	0.01596	0.0145	0.02276	0.01766	0.01609
350	0.02423	0.01923	0.0168	0.01526	0.02458	0.01908	0.01738
400	0.02533	0.02011	0.01756	0.01596	0.02628	0.0204	0.01858

Table 4.42: Thicknesses for CSSS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for CSSS							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$						
	t_{cD} (m)				t_{cE} (m)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
50	0.01267	0.01005	0.00878	0.00798	0.00929	0.00721	0.00657
100	0.01596	0.01267	0.01107	0.01005	0.01314	0.0102	0.00929
150	0.01827	0.0145	0.01267	0.01151	0.01609	0.01249	0.01138
200	0.02011	0.01596	0.01394	0.01267	0.01858	0.01442	0.01314
250	0.02166	0.01719	0.01502	0.01364	0.02077	0.01612	0.01469
300	0.02302	0.01827	0.01596	0.0145	0.02276	0.01766	0.01609
350	0.02423	0.01923	0.0168	0.01526	0.02458	0.01908	0.01738

400	0.02533	0.02011	0.01756	0.01596	0.02628	0.0204	0.01858
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Table 4.43: Thicknesses for CCCS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for CCCS							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$						
	t_{cD} (m)				t_{cE} (m)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
50	0.01233	0.01778	0.00855	0.00777	0.00897	0.00696	0.00634
100	0.01553	0.01872	0.01077	0.00978	0.01269	0.00985	0.00897
150	0.01778	0.01957	0.01233	0.0112	0.01554	0.01206	0.01099
200	0.01957	0.01778	0.01357	0.01233	0.01794	0.01393	0.01269
250	0.02108	0.01872	0.01461	0.01328	0.02006	0.01557	0.01419
300	0.0224	0.01957	0.01553	0.01411	0.02198	0.01706	0.01554
350	0.02358	0.01778	0.01635	0.01485	0.02374	0.01842	0.01678
400	0.02465	0.01872	0.01709	0.01553	0.02538	0.0197	0.01794

Table 4.44: Thicknesses for SSSS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for SSSS							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 2$						
	t_{cD} (m)				t_{cE} (m)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
50	0.01775	0.01409	0.0123	0.01118	0.01434	0.01113	0.01014
100	0.02236	0.01775	0.0155	0.01409	0.02028	0.01574	0.01434
150	0.02559	0.02031	0.01775	0.01612	0.02484	0.01928	0.01757
200	0.02817	0.02236	0.01953	0.01775	0.02869	0.02226	0.02028
250	0.03035	0.02409	0.02104	0.01912	0.03207	0.02489	0.02268
300	0.03225	0.02559	0.02236	0.02031	0.03513	0.02727	0.02484
350	0.03395	0.02694	0.02354	0.02139	0.03795	0.02945	0.02683

400	0.03549	0.02817	0.02461	0.02236	0.04057	0.03149	0.02869
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Table 4.45: Thicknesses for CCCC classical rectangular plate on aspect ratio of 2, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for CCCC							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 2$						
	t_{cD} (m)				t_{cE} (m)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
50	0.01144	0.00908	0.00793	0.00721	0.00868	0.00674	0.00614
100	0.01441	0.01144	0.00999	0.00908	0.01228	0.00953	0.00868
150	0.0165	0.0131	0.01144	0.01039	0.01504	0.01167	0.01063
200	0.01816	0.01441	0.01259	0.01144	0.01737	0.01348	0.01228
250	0.01956	0.01553	0.01356	0.01232	0.01941	0.01507	0.01373
300	0.02079	0.0165	0.01441	0.0131	0.02127	0.01651	0.01504
350	0.02188	0.01737	0.01517	0.01379	0.02297	0.01783	0.01624
400	0.02288	0.01816	0.01586	0.01441	0.02456	0.01906	0.01737

Table 4.46: Thicknesses for CSCS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for CSCS							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 2$						
	t_{cD} (m)				t_{cE} (m)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
50	0.01671	0.01326	0.01158	0.01053	0.01251	0.00971	0.00885
100	0.02105	0.01671	0.0146	0.01326	0.0177	0.01373	0.01251
150	0.0241	0.01913	0.01671	0.01518	0.02167	0.01682	0.01533
200	0.02652	0.02105	0.01839	0.01671	0.02503	0.01942	0.0177
250	0.02857	0.02268	0.01981	0.018	0.02798	0.02172	0.01978
300	0.03036	0.0241	0.02105	0.01913	0.03065	0.02379	0.02167
350	0.03196	0.02537	0.02216	0.02013	0.03311	0.02569	0.02341

400	0.03342	0.02652	0.02317	0.02105	0.03539	0.02747	0.02503
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Table 4.47: Thicknesses for CCSS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for CCSS							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 2$						
	t_{cD} (m)				t_{cE} (m)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
50	0.01368	0.01086	0.00948	0.00862	0.00999	0.00775	0.00706
100	0.01723	0.01368	0.01195	0.01086	0.01412	0.01096	0.00999
150	0.01973	0.01566	0.01368	0.01243	0.0173	0.01342	0.01223
200	0.02171	0.01723	0.01505	0.01368	0.01997	0.0155	0.01412
250	0.02339	0.01856	0.01622	0.01473	0.02233	0.01733	0.01579
300	0.02485	0.01973	0.01723	0.01566	0.02446	0.01898	0.0173
350	0.02616	0.02077	0.01814	0.01648	0.02642	0.02051	0.01868
400	0.02735	0.02171	0.01897	0.01723	0.02824	0.02192	0.01997

Table 4.48: Thicknesses for CSSS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for CSSS							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 2$						
	t_{cD} (m)				t_{cE} (m)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
50	0.01704	0.01352	0.01181	0.01073	0.00921	0.01003	0.00913
100	0.02147	0.01704	0.01489	0.01352	0.01303	0.01418	0.01292
150	0.02458	0.01951	0.01704	0.01548	0.01595	0.01737	0.01582
200	0.02705	0.02147	0.01875	0.01704	0.01842	0.02005	0.01827
250	0.02914	0.02313	0.0202	0.01836	0.0206	0.02242	0.02043
300	0.03096	0.02458	0.02147	0.01951	0.02256	0.02456	0.02237
350	0.0326	0.02587	0.0226	0.02053	0.02437	0.02653	0.02417

400	0.03408	0.02705	0.02363	0.02147	0.02605	0.02836	0.02584
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Table 4.49: Thicknesses for CCCS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, lateral loads and material strengths.

Thicknesses for CCCS							
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 2$						
	t_{cD} (m)				t_{cE} (m)		
	W_a 5mm	W_a 10 mm	W_a 15 mm	W_a 20 mm	f_y 250N/mm ²	f_y 415N/mm ²	f_y 500N/mm ²
50	0.01365	0.01969	0.00946	0.0086	0.00999	0.00776	0.00707
100	0.0172	0.02072	0.01192	0.01083	0.01413	0.01097	0.00999
150	0.01969	0.02167	0.01365	0.0124	0.01731	0.01343	0.01224
200	0.02167	0.01969	0.01502	0.01365	0.01998	0.01551	0.01413
250	0.02334	0.02072	0.01618	0.0147	0.02234	0.01734	0.0158
300	0.0248	0.02167	0.0172	0.01563	0.02447	0.019	0.01731
350	0.02611	0.01969	0.0181	0.01645	0.02643	0.02052	0.01869
400	0.0273	0.02072	0.01893	0.0172	0.02826	0.02193	0.01998

4.1.5.3 Critical Lateral Imposed Loads for Classical Rectangular Plates

Table 4.50: Critical lateral imposed loads for SSSS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1$					
	$W_a = 5\text{mm}$	$f_y = 250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 5\text{mm}$	$f_y = 415\text{N/mm}^2$	q_{ic} (kN)
5	2.4775	7.37484	2.4775	2.4775	12.4963	2.4775
10	22.13	30.2694	22.13	22.13	50.7553	22.13
15	76.1324	68.6835	68.6835	76.1324	114.777	76.1324
20	181.66	122.617	122.617	181.66	204.561	181.66
25	355.887	192.071	192.071	355.887	320.108	320.108
30	615.989	277.044	277.044	615.989	461.418	461.418
35	979.141	377.537	377.537	979.141	628.49	628.49

40	1462.52	493.55	493.55	1462.52	821.325	821.325
t(mm)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	2.4775	15.1347	2.4775	5.33999	7.37484	5.33999
10	22.13	61.3087	22.13	45.0299	30.2694	30.2694
15	76.1324	138.522	76.1324	153.42	68.6835	68.6835
20	181.66	246.775	181.66	364.86	122.617	122.617
25	355.887	386.067	355.887	713.699	192.071	192.071
30	615.989	556.398	556.398	1234.29	277.044	277.044
35	979.141	757.769	757.769	1960.98	377.537	377.537
40	1462.52	990.179	990.179	2928.12	493.55	493.55
t(mm)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	5.33999	12.4963	5.33999	5.33999	15.1347	5.33999
10	45.0299	50.7553	45.0299	45.0299	61.3087	45.0299
15	153.42	114.777	114.777	153.42	138.522	138.522
20	364.86	204.561	204.561	364.86	246.775	246.775
25	713.699	320.108	320.108	713.699	386.067	386.067
30	1234.29	461.418	461.418	1234.29	556.398	556.398
35	1960.98	628.49	628.49	1960.98	757.769	757.769
40	2928.12	821.325	821.325	2928.12	990.179	990.179
t(mm)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	8.20249	7.37484	7.37484	8.20249	12.4963	8.20249
10	67.9299	30.2694	30.2694	67.9299	50.7553	50.7553
15	230.707	68.6835	68.6835	230.707	114.777	114.777
20	548.059	122.617	122.617	548.059	204.561	204.561
25	1071.51	192.071	192.071	1071.51	320.108	320.108
30	1852.59	277.044	277.044	1852.59	461.418	461.418
35	2942.81	377.537	377.537	2942.81	628.49	628.49
40	4393.72	493.55	493.55	4393.72	821.325	821.325
t(mm)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	8.20249	15.1347	8.20249	11.065	7.37484	7.37484
10	67.9299	61.3087	61.3087	90.8299	30.2694	30.2694
15	230.707	138.522	138.522	307.995	68.6835	68.6835
20	548.059	246.775	246.775	731.259	122.617	122.617
25	1071.51	386.067	386.067	1429.32	192.071	192.071
30	1852.59	556.398	556.398	2470.89	277.044	277.044
35	2942.81	757.769	757.769	3924.65	377.537	377.537
40	4393.72	990.179	990.179	5859.31	493.55	493.55
t(mm)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	11.065	12.4963	11.065	11.065	15.1347	11.065
10	90.8299	50.7553	50.7553	90.8299	61.3087	61.3087
15	307.995	114.777	114.777	307.995	138.522	138.522

20	731.259	204.561	204.561	731.259	246.775	246.775
25	1429.32	320.108	320.108	1429.32	386.067	386.067
30	2470.89	461.418	461.418	2470.89	556.398	556.398
35	3924.65	628.49	628.49	3924.65	757.769	757.769
40	5859.31	821.325	821.325	5859.31	990.179	990.179

Table 4.51: Critical lateral imposed loads for CCCC classical rectangular plate on aspect ratio of 1, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1$					
	$W_a = 5\text{mm}$	$f_y = 250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 5\text{mm}$	$f_y = 415\text{N/mm}^2$	q_{ic} (kN)
5	8.52244	27.638	8.52244	8.52244	46.1331	8.52244
10	70.4896	111.322	70.4896	70.4896	185.303	70.4896
15	239.346	251.052	239.346	239.346	417.508	239.346
20	568.536	446.828	446.828	568.536	742.75	568.536
25	1111.51	698.649	698.649	1111.51	1161.03	1111.51
30	1921.7	1006.52	1006.52	1921.7	1672.34	1672.34
35	3052.56	1370.43	1370.43	3052.56	2276.69	2276.69
40	4557.53	1790.39	1790.39	4557.53	2974.08	2974.08
t(mm)	$W_a = 5\text{mm}$	$f_y = 500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y = 250\text{N/mm}^2$	q_{ic} (kN)
5	8.52244	55.661	8.52244	17.4299	27.638	17.4299
10	70.4896	223.414	70.4896	141.749	111.322	111.322
15	239.346	503.259	239.346	479.847	251.052	251.052
20	568.536	895.195	568.536	1138.61	446.828	446.828
25	1111.51	1399.22	1111.51	2224.94	698.649	698.649
30	1921.7	2015.34	1921.7	3845.71	1006.52	1006.52
35	3052.56	2743.56	2743.56	6107.81	1370.43	1370.43
40	4557.53	3583.86	3583.86	9118.14	1790.39	1790.39
t(mm)	$W_a = 10\text{mm}$	$f_y = 415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y = 500\text{N/mm}^2$	q_{ic} (kN)
5	17.4299	46.1331	17.4299	17.4299	55.661	17.4299
10	141.749	185.303	141.749	141.749	223.414	141.749
15	479.847	417.508	417.508	479.847	503.259	479.847
20	1138.61	742.75	742.75	1138.61	895.195	895.195
25	2224.94	1161.03	1161.03	2224.94	1399.22	1399.22
30	3845.71	1672.34	1672.34	3845.71	2015.34	2015.34
35	6107.81	2276.69	2276.69	6107.81	2743.56	2743.56
40	9118.14	2974.08	2974.08	9118.14	3583.86	3583.86

t(mm)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	26.3373	27.638	26.3373	26.3373	46.1331	26.3373
10	213.009	111.322	111.322	213.009	185.303	185.303
15	720.348	251.052	251.052	720.348	417.508	417.508
20	1708.69	446.828	446.828	1708.69	742.75	742.75
25	3338.37	698.649	698.649	3338.37	1161.03	1161.03
30	5769.71	1006.52	1006.52	5769.71	1672.34	1672.34
35	9163.07	1370.43	1370.43	9163.07	2276.69	2276.69
40	13678.8	1790.39	1790.39	13678.8	2974.08	2974.08
t(mm)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	26.3373	55.661	26.3373	35.2448	27.638	27.638
10	213.009	223.414	213.009	284.268	111.322	111.322
15	720.348	503.259	503.259	960.849	251.052	251.052
20	1708.69	895.195	895.195	2278.77	446.828	446.828
25	3338.37	1399.22	1399.22	4451.8	698.649	698.649
30	5769.71	2015.34	2015.34	7693.72	1006.52	1006.52
35	9163.07	2743.56	2743.56	12218.3	1370.43	1370.43
40	13678.8	3583.86	3583.86	18239.4	1790.39	1790.39
t(mm)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	35.2448	46.1331	35.2448	35.2448	55.661	35.2448
10	284.268	185.303	185.303	284.268	223.414	223.414
15	960.849	417.508	417.508	960.849	503.259	503.259
20	2278.77	742.75	742.75	2278.77	895.195	895.195
25	4451.8	1161.03	1161.03	4451.8	1399.22	1399.22
30	7693.72	1672.34	1672.34	7693.72	2015.34	2015.34
35	12218.3	2276.69	2276.69	12218.3	2743.56	2743.56
40	18239.4	2974.08	2974.08	18239.4	3583.86	3583.86

Table 4.52: Critical lateral imposed loads for CSCS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	5.58294	21.3094	5.58294	5.58294	35.6277	5.58294
10	46.9735	86.0075	46.9735	46.9735	143.281	46.9735
15	159.979	194.094	159.979	159.979	322.959	159.979

20	380.408	345.57	345.57	380.408	574.663	380.408
25	744.067	540.434	540.434	744.067	898.391	744.067
30	1286.77	778.687	778.687	1286.77	1294.15	1286.77
35	2044.31	1060.33	1060.33	2044.31	1761.93	1761.93
40	3052.51	1385.36	1385.36	3052.51	2301.73	2301.73
t(mm)	W _a = 5mm	f _y =500N/mm ²	q _{ic} (kN)	W _a = 10mm	f _y =250N/mm ²	q _{ic} (kN)
5	5.58294	43.0037	5.58294	11.5509	21.3094	11.5509
10	46.9735	172.785	46.9735	94.717	86.0075	86.0075
15	159.979	389.344	159.979	321.114	194.094	194.094
20	380.408	692.68	380.408	762.356	345.57	345.57
25	744.067	1082.79	744.067	1490.06	540.434	540.434
30	1286.77	1559.69	1286.77	2575.84	778.687	778.687
35	2044.31	2123.35	2044.31	4091.31	1060.33	1060.33
40	3052.51	2773.8	2773.8	6108.09	1385.36	1385.36
t(mm)	W _a = 10mm	f _y =415N/mm ²	q _{ic} (kN)	W _a = 10mm	f _y =500N/mm ²	q _{ic} (kN)
5	11.5509	35.6277	11.5509	11.5509	43.0037	11.5509
10	94.717	143.281	94.717	94.717	172.785	94.717
15	321.114	322.959	321.114	321.114	389.344	321.114
20	762.356	574.663	574.663	762.356	692.68	692.68
25	1490.06	898.391	898.391	1490.06	1082.79	1082.79
30	2575.84	1294.15	1294.15	2575.84	1559.69	1559.69
35	4091.31	1761.93	1761.93	4091.31	2123.35	2123.35
40	6108.09	2301.73	2301.73	6108.09	2773.8	2773.8
t(mm)	W _a = 15mm	f _y =250N/mm ²	q _{ic} (kN)	W _a = 15mm	f _y =415N/mm ²	q _{ic} (kN)
5	17.5188	21.3094	17.5188	17.5188	35.6277	17.5188
10	142.461	86.0075	86.0075	142.461	143.281	142.461
15	482.248	194.094	194.094	482.248	322.959	322.959
20	1144.3	345.57	345.57	1144.3	574.663	574.663
25	2236.05	540.434	540.434	2236.05	898.391	898.391
30	3864.92	778.687	778.687	3864.92	1294.15	1294.15
35	6138.31	1060.33	1060.33	6138.31	1761.93	1761.93
40	9163.68	1385.36	1385.36	9163.68	2301.73	2301.73
t(mm)	W _a = 15mm	f _y =500N/mm ²	q _{ic} (kN)	W _a = 20mm	f _y =250N/mm ²	q _{ic} (kN)
5	17.5188	21.3094	17.5188	17.5188	35.6277	17.5188
10	142.461	86.0075	86.0075	142.461	143.281	142.461
15	482.248	194.094	194.094	482.248	322.959	322.959
20	1144.3	345.57	345.57	1144.3	574.663	574.663
25	2236.05	540.434	540.434	2236.05	898.391	898.391
30	3864.92	778.687	778.687	3864.92	1294.15	1294.15
35	6138.31	1060.33	1060.33	6138.31	1761.93	1761.93
40	9163.68	1385.36	1385.36	9163.68	2301.73	2301.73

t(mm)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	23.4868	35.6277	23.4868	23.4868	43.0037	23.4868
10	190.204	143.281	143.281	190.204	172.785	172.785
15	643.382	322.959	322.959	643.382	389.344	389.344
20	1526.25	574.663	574.663	1526.25	692.68	692.68
25	2982.05	898.391	898.391	2982.05	1082.79	1082.79
30	5153.99	1294.15	1294.15	5153.99	1559.69	1559.69
35	8185.32	1761.93	1761.93	8185.32	2123.35	2123.35
40	12219.3	2301.73	2301.73	12219.3	2773.8	2773.8

Table 4.53: Critical lateral imposed loads for CCSS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	5.26347	20.7224	5.26347	5.26347	34.6533	5.26347
10	44.4178	83.6597	44.4178	44.4178	139.383	44.4178
15	151.354	188.812	151.354	151.354	314.19	151.354
20	359.962	336.179	336.179	359.962	559.073	359.962
25	704.134	525.76	525.76	704.134	874.033	704.134
30	1217.76	757.557	757.557	1217.76	1259.07	1217.76
35	1934.73	1031.57	1031.57	1934.73	1714.18	1714.18
40	2888.94	1347.79	1347.79	2888.94	2239.37	2239.37
t(mm)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	5.26347	41.8298	5.26347	10.9119	20.7224	10.9119
10	44.4178	168.089	44.4178	89.6055	83.6597	83.6597
15	151.354	378.778	151.354	303.862	188.812	188.812
20	359.962	673.897	359.962	721.464	336.179	336.179
25	704.134	1053.45	704.134	1410.19	525.76	525.76
30	1217.76	1517.42	1217.76	2437.83	757.557	757.557
35	1934.73	2065.83	1934.73	3872.16	1031.57	1031.57
40	2888.94	2698.67	2698.67	5780.95	1347.79	1347.79
t(mm)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	10.9119	34.6533	10.9119	10.9119	41.8298	10.9119
10	89.6055	139.383	89.6055	89.6055	168.089	89.6055
15	303.862	314.19	303.862	303.862	378.778	303.862

20	721.464	559.073	559.073	721.464	673.897	673.897
25	1410.19	874.033	874.033	1410.19	1053.45	1053.45
30	2437.83	1259.07	1259.07	2437.83	1517.42	1517.42
35	3872.16	1714.18	1714.18	3872.16	2065.83	2065.83
40	5780.95	2239.37	2239.37	5780.95	2698.67	2698.67
t(mm)	W _a = 15mm	f _y =250N/mm ²	q _{ic} (kN)	W _a = 15mm	f _y =415N/mm ²	q _{ic} (kN)
5	16.5604	20.7224	16.5604	16.5604	34.6533	16.5604
10	134.793	83.6597	83.6597	134.793	139.383	134.793
15	456.371	188.812	188.812	456.371	314.19	314.19
20	1082.97	336.179	336.179	1082.97	559.073	559.073
25	2116.25	525.76	525.76	2116.25	874.033	874.033
30	3657.9	757.557	757.557	3657.9	1259.07	1259.07
35	5809.58	1031.57	1031.57	5809.58	1714.18	1714.18
40	5780.95	1347.79	1347.79	5780.95	2239.37	2239.37
t(mm)	W _a = 15mm	f _y =500N/mm ²	q _{ic} (kN)	W _a = 20mm	f _y =250N/mm ²	q _{ic} (kN)
5	16.5604	41.8298	16.5604	22.2089	20.7224	20.7224
10	134.793	168.089	134.793	179.981	83.6597	83.6597
15	456.371	378.778	378.778	608.88	188.812	188.812
20	1082.97	673.897	673.897	1444.47	336.179	336.179
25	2116.25	1053.45	1053.45	2822.31	525.76	525.76
30	3657.9	1517.42	1517.42	4877.97	757.557	757.557
35	5809.58	2065.83	2065.83	7747.01	1031.57	1031.57
40	5780.95	2698.67	2698.67	11565	1347.79	1347.79
t(mm)	W _a = 20mm	f _y =415N/mm ²	q _{ic} (kN)	W _a = 20mm	f _y =500N/mm ²	q _{ic} (kN)
5	22.2089	34.6533	22.2089	22.2089	41.8298	22.2089
10	179.981	139.383	139.383	179.981	168.089	168.089
15	608.88	314.19	314.19	608.88	378.778	378.778
20	1444.47	559.073	559.073	1444.47	673.897	673.897
25	2822.31	874.033	874.033	2822.31	1053.45	1053.45
30	4877.97	1259.07	1259.07	4877.97	1517.42	1517.42
35	7747.01	1714.18	1714.18	7747.01	2065.83	2065.83
40	11565	2239.37	2239.37	11565	2698.67	2698.67

Table 4.54: Critical lateral imposed loads for CSSS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q _{ic})	
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1$

	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	3.81723	14.3506	3.81723	3.81723	24.0761	3.81723
10	32.8478	58.1723	32.8478	32.8478	97.0743	32.8478
15	112.305	131.465	112.305	112.305	218.995	112.305
20	267.403	234.229	234.229	267.403	389.837	267.403
25	523.353	366.465	366.465	523.353	609.602	523.353
30	905.371	528.171	528.171	905.371	878.288	878.288
35	1438.67	719.349	719.349	1438.67	1195.9	1195.9
40	2148.46	939.997	939.997	2148.46	1562.43	1562.43
t(mm)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	3.81723	29.0862	3.81723	8.01945	14.3506	8.01945
10	32.8478	117.115	32.8478	66.4656	58.1723	58.1723
15	112.305	264.086	112.305	225.765	131.465	131.465
20	267.403	469.999	267.403	536.345	234.229	234.229
25	523.353	734.854	523.353	1048.63	366.465	366.465
30	905.371	1058.65	905.371	1813.05	528.171	528.171
35	1438.67	1441.39	1438.67	2880.03	719.349	719.349
40	2148.46	1883.07	1438.67	4300	939.997	939.997
t(mm)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	8.01945	24.0761	8.01945	8.01945	29.0862	8.01945
10	66.4656	97.0743	66.4656	66.4656	117.115	66.4656
15	225.765	218.995	218.995	225.765	264.086	225.765
20	536.345	389.837	389.837	536.345	469.999	469.999
25	1048.63	609.602	609.602	1048.63	734.854	734.854
30	1813.05	878.288	878.288	1813.05	1058.65	1058.65
35	2880.03	1195.9	1195.9	2880.03	1441.39	1441.39
40	4300	1562.43	1562.43	4300	1883.07	1883.07
t(mm)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	12.2217	14.3506	12.2217	12.2217	24.0761	12.2217
10	100.083	58.1723	58.1723	100.083	97.0743	97.0743
15	339.225	131.465	131.465	339.225	218.995	218.995
20	805.287	234.229	234.229	805.287	389.837	389.837
25	1573.91	366.465	366.465	1573.91	609.602	609.602
30	2720.73	528.171	528.171	2720.73	878.288	878.288
35	4321.4	719.349	719.349	4321.4	1195.9	1195.9
40	6451.54	939.997	939.997	6451.54	1562.43	1562.43
t(mm)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	12.2217	29.0862	12.2217	16.4239	14.3506	16.4239
10	100.083	117.115	100.083	133.701	58.1723	58.1723
15	339.225	264.086	264.086	452.685	131.465	131.465
20	805.287	469.999	469.999	1074.23	234.229	234.229

25	1573.91	734.854	734.854	2099.19	366.465	366.465
30	2720.73	1058.65	1058.65	3628.41	528.171	528.171
35	4321.4	1441.39	1441.39	5762.76	719.349	719.349
40	6451.54	1883.07	1883.07	8603.08	939.997	939.997
t(mm)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	16.4239	24.0761	16.4239	16.4239	29.0862	16.4239
10	133.701	97.0743	97.0743	133.701	117.115	117.115
15	452.685	218.995	218.995	452.685	264.086	264.086
20	1074.23	389.837	389.837	1074.23	469.999	469.999
25	2099.19	609.602	609.602	2099.19	734.854	734.854
30	3628.41	878.288	878.288	3628.41	1058.65	1058.65
35	5762.76	1195.9	1195.9	5762.76	1441.39	1441.39
40	8603.08	1562.43	1562.43	8603.08	1883.07	1883.07

Table 4.55: Critical lateral imposed loads for CCCS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	6.9905	26.7202	6.9905	6.9905	44.6097	6.9905
10	58.234	107.651	58.234	58.234	179.209	58.234
15	197.984	242.792	197.984	197.984	403.797	197.984
20	470.492	432.144	432.144	470.492	718.375	470.492
25	920.013	675.706	675.706	920.013	1122.94	920.013
30	1590.8	973.478	973.478	1590.8	1617.5	1590.8
35	2527.1	1325.46	1325.46	2527.1	2202.04	2202.04
40	3773.18	1731.65	1731.65	3773.18	2876.58	2876.58
t(mm)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	6.9905	53.8255	6.9905	14.366	26.7202	14.366
10	58.234	216.072	58.234	117.238	107.651	107.651
15	197.984	486.739	197.984	397.122	242.792	242.792
20	470.492	865.827	470.492	942.524	432.144	432.144
25	920.013	1353.34	920.013	1841.95	675.706	675.706
30	1590.8	1949.27	1590.8	3183.91	973.478	973.478
35	2527.1	2653.62	2527.1	5056.9	1325.46	1325.46
40	3773.18	3466.39	3466.39	7549.44	1731.65	1731.65

t(mm)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	14.366	44.6097	14.366	14.366	53.8255	14.366
10	117.238	179.209	117.238	117.238	216.072	117.238
15	397.122	403.797	397.122	397.122	486.739	397.122
20	942.524	718.375	718.375	942.524	865.827	865.827
25	1841.95	1122.94	1122.94	1841.95	1353.34	1353.34
30	3183.91	1617.5	1617.5	3183.91	1949.27	1949.27
35	5056.9	2202.04	2202.04	5056.9	2653.62	2653.62
40	7549.44	2876.58	2876.58	7549.44	3466.39	3466.39
t(mm)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	21.7415	26.7202	21.7415	21.7415	44.6097	21.7415
10	176.242	107.651	107.651	176.242	179.209	176.242
15	596.261	242.792	242.792	596.261	403.797	403.797
20	1414.56	432.144	432.144	1414.56	718.375	718.375
25	2763.89	675.706	675.706	2763.89	1122.94	1122.94
30	4777.02	973.478	973.478	4777.02	1617.5	1617.5
35	7586.7	1325.46	1325.46	7586.7	2202.04	2202.04
40	11325.7	1731.65	1731.65	11325.7	2876.58	2876.58
t(mm)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	21.7415	53.8255	21.7415	29.117	26.7202	26.7202
10	176.242	216.072	176.242	235.246	107.651	107.651
15	596.261	486.739	486.739	795.399	242.792	242.792
20	1414.56	865.827	865.827	1886.59	432.144	432.144
25	2763.89	1353.34	1353.34	3685.83	675.706	675.706
30	4777.02	1949.27	1949.27	6370.13	973.478	973.478
35	7586.7	2653.62	2653.62	10116.5	1325.46	1325.46
40	11325.7	3466.39	3466.39	15102	1731.65	1731.65
t(mm)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	29.117	44.6097	29.117	29.117	53.8255	29.117
10	235.246	179.209	179.209	235.246	216.072	216.072
15	795.399	403.797	403.797	795.399	486.739	486.739
20	1886.59	718.375	718.375	1886.59	865.827	865.827
25	3685.83	1122.94	1122.94	3685.83	1353.34	1353.34
30	6370.13	1617.5	1617.5	6370.13	1949.27	1949.27
35	10116.5	2202.04	2202.04	10116.5	2653.62	2653.62
40	15102	2876.58	2876.58	15102	3466.39	3466.39

Table 4.56: Critical lateral imposed loads for SSSS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	Aspect ration $\alpha = \frac{b}{a} = 1.5$					
	$W_a = 5\text{mm}$	$f_y = 250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 5\text{mm}$	$f_y = 415\text{N/mm}^2$	q_{ic} (kN)
5	1.10822	6.22784	1.10822	1.10822	10.5923	1.10822
10	11.1758	25.6814	11.1758	11.1758	43.1393	11.1758
15	39.1619	58.3606	39.1619	39.1619	97.6408	39.1619
20	94.0261	104.265	94.0261	94.0261	174.097	94.0261
25	184.728	163.396	163.396	184.728	272.508	184.728
30	320.226	235.752	235.752	320.226	392.873	320.226
35	509.479	321.334	321.334	509.479	535.193	509.479
40	761.449	420.142	420.142	761.449	699.468	699.468
t(mm)	$W_a = 5\text{mm}$	$f_y = 500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y = 250\text{N/mm}^2$	q_{ic} (kN)
5	1.10822	12.8407	1.10822	2.60144	6.22784	2.60144
10	11.1758	52.1327	11.1758	23.1215	25.6814	23.1215
15	39.1619	117.876	39.1619	79.4789	58.3606	58.3606
20	94.0261	210.071	94.0261	189.592	104.265	104.265
25	184.728	328.717	184.728	371.38	163.396	163.396
30	320.226	473.815	320.226	642.761	235.752	235.752
35	509.479	645.363	509.479	1021.65	321.334	321.334
40	761.449	843.364	761.449	1525.98	420.142	420.142
t(mm)	$W_a = 10\text{mm}$	$f_y = 415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y = 500\text{N/mm}^2$	q_{ic} (kN)
5	2.60144	10.5923	2.60144	2.60144	12.8407	2.60144
10	23.1215	43.1393	23.1215	23.1215	52.1327	23.1215
15	79.4789	97.6408	79.4789	79.4789	117.876	79.4789
20	189.592	174.097	174.097	189.592	210.071	189.592
25	371.38	272.508	272.508	371.38	328.717	328.717
30	642.761	392.873	392.873	642.761	473.815	473.815
35	1021.65	535.193	535.193	1021.65	645.363	645.363
40	1525.98	699.468	699.468	1525.98	843.364	843.364
t(mm)	$W_a = 15\text{mm}$	$f_y = 250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 15\text{mm}$	$f_y = 415\text{N/mm}^2$	q_{ic} (kN)
5	4.09466	6.22784	4.09466	4.09466	10.5923	4.09466
10	35.0673	25.6814	25.6814	35.0673	43.1393	35.0673
15	119.796	58.3606	58.3606	119.796	97.6408	97.6408
20	285.158	104.265	104.265	285.158	174.097	174.097
25	558.032	163.396	163.396	558.032	272.508	272.508
30	965.296	235.752	235.752	965.296	392.873	392.873
35	1533.83	321.334	321.334	1533.83	535.193	535.193
40	2290.51	420.142	420.142	2290.51	699.468	699.468

t(mm)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	4.09466	12.8407	4.09466	5.58788	6.22784	5.58788
10	35.0673	52.1327	35.0673	47.013	25.6814	25.6814
15	119.796	117.876	117.876	160.113	58.3606	58.3606
20	285.158	210.071	210.071	380.724	104.265	104.265
25	558.032	328.717	328.717	744.685	163.396	163.396
30	965.296	473.815	473.815	1287.83	235.752	235.752
35	1533.83	645.363	645.363	2046	321.334	321.334
40	2290.51	843.364	843.364	3055.03	420.142	420.142
t(mm)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	5.58788	10.5923	5.58788	5.58788	12.8407	5.58788
10	47.013	43.1393	43.1393	47.013	52.1327	47.013
15	160.113	97.6408	97.6408	160.113	117.876	117.876
20	380.724	174.097	174.097	380.724	210.071	210.071
25	744.685	272.508	272.508	744.685	328.717	328.717
30	1287.83	392.873	392.873	1287.83	473.815	473.815
35	2046	535.193	535.193	2046	645.363	645.363
40	3055.03	699.468	699.468	3055.03	843.364	843.364

Table 4.57: Critical lateral imposed loads for CCCC classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	Aspect ration $\alpha = \frac{b}{a} = 1.5$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	4.64291	18.3457	4.64291	4.64291	30.708	4.64291
10	39.4533	74.1529	39.4533	39.4533	123.602	39.4533
15	134.599	167.422	134.599	134.599	278.682	134.599
20	320.246	298.152	298.152	320.246	495.948	320.246
25	626.564	466.343	466.343	626.564	775.4	626.564
30	1083.72	671.996	671.996	1083.72	1117.04	1083.72
35	1721.88	915.111	915.111	1721.88	1520.86	1520.86
40	2571.21	1195.69	1195.69	2571.21	1986.87	1986.87
t(mm)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	4.64291	37.0765	4.64291	9.67082	18.3457	9.67082
10	39.4533	149.076	39.4533	79.6765	74.1529	74.1529
15	134.599	335.998	134.599	270.352	167.422	167.422

20	320.246	597.844	320.246	642.032	298.152	298.152
25	626.564	934.612	626.564	1255.05	466.343	466.343
30	1083.72	1346.3	1083.72	2169.75	671.996	671.996
35	1721.88	1832.92	1721.88	3446.45	915.111	915.111
40	2571.21	2394.45	2394.45	5145.5	1195.69	1195.69
t(mm)	W _a = 10mm	f _y =415N/mm ²	q _{ic} (kN)	W _a = 10mm	f _y =500N/mm ²	q _{ic} (kN)
5	9.67082	30.708	9.67082	9.67082	37.0765	9.67082
10	79.6765	123.602	79.6765	79.6765	149.076	79.6765
15	270.352	278.682	270.352	270.352	335.998	270.352
20	642.032	495.948	495.948	642.032	597.844	597.844
25	1255.05	775.4	775.4	1255.05	934.612	934.612
30	2169.75	1117.04	1117.04	2169.75	1346.3	1346.3
35	3446.45	1520.86	1520.86	3446.45	1832.92	1832.92
40	5145.5	1986.87	1986.87	5145.5	2394.45	2394.45
t(mm)	W _a = 15mm	f _y =250N/mm ²	q _{ic} (kN)	W _a = 15mm	f _y =415N/mm ²	q _{ic} (kN)
5	14.6987	18.3457	14.6987	14.6987	30.708	14.6987
10	119.9	74.1529	74.1529	119.9	123.602	119.9
15	406.106	167.422	167.422	406.106	278.682	278.682
20	963.818	298.152	298.152	963.818	495.948	495.948
25	1883.54	466.343	466.343	1883.54	775.4	775.4
30	3255.77	671.996	671.996	3255.77	1117.04	1117.04
35	5171.02	915.111	915.111	5171.02	1520.86	1520.86
40	7719.79	1195.69	1195.69	7719.79	1986.87	1986.87
t(mm)	W _a = 15mm	f _y =500N/mm ²	q _{ic} (kN)	W _a = 20mm	f _y =250N/mm ²	q _{ic} (kN)
5	14.6987	37.0765	14.6987	19.7266	18.3457	18.3457
10	119.9	149.076	119.9	160.123	74.1529	74.1529
15	406.106	335.998	335.998	541.859	167.422	167.422
20	963.818	597.844	597.844	1285.6	298.152	298.152
25	1883.54	934.612	934.612	2512.03	466.343	466.343
30	3255.77	1346.3	1346.3	4341.8	671.996	671.996
35	5171.02	1832.92	1832.92	6895.6	915.111	915.111
40	7719.79	2394.45	2394.45	10294.1	1195.69	1195.69
t(mm)	W _a = 20mm	f _y =415N/mm ²	q _{ic} (kN)	W _a = 20mm	f _y =500N/mm ²	q _{ic} (kN)
5	19.7266	30.708	19.7266	19.7266	37.0765	19.7266
10	160.123	123.602	123.602	160.123	149.076	149.076
15	541.859	278.682	278.682	541.859	335.998	335.998
20	1285.6	495.948	495.948	1285.6	597.844	597.844
25	2512.03	775.4	775.4	2512.03	934.612	934.612
30	4341.8	1117.04	1117.04	4341.8	1346.3	1346.3
35	6895.6	1520.86	1520.86	6895.6	1832.92	1832.92
40	10294.1	1986.87	1986.87	10294.1	2394.45	2394.45

Table 4.58: Critical lateral imposed loads for CSCS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	1.76739	10.7855	1.76739	1.76739	18.1581	1.76739
10	16.4491	43.9121	16.4491	16.4491	73.4023	16.4491
15	56.9595	99.3797	56.9595	56.9595	165.733	56.9595
20	136.213	177.188	136.213	136.213	295.149	136.213
25	267.124	277.338	267.124	267.124	461.652	267.124
30	462.606	399.829	399.829	462.606	665.241	462.606
35	735.574	544.661	544.661	735.574	905.916	735.574
40	1098.94	711.834	711.834	1098.94	1183.68	1098.94
t(mm)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	1.76739	21.9561	1.76739	3.91978	10.7855	3.91978
10	16.4491	88.5942	16.4491	33.6682	43.9121	33.6682
15	56.9595	199.915	56.9595	115.074	99.3797	99.3797
20	136.213	355.917	136.213	273.966	177.188	177.188
25	267.124	556.601	267.124	536.172	277.338	277.338
30	462.606	801.968	462.606	927.522	399.829	399.829
35	735.574	1092.02	735.574	1473.84	544.661	544.661
40	1098.94	1426.75	1098.94	2200.97	711.834	711.834
t(mm)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	3.91978	18.1581	3.91978	3.91978	21.9561	3.91978
10	33.6682	73.4023	33.6682	33.6682	88.5942	33.6682
15	115.074	165.733	115.074	115.074	199.915	115.074
20	273.966	295.149	273.966	273.966	355.917	273.966
25	536.172	461.652	461.652	536.172	556.601	536.172
30	927.522	665.241	665.241	927.522	801.968	801.968
35	1473.84	905.916	905.916	1473.84	1092.02	1092.02
40	2200.97	1183.68	1183.68	2200.97	1426.75	1426.75
t(mm)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	6.07216	10.7855	6.07216	6.07216	18.1581	6.07216
10	50.8873	43.9121	43.9121	50.8873	73.4023	50.8873
15	173.188	99.3797	99.3797	173.188	165.733	165.733
20	411.719	177.188	177.188	411.719	295.149	295.149
25	805.22	277.338	277.338	805.22	461.652	461.652

30	1392.44	399.829	399.829	1392.44	665.241	665.241
35	2212.11	544.661	544.661	2212.11	905.916	905.916
40	3302.99	711.834	711.834	3302.99	1183.68	1183.68
t(mm)	W _a = 15mm	f _y =500N/mm ²	q _{ic} (kN)	W _a = 20mm	f _y =250N/mm ²	q _{ic} (kN)
5	6.07216	21.9561	6.07216	8.22455	10.7855	8.22455
10	50.8873	88.5942	50.8873	68.1064	43.9121	43.9121
15	173.188	199.915	173.188	231.303	99.3797	99.3797
20	411.719	355.917	355.917	549.471	177.188	177.188
25	805.22	556.601	556.601	1074.27	277.338	277.338
30	1392.44	801.968	801.968	1857.35	399.829	399.829
35	2212.11	1092.02	1092.02	2950.38	544.661	544.661
40	3302.99	1426.75	1426.75	4405.01	711.834	711.834
t(mm)	W _a = 20mm	f _y =415N/mm ²	q _{ic} (kN)	W _a = 20mm	f _y =500N/mm ²	q _{ic} (kN)
5	8.22455	18.1581	8.22455	8.22455	21.9561	8.22455
10	68.1064	73.4023	68.1064	68.1064	88.5942	68.1064
15	231.303	165.733	165.733	231.303	199.915	199.915
20	549.471	295.149	295.149	549.471	355.917	355.917
25	1074.27	461.652	461.652	1074.27	556.601	556.601
30	1857.35	665.241	665.241	1857.35	801.968	801.968
35	2950.38	905.916	905.916	2950.38	1092.02	1092.02
40	4405.01	1183.68	1183.68	4405.01	1426.75	1426.75

Table 4.59: Critical lateral imposed loads for CCSS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q _{ic})						
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$					
	W _a = 5mm	f _y =250N/mm ²	q _{ic} (kN)	W _a = 5mm	f _y =415N/mm ²	q _{ic} (kN)
5	2.69095	14.0974	2.69095	2.69095	23.6557	2.69095
10	23.8376	57.1595	23.8376	23.8376	95.3929	23.8376
15	81.8957	129.186	81.8957	81.8957	215.212	81.8957
20	195.321	230.178	195.321	195.321	383.112	195.321
25	382.569	360.134	360.134	382.569	599.093	382.569
30	662.095	519.055	519.055	662.095	863.156	662.095
35	1052.36	706.941	706.941	1052.36	1175.3	1052.36
40	1571.81	923.791	923.791	1571.81	1535.53	1535.53
t(mm)	W _a = 5mm	f _y =500N/mm ²	q _{ic} (kN)	W _a = 10mm	f _y =250N/mm ²	q _{ic} (kN)

5	2.69095	28.5797	2.69095	5.7669	14.0974	5.7669
10	23.8376	115.089	23.8376	48.4452	57.1595	48.4452
15	81.8957	259.528	81.8957	164.946	129.186	129.186
20	195.321	461.896	195.321	392.182	230.178	230.178
25	382.569	722.193	382.569	767.063	360.134	360.134
30	662.095	1040.42	662.095	1326.5	519.055	519.055
35	1052.36	1416.58	1052.36	2107.41	706.941	706.941
40	1571.81	1850.66	1571.81	3146.69	923.791	923.791
t(mm)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	5.7669	23.6557	5.7669	5.7669	28.5797	5.7669
10	48.4452	95.3929	48.4452	48.4452	115.089	48.4452
15	164.946	215.212	164.946	164.946	259.528	164.946
20	392.182	383.112	383.112	392.182	461.896	392.182
25	767.063	599.093	599.093	767.063	722.193	722.193
30	1326.5	863.156	863.156	1326.5	1040.42	1040.42
35	2107.41	1175.3	1175.3	2107.41	1416.58	1416.58
40	3146.69	1535.53	1535.53	3146.69	1850.66	1850.66
t(mm)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	8.84285	14.0974	8.84285	8.84285	23.6557	8.84285
10	73.0528	57.1595	57.1595	73.0528	95.3929	73.0528
15	247.997	129.186	129.186	247.997	215.212	215.212
20	589.042	230.178	230.178	589.042	383.112	383.112
25	1151.56	360.134	360.134	1151.56	599.093	599.093
30	1990.91	519.055	519.055	1990.91	863.156	863.156
35	3162.46	706.941	706.941	3162.46	1175.3	1175.3
40	4721.58	923.791	923.791	4721.58	1535.53	1535.53
t(mm)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	8.84285	28.5797	8.84285	11.9188	14.0974	11.9188
10	73.0528	115.089	73.0528	97.6604	57.1595	57.1595
15	247.997	259.528	247.997	331.048	129.186	129.186
20	589.042	461.896	461.896	785.903	230.178	230.178
25	1151.56	722.193	722.193	1536.05	360.134	360.134
30	1990.91	1040.42	1040.42	2655.31	519.055	519.055
35	3162.46	1416.58	1416.58	4217.51	706.941	706.941
40	4721.58	1850.66	1850.66	6296.47	923.791	923.791
t(mm)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	11.9188	23.6557	11.9188	11.9188	28.5797	11.9188
10	97.6604	95.3929	95.3929	97.6604	115.089	97.6604
15	331.048	215.212	215.212	331.048	259.528	259.528
20	785.903	383.112	383.112	785.903	461.896	461.896
25	1536.05	599.093	599.093	1536.05	722.193	722.193
30	2655.31	863.156	863.156	2655.31	1040.42	1040.42

35	4217.51	1175.3	1175.3	4217.51	1416.58	1416.58
40	6296.47	1535.53	1535.53	6296.47	1850.66	1850.66

Table 4.60: Critical lateral imposed loads for CSSS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$					
	$W_a = 5\text{mm}$	$f_y = 250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 5\text{mm}$	$f_y = 415\text{N/mm}^2$	q_{ic} (kN)
5	1.44703	9.00281	1.44703	1.44703	15.1988	1.44703
10	13.8862	36.7812	13.8862	13.8862	61.565	13.8862
15	48.3097	83.3353	48.3097	48.3097	139.099	48.3097
20	115.71	148.665	115.71	115.71	247.8	115.71
25	227.078	232.77	227.078	227.078	387.669	227.078
30	393.408	335.651	335.651	393.408	558.705	393.408
35	625.69	457.308	457.308	625.69	760.909	625.69
40	934.917	597.74	597.74	934.917	994.28	934.917
t(mm)	$W_a = 5\text{mm}$	$f_y = 500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y = 250\text{N/mm}^2$	q_{ic} (kN)
5	1.44703	18.3906	1.44703	3.27905	9.00281	3.27905
10	13.8862	74.3324	13.8862	28.5424	36.7812	28.5424
15	48.3097	167.826	48.3097	97.7744	83.3353	83.3353
20	115.71	298.87	115.71	232.959	148.665	148.665
25	227.078	467.465	227.078	456.082	232.77	232.77
30	393.408	673.612	393.408	789.125	335.651	335.651
35	625.69	917.31	625.69	1254.08	457.308	457.308
40	934.917	1198.56	934.917	1872.92	597.74	597.74
t(mm)	$W_a = 10\text{mm}$	$f_y = 415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y = 500\text{N/mm}^2$	q_{ic} (kN)
5	3.27905	15.1988	3.27905	3.27905	18.3906	3.27905
10	28.5424	61.565	28.5424	28.5424	74.3324	28.5424
15	97.7744	139.099	97.7744	97.7744	167.826	97.7744
20	232.959	247.8	232.959	232.959	298.87	232.959
25	456.082	387.669	387.669	456.082	467.465	456.082
30	789.125	558.705	558.705	789.125	673.612	673.612
35	1254.08	760.909	760.909	1254.08	917.31	917.31
40	1872.92	994.28	994.28	1872.92	1198.56	1198.56
t(mm)	$W_a = 15\text{mm}$	$f_y = 250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 15\text{mm}$	$f_y = 415\text{N/mm}^2$	q_{ic} (kN)
5	5.11108	9.00281	5.11108	5.11108	15.1988	5.11108

10	43.1986	36.7812	36.7812	43.1986	61.565	43.1986
15	147.239	83.3353	83.3353	147.239	139.099	139.099
20	350.209	148.665	148.665	350.209	247.8	247.8
25	685.085	232.77	232.77	685.085	387.669	387.669
30	1184.84	335.651	335.651	1184.84	558.705	558.705
35	1882.46	457.308	457.308	1882.46	760.909	760.909
40	2810.91	597.74	597.74	2810.91	994.28	994.28
t(mm)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	5.11108	18.3906	5.11108	6.9431	9.00281	6.9431
10	43.1986	74.3324	43.1986	57.8548	36.7812	36.7812
15	147.239	167.826	147.239	196.704	83.3353	83.3353
20	350.209	298.87	298.87	467.459	148.665	148.665
25	685.085	467.465	467.465	914.088	232.77	232.77
30	1184.84	673.612	673.612	1580.56	335.651	335.651
35	1882.46	917.31	917.31	2510.85	457.308	457.308
40	2810.91	1198.56	1198.56	3748.91	597.74	597.74
t(mm)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	6.9431	15.1988	6.9431	6.9431	18.3906	6.9431
10	57.8548	61.565	57.8548	57.8548	74.3324	57.8548
15	196.704	139.099	139.099	196.704	167.826	167.826
20	467.459	247.8	247.8	467.459	298.87	298.87
25	914.088	387.669	387.669	914.088	467.465	467.465
30	1580.56	558.705	558.705	1580.56	673.612	673.612
35	2510.85	760.909	760.909	2510.85	917.31	917.31
40	3748.91	994.28	994.28	3748.91	1198.56	1198.56

Table 4.61: Critical lateral imposed loads for CCCS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	Aspect ration $\alpha = \frac{b}{a} = 1.5$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	2.95237	15.1456	2.95237	2.95237	25.3959	2.95237
10	25.929	61.3525	25.929	25.929	102.353	25.929
15	88.954	138.621	88.954	88.954	230.873	88.954
20	212.052	246.95	212.052	212.052	410.954	212.052
25	415.246	386.341	386.341	415.246	642.596	415.246
30	718.562	556.793	556.793	718.562	925.801	718.562

35	1142.02	758.306	758.306	1142.02	1260.57	1142.02
40	1705.65	990.88	990.88	1705.65	1646.89	1646.89
t(mm)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	2.95237	30.6763	2.95237	6.28974	15.1456	6.28974
10	25.929	123.475	25.929	52.6279	61.3525	52.6279
15	88.954	278.396	88.954	179.063	138.621	138.621
20	212.052	495.44	212.052	425.643	246.95	246.95
25	415.246	774.607	415.246	832.417	386.341	386.341
30	718.562	1115.9	718.562	1439.43	556.793	556.793
35	1142.02	1519.31	1142.02	2286.74	758.306	758.306
40	1705.65	1984.84	1705.65	3414.39	990.88	990.88
t(mm)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	6.28974	25.3959	6.28974	6.28974	30.6763	6.28974
10	52.6279	102.353	52.6279	52.6279	123.475	52.6279
15	179.063	230.873	179.063	179.063	278.396	179.063
20	425.643	410.954	410.954	425.643	495.44	425.643
25	832.417	642.596	642.596	832.417	774.607	774.607
30	1439.43	925.801	925.801	1439.43	1115.9	1115.9
35	2286.74	1260.57	1260.57	2286.74	1519.31	1519.31
40	3414.39	1646.89	1646.89	3414.39	1984.84	1984.84
t(mm)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	9.62711	15.1456	9.62711	9.62711	25.3959	9.62711
10	79.3269	61.3525	61.3525	79.3269	102.353	79.3269
15	269.172	138.621	138.621	269.172	230.873	230.873
20	639.235	246.95	246.95	639.235	410.954	410.954
25	1249.59	386.341	386.341	1249.59	642.596	642.596
30	2160.31	556.793	556.793	2160.31	925.801	925.801
35	3431.46	758.306	758.306	3431.46	1260.57	1260.57
40	5123.12	990.88	990.88	5123.12	1646.89	1646.89
t(mm)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	9.62711	30.6763	9.62711	12.9645	15.1456	12.9645
10	79.3269	123.475	79.3269	106.026	61.3525	61.3525
15	269.172	278.396	269.172	359.281	138.621	138.621
20	639.235	495.44	495.44	852.827	246.95	246.95
25	1249.59	774.607	774.607	1666.76	386.341	386.341
30	2160.31	1115.9	1115.9	2881.18	556.793	556.793
35	3431.46	1519.31	1519.31	4576.18	758.306	758.306
40	5123.12	1984.84	1984.84	6831.85	990.88	990.88
t(mm)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	12.9645	25.3959	12.9645	12.9645	30.6763	12.9645
10	106.026	102.353	102.353	106.026	123.475	106.026

15	359.281	230.873	230.873	359.281	278.396	278.396
20	852.827	410.954	410.954	852.827	495.44	495.44
25	1666.76	642.596	642.596	1666.76	774.607	774.607
30	2881.18	925.801	925.801	2881.18	1115.9	1115.9
35	4576.18	1260.57	1260.57	4576.18	1519.31	1519.31
40	6831.85	1646.89	1646.89	6831.85	1984.84	1984.84

Table 4.62: Critical lateral imposed loads for SSSS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 2$					
	$W_a = 5\text{mm}$	$f_y = 250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 5\text{mm}$	$f_y = 415\text{N/mm}^2$	q_{ic} (kN)
5	0.7334	5.69159	0.7334	0.7334	9.70215	0.7334
10	8.17716	23.5364	8.17716	8.17716	39.5786	8.17716
15	29.0417	53.5343	29.0417	29.0417	89.6293	29.0417
20	70.0373	95.6855	70.0373	70.0373	159.854	70.0373
25	137.874	149.99	137.874	137.874	250.254	137.874
30	239.263	216.447	216.447	239.263	360.827	239.263
35	380.915	295.058	295.058	380.915	491.575	380.915
40	569.538	385.822	385.822	569.538	642.497	569.538
t(mm)	$W_a = 5\text{mm}$	$f_y = 500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y = 250\text{N/mm}^2$	q_{ic} (kN)
5	0.7334	11.7682	0.7334	1.85179	5.69159	1.85179
10	8.17716	47.8427	8.17716	17.1243	23.5364	17.1243
15	29.0417	108.224	29.0417	59.2383	53.5343	53.5343
20	70.0373	192.911	70.0373	141.615	95.6855	95.6855
25	137.874	301.905	137.874	277.674	149.99	149.99
30	239.263	435.205	239.263	480.837	216.447	216.447
35	380.915	592.811	380.915	764.524	295.058	295.058
40	569.538	774.724	569.538	1142.16	385.822	385.822
t(mm)	$W_a = 10\text{mm}$	$f_y = 415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y = 500\text{N/mm}^2$	q_{ic} (kN)
5	1.85179	9.70215	1.85179	1.85179	11.7682	1.85179
10	17.1243	39.5786	17.1243	17.1243	47.8427	17.1243
15	59.2383	89.6293	59.2383	59.2383	108.224	59.2383
20	141.615	159.854	141.615	141.615	192.911	141.615
25	277.674	250.254	250.254	277.674	301.905	277.674
30	480.837	360.827	360.827	480.837	435.205	435.205
35	764.524	491.575	491.575	764.524	592.811	592.811

40	1142.16	642.497	642.497	1142.16	774.724	774.724
t(mm)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	2.97019	5.69159	2.97019	2.97019	9.70215	2.97019
10	26.0715	23.5364	23.5364	26.0715	39.5786	26.0715
15	89.435	53.5343	53.5343	89.435	89.6293	89.435
20	213.192	95.6855	95.6855	213.192	159.854	159.854
25	417.473	149.99	149.99	417.473	250.254	250.254
30	722.41	216.447	216.447	722.41	360.827	360.827
35	1148.13	295.058	295.058	1148.13	491.575	491.575
40	1714.78	385.822	385.822	1714.78	642.497	642.497
t(mm)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	2.97019	11.7682	2.97019	4.08858	5.69159	4.08858
10	26.0715	47.8427	26.0715	35.0186	23.5364	23.5364
15	89.435	108.224	89.435	119.632	53.5343	53.5343
20	213.192	192.911	192.911	284.769	95.6855	95.6855
25	417.473	301.905	301.905	557.273	149.99	149.99
30	722.41	435.205	435.205	963.983	216.447	216.447
35	1148.13	592.811	592.811	1531.74	295.058	295.058
40	1714.78	774.724	774.724	2287.39	385.822	385.822
t(mm)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	4.08858	9.70215	4.08858	4.08858	11.7682	4.08858
10	35.0186	39.5786	35.0186	35.0186	47.8427	35.0186
15	119.632	89.6293	89.6293	119.632	108.224	108.224
20	284.769	159.854	159.854	284.769	192.911	192.911
25	557.273	250.254	250.254	557.273	301.905	301.905
30	963.983	360.827	360.827	963.983	435.205	435.205
35	1531.74	491.575	491.575	1531.74	592.811	592.811
40	2287.39	642.497	642.497	2287.39	774.724	774.724

Table 4.63: Critical lateral imposed loads for CCCC classical rectangular plate on aspect ratio of 2, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 2$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	3.79019	16.197	3.79019	3.79019	27.1411	3.79019
10	32.6316	65.5579	32.6316	32.6316	109.334	32.6316

15	111.575	148.083	111.575	111.575	246.58	111.575
20	265.672	263.772	263.772	265.672	438.877	265.672
25	519.974	412.625	412.625	519.974	686.227	519.974
30	899.532	594.641	594.641	899.532	988.629	899.532
35	1429.4	809.822	809.822	1429.4	1346.08	1346.08
40	2134.62	1058.17	1058.17	2134.62	1758.59	1758.59
t(mm)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	3.79019	32.779	3.79019	7.96539	16.197	7.96539
10	32.6316	131.886	32.6316	66.0331	65.5579	65.5579
15	111.575	297.321	111.575	224.306	148.083	148.083
20	265.672	529.083	265.672	532.885	263.772	263.772
25	519.974	827.174	519.974	1041.87	412.625	412.625
30	899.532	1191.59	899.532	1801.37	594.641	594.641
35	1429.4	1622.34	1429.4	2861.49	809.822	809.822
40	2134.62	2119.41	2119.41	4272.32	1058.17	1058.17
t(mm)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	7.96539	27.1411	7.96539	7.96539	32.779	7.96539
10	66.0331	109.334	66.0331	66.0331	131.886	66.0331
15	224.306	246.58	224.306	224.306	297.321	224.306
20	532.885	438.877	438.877	532.885	529.083	529.083
25	1041.87	686.227	686.227	1041.87	827.174	827.174
30	1801.37	988.629	988.629	1801.37	1191.59	1191.59
35	2861.49	1346.08	1346.08	2861.49	1622.34	1622.34
40	4272.32	1758.59	1758.59	4272.32	2119.41	2119.41
t(mm)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	12.1406	16.197	12.1406	12.1406	27.1411	12.1406
10	99.4347	65.5579	65.5579	99.4347	109.334	99.4347
15	337.036	148.083	148.083	337.036	246.58	246.58
20	800.097	263.772	263.772	800.097	438.877	438.877
25	1563.77	412.625	412.625	1563.77	686.227	686.227
30	2703.22	594.641	594.641	2703.22	988.629	988.629
35	4293.58	809.822	809.822	4293.58	1346.08	1346.08
40	6410.02	1058.17	1058.17	6410.02	1758.59	1758.59
t(mm)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	12.1406	32.779	12.1406	16.3158	16.197	16.197
10	99.4347	131.886	99.4347	132.836	65.5579	65.5579
15	337.036	297.321	297.321	449.766	148.083	148.083
20	800.097	529.083	529.083	1067.31	263.772	263.772
25	1563.77	827.174	827.174	2085.67	412.625	412.625
30	2703.22	1191.59	1191.59	3605.06	594.641	594.641
35	4293.58	1622.34	1622.34	5725.67	809.822	809.822
40	6410.02	2119.41	2119.41	8547.72	1058.17	1058.17

t(mm)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	16.3158	27.1411	16.3158	16.3158	32.779	16.3158
10	132.836	109.334	109.334	132.836	131.886	131.886
15	449.766	246.58	246.58	449.766	297.321	297.321
20	1067.31	438.877	438.877	1067.31	529.083	529.083
25	2085.67	686.227	686.227	2085.67	827.174	827.174
30	3605.06	988.629	988.629	3605.06	1191.59	1191.59
35	5725.67	1346.08	1346.08	5725.67	1622.34	1622.34
40	8547.72	1758.59	1758.59	8547.72	2119.41	2119.41

Table 4.64: Critical lateral imposed loads for CSCS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 2$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	0.9551	7.59906	0.9551	0.9551	12.8685	0.9551
10	9.95079	31.1662	9.95079	9.95079	52.2441	9.95079
15	35.0277	70.7015	35.0277	35.0277	118.127	35.0277
20	84.2263	126.205	84.2263	84.2263	210.517	84.2263
25	165.587	197.676	165.587	165.587	329.413	165.587
30	287.151	285.116	285.116	287.151	474.817	287.151
35	456.959	388.524	388.524	456.959	646.728	456.959
40	683.051	507.9	507.9	683.051	845.146	683.051
t(mm)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	0.9551	15.5831	0.9551	2.2952	7.59906	2.2952
10	9.95079	63.1024	9.95079	20.6716	31.1662	20.6716
15	35.0277	142.558	35.0277	71.2104	70.7015	70.7015
20	84.2263	253.95	84.2263	169.993	126.205	126.205
25	165.587	397.278	165.587	333.1	197.676	197.676
30	287.151	572.542	287.151	576.613	285.116	285.116
35	456.959	779.742	456.959	916.613	388.524	388.524
40	683.051	1018.88	683.051	1369.18	507.9	507.9
t(mm)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	2.2952	12.8685	2.2952	2.2952	15.5831	2.2952
10	20.6716	52.2441	20.6716	20.6716	63.1024	20.6716
15	71.2104	118.127	71.2104	71.2104	142.558	71.2104

20	169.993	210.517	169.993	169.993	253.95	169.993
25	333.1	329.413	329.413	333.1	397.278	333.1
30	576.613	474.817	474.817	576.613	572.542	572.542
35	916.613	646.728	646.728	916.613	779.742	779.742
40	1369.18	845.146	845.146	1369.18	1018.88	1018.88
t(mm)	W _a = 15mm	f _y =250N/mm ²	q _{ic} (kN)	W _a = 15mm	f _y =415N/mm ²	q _{ic} (kN)
5	3.6353	7.59906	3.6353	3.6353	12.8685	3.6353
10	31.3924	31.1662	31.1662	31.3924	52.2441	31.3924
15	107.393	70.7015	70.7015	107.393	118.127	107.393
20	255.759	126.205	126.205	255.759	210.517	210.517
25	500.612	197.676	197.676	500.612	329.413	329.413
30	866.074	285.116	285.116	866.074	474.817	474.817
35	1376.27	388.524	388.524	1376.27	646.728	646.728
40	2055.31	507.9	507.9	2055.31	845.146	845.146
t(mm)	W _a = 15mm	f _y =500N/mm ²	q _{ic} (kN)	W _a = 20mm	f _y =250N/mm ²	q _{ic} (kN)
5	3.6353	15.5831	3.6353	4.9754	7.59906	4.9754
10	31.3924	63.1024	31.3924	42.1132	31.1662	31.1662
15	107.393	142.558	107.393	143.576	70.7015	70.7015
20	255.759	253.95	253.95	341.525	126.205	126.205
25	500.612	397.278	397.278	668.125	197.676	197.676
30	866.074	572.542	572.542	1155.54	285.116	285.116
35	1376.27	779.742	779.742	1835.92	388.524	388.524
40	2055.31	1018.88	1018.88	2741.44	507.9	507.9
t(mm)	W _a = 20mm	f _y =415N/mm ²	q _{ic} (kN)	W _a = 20mm	f _y =500N/mm ²	q _{ic} (KN)
5	4.9754	12.8685	4.9754	4.9754	15.5831	4.9754
10	42.1132	52.2441	42.1132	42.1132	63.1024	42.1132
15	143.576	118.127	118.127	143.576	142.558	142.558
20	341.525	210.517	210.517	341.525	253.95	253.95
25	668.125	329.413	329.413	668.125	397.278	397.278
30	1155.54	474.817	474.817	1155.54	572.542	572.542
35	1835.92	646.728	646.728	1835.92	779.742	779.742
40	2741.44	845.146	845.146	2741.44	1018.88	1018.88

Table 4.65: Critical lateral imposed loads for CCSS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q _{ic})	
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 2$

	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	2.05778	12.152	2.05778	2.05778	20.4264	2.05778
10	18.7722	49.3779	18.7722	18.7722	82.4755	18.7722
15	64.8	111.678	64.8	64.8	186.147	64.8
20	154.798	199.052	154.798	154.798	331.442	154.798
25	303.422	311.499	303.422	303.422	518.359	303.422
30	525.33	449.021	449.021	525.33	746.899	525.33
35	835.177	611.617	611.617	835.177	1017.06	835.177
40	1247.62	799.286	799.286	1247.62	1328.85	1247.62
t(mm)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	2.05778	24.6889	2.05778	4.50055	12.152	4.50055
10	18.7722	99.5258	18.7722	38.3144	49.3779	38.3144
15	64.8	224.511	64.8	130.755	111.678	111.678
20	154.798	399.643	154.798	311.135	199.052	199.052
25	303.422	624.924	303.422	608.769	311.499	311.499
30	525.33	900.352	525.33	1052.97	449.021	449.021
35	835.177	1225.93	835.177	1673.05	611.617	611.617
40	1247.62	1601.65	1247.62	2498.32	799.286	799.286
t(mm)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	4.50055	20.4264	4.50055	4.50055	24.6889	4.50055
10	38.3144	82.4755	38.3144	38.3144	99.5258	38.3144
15	130.755	186.147	130.755	130.755	224.511	130.755
20	311.135	331.442	311.135	311.135	399.643	311.135
25	608.769	518.359	518.359	608.769	624.924	608.769
30	1052.97	746.899	746.899	1052.97	900.352	900.352
35	1673.05	1017.06	1017.06	1673.05	1225.93	1225.93
40	2498.32	1328.85	1328.85	2498.32	1601.65	1601.65
t(mm)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	6.94333	12.152	6.94333	6.94333	20.4264	6.94333
10	57.8566	49.3779	49.3779	57.8566	82.4755	57.8566
15	196.71	111.678	111.678	196.71	186.147	186.147
20	467.473	199.052	199.052	467.473	331.442	331.442
25	914.116	311.499	311.499	914.116	518.359	518.359
30	1580.61	449.021	449.021	1580.61	746.899	746.899
35	2510.92	611.617	611.617	2510.92	1017.06	1017.06
40	3749.02	799.286	799.286	3749.02	1328.85	1328.85
t(mm)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	6.94333	24.6889	6.94333	9.38611	12.152	9.38611
10	57.8566	99.5258	57.8566	77.3988	49.3779	49.3779
15	196.71	224.511	196.71	262.665	111.678	111.678
20	467.473	399.643	399.643	623.811	199.052	199.052

25	914.116	624.924	624.924	1219.46	311.499	311.499
30	1580.61	900.352	900.352	2108.25	449.021	449.021
35	2510.92	1225.93	1225.93	3348.79	611.617	611.617
40	3749.02	1601.65	1601.65	4999.73	799.286	799.286
t(mm)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	9.38611	20.4264	9.38611	9.38611	24.6889	9.38611
10	77.3988	82.4755	77.3988	77.3988	99.5258	77.3988
15	262.665	186.147	186.147	262.665	224.511	224.511
20	623.811	331.442	331.442	623.811	399.643	399.643
25	1219.46	518.359	518.359	1219.46	624.924	624.924
30	2108.25	746.899	746.899	2108.25	900.352	900.352
35	3348.79	1017.06	1017.06	3348.79	1225.93	1225.93
40	4999.73	1328.85	1328.85	4999.73	1601.65	1601.65

Table 4.66: Critical lateral imposed loads for CSSS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 2$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	0.87838	7.10598	0.87838	0.87838	12.05	0.87838
10	9.33701	29.1939	9.33701	9.33701	48.9701	9.33701
15	32.9562	66.2638	32.9562	32.9562	110.76	32.9562
20	79.3161	118.316	79.3161	79.3161	197.421	79.3161
25	155.997	185.35	155.997	155.997	308.951	155.997
30	270.579	267.365	267.365	270.579	445.351	270.579
35	430.643	364.363	364.363	430.643	606.622	430.643
40	643.769	476.343	476.343	643.769	792.762	643.769
t(mm)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	0.87838	14.597	0.87838	2.14175	7.10598	2.14175
10	9.33701	59.1579	9.33701	19.444	29.1939	19.444
15	32.9562	133.683	32.9562	67.0673	66.2638	67.0673
20	79.3161	238.171	79.3161	160.172	118.316	160.172
25	155.997	372.624	155.997	313.919	185.35	185.35
30	270.579	537.041	270.579	543.469	267.365	267.365
35	430.643	731.421	430.643	863.981	364.363	364.363
40	643.769	955.766	643.769	1290.62	476.343	476.343
t(mm)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)

5	2.14175	12.05	2.14175	2.14175	14.597	2.14175
10	19.444	48.9701	19.444	19.444	59.1579	19.444
15	67.0673	110.76	67.0673	67.0673	133.683	67.0673
20	160.172	197.421	160.172	160.172	238.171	160.172
25	313.919	308.951	308.951	313.919	372.624	313.919
30	543.469	445.351	445.351	543.469	537.041	537.041
35	863.981	606.622	606.622	863.981	731.421	731.421
40	1290.62	792.762	792.762	1290.62	955.766	955.766
t(mm)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	3.40513	7.10598	3.40513	3.40513	12.05	3.40513
10	29.551	29.1939	29.1939	29.551	48.9701	29.551
15	101.179	66.2638	66.2638	101.179	110.76	101.179
20	241.028	118.316	118.316	241.028	197.421	197.421
25	471.841	185.35	185.35	471.841	308.951	308.951
30	816.358	267.365	267.365	816.358	445.351	445.351
35	1297.32	364.363	364.363	1297.32	606.622	606.622
40	1937.47	476.343	476.343	1937.47	792.762	792.762
t(mm)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	3.40513	14.597	3.40513	4.66851	7.10598	4.66851
10	29.551	59.1579	29.551	39.6581	29.1939	29.1939
15	101.179	133.683	101.179	135.29	66.2638	66.2638
20	241.028	238.171	238.171	321.884	118.316	118.316
25	471.841	372.624	372.624	629.763	185.35	185.35
30	816.358	537.041	537.041	1089.25	267.365	267.365
35	1297.32	731.421	731.421	1730.66	364.363	364.363
40	1937.47	955.766	955.766	2584.32	476.343	476.343
t(mm)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	4.668506	12.05003	4.668506	4.668506	14.59696	4.668506
10	39.65805	48.97012	39.65805	39.65805	59.15785	39.65805
15	135.2897	110.7603	110.7603	135.2897	133.6827	133.6827
20	321.8844	197.4205	197.4205	321.8844	238.1714	238.1714
25	629.7632	308.9507	308.9507	629.7632	372.6241	372.6241
30	1089.247	445.3511	445.3511	1089.247	537.0407	537.0407
35	1730.657	606.6215	606.6215	1730.657	731.4212	731.4212
40	2584.315	792.7619	792.7619	2584.315	955.7657	955.7657

Table 4.67: Critical lateral imposed loads for CCCS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	Aspect ratio $\alpha = \frac{b}{a} = 2$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	2.07279	12.1371	2.07279	2.07279	20.4017	2.07279
10	18.8923	49.3184	18.8923	18.8923	82.3768	18.8923
15	65.2054	111.544	65.2054	65.2054	185.925	65.2054
20	155.759	198.814	155.759	155.759	331.047	155.759
25	305.299	311.128	305.299	305.299	517.742	305.299
30	528.573	448.486	448.486	528.573	746.011	528.573
35	840.328	610.888	610.888	840.328	1015.85	840.328
40	1255.31	798.335	798.335	1255.31	1327.27	1255.31
t(mm)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	2.07279	24.6592	2.07279	4.53059	12.1371	4.53059
10	18.8923	99.4068	18.8923	38.5547	49.3184	38.5547
15	65.2054	224.243	65.2054	131.566	111.544	111.544
20	155.759	399.167	155.759	313.058	198.814	198.814
25	305.299	624.18	305.299	612.523	311.128	311.128
30	528.573	899.281	528.573	1059.46	448.486	448.486
35	840.328	1224.47	840.328	1683.35	610.888	610.888
40	1255.31	1599.75	1255.31	2513.7	798.335	798.335
t(mm)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	4.53059	20.4017	4.53059	4.53059	24.6592	4.53059
10	38.5547	82.3768	38.5547	38.5547	99.4068	38.5547
15	131.566	185.925	131.566	131.566	224.243	131.566
20	313.058	331.047	313.058	313.058	399.167	313.058
25	612.523	517.742	517.742	612.523	624.18	612.523
30	1059.46	746.011	746.011	1059.46	899.281	899.281
35	1683.35	1015.85	1015.85	1683.35	1224.47	1224.47
40	2513.7	1327.27	1327.27	2513.7	1599.75	1599.75
t(mm)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)
5	6.98838	12.1371	6.98838	6.98838	20.4017	6.98838
10	58.217	49.3184	49.3184	58.217	82.3768	58.217
15	197.926	111.544	111.544	197.926	185.925	185.925
20	470.356	198.814	198.814	470.356	331.047	331.047
25	919.747	311.128	311.128	919.747	517.742	517.742

30	1590.34	448.486	448.486	1590.34	746.011	746.011
35	2526.37	610.888	610.888	2526.37	1015.85	1015.85
40	3772.09	798.335	798.335	3772.09	1327.27	1327.27
t(mm)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	q_{ic} (kN)
5	6.98838	24.6592	6.98838	9.44617	12.1371	9.44617
10	58.217	99.4068	58.217	77.8794	49.3184	49.3184
15	197.926	224.243	197.926	264.287	111.544	111.544
20	470.356	399.167	399.167	627.655	198.814	198.814
25	919.747	624.18	624.18	1226.97	311.128	311.128
30	1590.34	899.281	899.281	2121.22	448.486	448.486
35	2526.37	1224.47	1224.47	3369.4	610.888	610.888
40	3772.09	1599.75	1599.75	5030.48	798.335	798.335
t(mm)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	q_{ic} (kN)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	q_{ic} (kN)
5	9.44617	20.4017	9.44617	9.44617	24.6592	9.44617
10	77.8794	82.3768	77.8794	77.8794	99.4068	77.8794
15	264.287	185.925	185.925	264.287	224.243	224.243
20	627.655	331.047	331.047	627.655	399.167	399.167
25	1226.97	517.742	517.742	1226.97	624.18	624.18
30	2121.22	746.011	746.011	2121.22	899.281	899.281
35	3369.4	1015.85	1015.85	3369.4	1224.47	1224.47
40	5030.48	1327.27	1327.27	5030.48	1599.75	1599.75

4.1.5.4 Critical Thicknesses for Classical Rectangular Plates

Table 4.68: Critical thicknesses for SSSS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t_c)						
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.01297	0.01269	0.01297	0.01297	0.00985	0.01297
100	0.01635	0.01795	0.01795	0.01635	0.01393	0.01635
150	0.01871	0.02198	0.02198	0.01871	0.01706	0.01871
200	0.02059	0.02538	0.02538	0.02059	0.0197	0.02059
250	0.02218	0.02838	0.02838	0.02218	0.02203	0.02218
300	0.02357	0.03109	0.03109	0.02357	0.02413	0.02413
350	0.02482	0.03358	0.03358	0.02482	0.02606	0.02606
400	0.02595	0.0359	0.0359	0.02595	0.02786	0.02786

q(kN)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.01297	0.00898	0.01297	0.0103	0.01269	0.01269
100	0.01635	0.01269	0.01635	0.01297	0.01795	0.01795
150	0.01871	0.01554	0.01871	0.01485	0.02198	0.02198
200	0.02059	0.01795	0.02059	0.01635	0.02538	0.02538
250	0.02218	0.02007	0.02218	0.01761	0.02838	0.02838
300	0.02357	0.02198	0.02357	0.01871	0.03109	0.03109
350	0.02482	0.02374	0.02482	0.0197	0.03358	0.03358
400	0.02595	0.02538	0.02595	0.02059	0.0359	0.0359
q(kN)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.0103	0.00985	0.0103	0.0103	0.00898	0.0103
100	0.01297	0.01393	0.01393	0.01297	0.01269	0.01297
150	0.01485	0.01706	0.01706	0.01485	0.01554	0.01554
200	0.01635	0.0197	0.0197	0.01635	0.01795	0.01795
250	0.01761	0.02203	0.02203	0.01761	0.02007	0.02007
300	0.01871	0.02413	0.02413	0.01871	0.02198	0.02198
350	0.0197	0.02606	0.02606	0.0197	0.02374	0.02374
400	0.02059	0.02786	0.02786	0.02059	0.02538	0.02538
q(kN)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.009	0.01269	0.01269	0.009	0.00985	0.00985
100	0.01133	0.01795	0.01795	0.01133	0.01393	0.01393
150	0.01297	0.02198	0.02198	0.01297	0.01706	0.01706
200	0.01428	0.02538	0.02538	0.01428	0.0197	0.0197
250	0.01538	0.02838	0.02838	0.01538	0.02203	0.02203
300	0.01635	0.03109	0.03109	0.01635	0.02413	0.02413
350	0.01721	0.03358	0.03358	0.01721	0.02606	0.02606
400	0.01799	0.0359	0.0359	0.01799	0.02786	0.02786
q(kN)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.009	0.00898	0.009	0.00817	0.01269	0.01269
100	0.01133	0.01269	0.01269	0.0103	0.01795	0.01795
150	0.01297	0.01554	0.01554	0.01179	0.02198	0.02198
200	0.01428	0.01795	0.01795	0.01297	0.02538	0.02538
250	0.01538	0.02007	0.02007	0.01398	0.02838	0.02838
300	0.01635	0.02198	0.02198	0.01485	0.03109	0.03109
350	0.01721	0.02374	0.02374	0.01563	0.03358	0.03358
400	0.01799	0.02538	0.02538	0.01635	0.0359	0.0359
q(kN)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.00817	0.00985	0.00985	0.00817	0.00898	0.00898
100	0.0103	0.01393	0.01393	0.0103	0.01269	0.01269
150	0.01179	0.01706	0.01706	0.01179	0.01554	0.01554
200	0.01297	0.0197	0.0197	0.01297	0.01795	0.01795

250	0.01398	0.02203	0.02203	0.01398	0.02007	0.02007
300	0.01485	0.02413	0.02413	0.01485	0.02198	0.02198
350	0.01563	0.02606	0.02606	0.01563	0.02374	0.02374
400	0.01635	0.02786	0.02786	0.01635	0.02538	0.02538

Table 4.69: Critical thicknesses for CCCC classical rectangular plate on aspect ratio of 1, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t_c)						
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1$					
	$W_a = 5\text{mm}$	$f_y = 250\text{N/mm}^2$	t_c (m)	$W_a = 5\text{mm}$	$f_y = 415\text{N/mm}^2$	t_c (m)
50	0.00889	0.00668	0.00889	0.00889	0.00518	0.00889
100	0.0112	0.00945	0.0112	0.0112	0.00733	0.0112
150	0.01282	0.01157	0.01282	0.01282	0.00898	0.01282
200	0.01411	0.01336	0.01411	0.01411	0.01037	0.01411
250	0.0152	0.01493	0.0152	0.0152	0.01159	0.0152
300	0.01615	0.01636	0.01615	0.01615	0.0127	0.01615
350	0.017	0.01767	0.017	0.017	0.01372	0.017
400	0.01777	0.01889	0.01777	0.01777	0.01466	0.01777
q(kN)	$W_a = 5\text{mm}$	$f_y = 500\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y = 250\text{N/mm}^2$	t_c (m)
50	0.00889	0.00472	0.00889	0.00705	0.00668	0.00705
100	0.0112	0.00668	0.0112	0.00889	0.00945	0.00945
150	0.01282	0.00818	0.01282	0.01017	0.01157	0.01157
200	0.01411	0.00945	0.01411	0.0112	0.01336	0.01336
250	0.0152	0.01056	0.0152	0.01206	0.01493	0.01493
300	0.01615	0.01157	0.01615	0.01282	0.01636	0.01636
350	0.017	0.0125	0.017	0.01349	0.01767	0.01767
400	0.01777	0.01336	0.01777	0.01411	0.01889	0.01889
q(kN)	$W_a = 10\text{mm}$	$f_y = 415\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y = 500\text{N/mm}^2$	t_c (m)
50	0.00705	0.00518	0.00705	0.00705	0.00472	0.00705
100	0.00889	0.00733	0.00889	0.00889	0.00668	0.00889
150	0.01017	0.00898	0.01017	0.01017	0.00818	0.01017
200	0.0112	0.01037	0.0112	0.0112	0.00945	0.0112
250	0.01206	0.01159	0.01206	0.01206	0.01056	0.01206
300	0.01282	0.0127	0.01282	0.01282	0.01157	0.01282
350	0.01349	0.01372	0.01372	0.01349	0.0125	0.01349
400	0.01411	0.01466	0.01466	0.01411	0.01336	0.01411
q(kN)	$W_a = 15\text{mm}$	$f_y = 250\text{N/mm}^2$	t_c (m)	$W_a = 15\text{mm}$	$f_y = 415\text{N/mm}^2$	t_c (m)

50	0.00616	0.00668	0.00668	0.00616	0.00518	0.00616
100	0.00776	0.00945	0.00945	0.00776	0.00733	0.00776
150	0.00889	0.01157	0.01157	0.00889	0.00898	0.00898
200	0.00978	0.01336	0.01336	0.00978	0.01037	0.01037
250	0.01054	0.01493	0.01493	0.01054	0.01159	0.01159
300	0.0112	0.01636	0.01636	0.0112	0.0127	0.0127
350	0.01179	0.01767	0.01767	0.01179	0.01372	0.01372
400	0.01232	0.01889	0.01889	0.01232	0.01466	0.01466
q(kN)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.00616	0.00472	0.00616	0.0056	0.00668	0.00668
100	0.00776	0.00668	0.00776	0.00705	0.00945	0.00945
150	0.00889	0.00818	0.00889	0.00807	0.01157	0.01157
200	0.00978	0.00945	0.00978	0.00889	0.01336	0.01336
250	0.01054	0.01056	0.01056	0.00957	0.01493	0.01493
300	0.0112	0.01157	0.01157	0.01017	0.01636	0.01636
350	0.01179	0.0125	0.0125	0.01071	0.01767	0.01767
400	0.01232	0.01336	0.01336	0.0112	0.01889	0.01889
q(kN)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.0056	0.00518	0.0056	0.0056	0.00472	0.0056
100	0.00705	0.00733	0.00733	0.00705	0.00668	0.00705
150	0.00807	0.00898	0.00898	0.00807	0.00818	0.00818
200	0.00889	0.01037	0.01037	0.00889	0.00945	0.00945
250	0.00957	0.01159	0.01159	0.00957	0.01056	0.01056
300	0.01017	0.0127	0.0127	0.01017	0.01157	0.01157
350	0.01071	0.01372	0.01372	0.01071	0.0125	0.0125
400	0.0112	0.01466	0.01466	0.0112	0.01336	0.01336

Table 4.70: Critical thicknesses for CSCS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t_c)						
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.01016	0.00759	0.01016	0.01016	0.00589	0.01016
100	0.0128	0.01074	0.0128	0.0128	0.00833	0.0128
150	0.01465	0.01315	0.01465	0.01465	0.0102	0.01465
200	0.01612	0.01518	0.01612	0.01612	0.01178	0.01612
250	0.01737	0.01697	0.01737	0.01737	0.01317	0.01737

300	0.01845	0.01859	0.01859	0.01845	0.01443	0.01845
350	0.01943	0.02008	0.02008	0.01943	0.01559	0.01943
400	0.02031	0.02147	0.02147	0.02031	0.01666	0.02031
q(kN)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.01016	0.00537	0.01016	0.00806	0.00759	0.00806
100	0.0128	0.00759	0.0128	0.01016	0.01074	0.01074
150	0.01465	0.0093	0.01465	0.01163	0.01315	0.01315
200	0.01612	0.01074	0.01612	0.0128	0.01518	0.01518
250	0.01737	0.012	0.01737	0.01378	0.01697	0.01697
300	0.01845	0.01315	0.01845	0.01465	0.01859	0.01859
350	0.01943	0.0142	0.01943	0.01542	0.02008	0.02008
400	0.02031	0.01518	0.02031	0.01612	0.02147	0.02147
q(kN)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.00806	0.00589	0.00806	0.00806	0.00537	0.00806
100	0.01016	0.00833	0.01016	0.01016	0.00759	0.01016
150	0.01163	0.0102	0.01163	0.01163	0.0093	0.01163
200	0.0128	0.01178	0.0128	0.0128	0.01074	0.0128
250	0.01378	0.01317	0.01378	0.01378	0.012	0.01378
300	0.01465	0.01443	0.01465	0.01465	0.01315	0.01465
350	0.01542	0.01559	0.01559	0.01542	0.0142	0.01542
400	0.01612	0.01666	0.01666	0.01612	0.01518	0.01612
q(kN)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.00704	0.00759	0.00759	0.00704	0.00589	0.00704
100	0.00887	0.01074	0.01074	0.00887	0.00833	0.00887
150	0.01016	0.01315	0.01315	0.01016	0.0102	0.0102
200	0.01118	0.01518	0.01518	0.01118	0.01178	0.01178
250	0.01204	0.01697	0.01697	0.01204	0.01317	0.01317
300	0.0128	0.01859	0.01859	0.0128	0.01443	0.01443
350	0.01347	0.02008	0.02008	0.01347	0.01559	0.01559
400	0.01408	0.02147	0.02147	0.01408	0.01666	0.01666
q(kN)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.00704	0.00537	0.00704	0.0064	0.00759	0.00759
100	0.00887	0.00759	0.00887	0.00806	0.01074	0.01074
150	0.01016	0.0093	0.01016	0.00923	0.01315	0.01315
200	0.01118	0.01074	0.01118	0.01016	0.01518	0.01518
250	0.01204	0.012	0.01204	0.01094	0.01697	0.01697
300	0.0128	0.01315	0.01315	0.01163	0.01859	0.01859
350	0.01347	0.0142	0.0142	0.01224	0.02008	0.02008
400	0.01408	0.01518	0.01518	0.0128	0.02147	0.02147
q(kN)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.0064	0.00589	0.0064	0.0064	0.00537	0.0064

100	0.00806	0.00833	0.00833	0.00806	0.00759	0.00806
150	0.00923	0.0102	0.0102	0.00923	0.0093	0.0093
200	0.01016	0.01178	0.01178	0.01016	0.01074	0.01074
250	0.01094	0.01317	0.01317	0.01094	0.012	0.012
300	0.01163	0.01443	0.01443	0.01163	0.01315	0.01315
350	0.01224	0.01559	0.01559	0.01224	0.0142	0.0142
400	0.0128	0.01666	0.01666	0.0128	0.01518	0.01518

Table 4.71: Critical thicknesses for CCSS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t_c)						
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1$					
	$W_a = 5\text{mm}$	$f_y = 250\text{N/mm}^2$	t_c (m)	$W_a = 5\text{mm}$	$f_y = 415\text{N/mm}^2$	t_c (m)
50	0.01034	0.0077	0.01034	0.01034	0.00597	0.01034
100	0.01303	0.01088	0.01303	0.01303	0.00845	0.01303
150	0.01492	0.01333	0.01492	0.01492	0.01035	0.01492
200	0.01642	0.01539	0.01642	0.01642	0.01195	0.01642
250	0.01769	0.01721	0.01769	0.01769	0.01336	0.01769
300	0.0188	0.01885	0.01885	0.0188	0.01463	0.0188
350	0.01979	0.02036	0.02036	0.01979	0.0158	0.01979
400	0.02069	0.02177	0.02177	0.02069	0.01689	0.02069
q(kN)	$W_a = 5\text{mm}$	$f_y = 500\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y = 250\text{N/mm}^2$	t_c (m)
50	0.01034	0.00544	0.01034	0.00821	0.0077	0.00821
100	0.01303	0.0077	0.01303	0.01034	0.01088	0.01088
150	0.01492	0.00943	0.01492	0.01184	0.01333	0.01333
200	0.01642	0.01088	0.01642	0.01303	0.01539	0.01539
250	0.01769	0.01217	0.01769	0.01404	0.01721	0.01721
300	0.0188	0.01333	0.0188	0.01492	0.01885	0.01885
350	0.01979	0.0144	0.01979	0.0157	0.02036	0.02036
400	0.02069	0.01539	0.02069	0.01642	0.02177	0.02177
q(kN)	$W_a = 10\text{mm}$	$f_y = 415\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y = 500\text{N/mm}^2$	t_c (m)
50	0.00821	0.00597	0.00821	0.00821	0.00544	0.00821
100	0.01034	0.00845	0.01034	0.01034	0.0077	0.01034
150	0.01184	0.01035	0.01184	0.01184	0.00943	0.01184
200	0.01303	0.01195	0.01303	0.01303	0.01088	0.01303
250	0.01404	0.01336	0.01404	0.01404	0.01217	0.01404
300	0.01492	0.01463	0.01492	0.01492	0.01333	0.01492

350	0.0157	0.0158	0.0158	0.0157	0.0144	0.0157
400	0.01642	0.01689	0.01689	0.01642	0.01539	0.01642
q(kN)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.00717	0.0077	0.0077	0.00717	0.00597	0.00717
100	0.00904	0.01088	0.01088	0.00904	0.00845	0.00904
150	0.01034	0.01333	0.01333	0.01034	0.01035	0.01035
200	0.01138	0.01539	0.01539	0.01138	0.01195	0.01195
250	0.01226	0.01721	0.01721	0.01226	0.01336	0.01336
300	0.01303	0.01885	0.01885	0.01303	0.01463	0.01463
350	0.01372	0.02036	0.02036	0.01372	0.0158	0.0158
400	0.01434	0.02177	0.02177	0.01434	0.01689	0.01689
q(kN)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.00717	0.00544	0.00717	0.00652	0.0077	0.0077
100	0.00904	0.0077	0.00904	0.00821	0.01088	0.01088
150	0.01034	0.00943	0.01034	0.0094	0.01333	0.01333
200	0.01138	0.01088	0.01138	0.01034	0.01539	0.01539
250	0.01226	0.01217	0.01226	0.01114	0.01721	0.01721
300	0.01303	0.01333	0.01333	0.01184	0.01885	0.01885
350	0.01372	0.0144	0.0144	0.01246	0.02036	0.02036
400	0.01434	0.01539	0.01539	0.01303	0.02177	0.02177
q(kN)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.00652	0.00597	0.00652	0.00652	0.00544	0.00652
100	0.00821	0.00845	0.00845	0.00821	0.0077	0.00821
150	0.0094	0.01035	0.01035	0.0094	0.00943	0.00943
200	0.01034	0.01195	0.01195	0.01034	0.01088	0.01088
250	0.01114	0.01336	0.01336	0.01114	0.01217	0.01217
300	0.01184	0.01463	0.01463	0.01184	0.01333	0.01333
350	0.01246	0.0158	0.0158	0.01246	0.0144	0.0144
400	0.01303	0.01689	0.01689	0.01303	0.01539	0.01539

Table 4.72: Critical thicknesses for CSSS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t_c)						
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.01142	0.01154	0.01154	0.01142	0.00715	0.01142

100	0.01438	0.01632	0.01632	0.01438	0.01011	0.01438
150	0.01646	0.01999	0.01999	0.01646	0.01238	0.01646
200	0.01812	0.02308	0.02308	0.01812	0.0143	0.01812
250	0.01952	0.0258	0.0258	0.01952	0.01599	0.01952
300	0.02074	0.02827	0.02827	0.02074	0.01751	0.02074
350	0.02184	0.03053	0.03053	0.02184	0.01891	0.02184
400	0.02283	0.03264	0.03264	0.02283	0.02022	0.02283
q(kN)	W _a = 5mm	f _y =500N/mm ²	t _c (m)	W _a = 10mm	f _y =250N/mm ²	t _c (m)
50	0.01142	0.00651	0.01142	0.00906	0.01154	0.01154
100	0.01438	0.00921	0.01438	0.01142	0.01632	0.01632
150	0.01646	0.01128	0.01646	0.01307	0.01999	0.01999
200	0.01812	0.01303	0.01812	0.01438	0.02308	0.02308
250	0.01952	0.01456	0.01952	0.01549	0.0258	0.0258
300	0.02074	0.01595	0.02074	0.01646	0.02827	0.02827
350	0.02184	0.01723	0.02184	0.01733	0.03053	0.03053
400	0.02283	0.01842	0.02283	0.01812	0.03264	0.03264
q(kN)	W _a = 10mm	f _y =415N/mm ²	t _c (m)	W _a = 10mm	f _y =500N/mm ²	t _c (m)
50	0.00906	0.00715	0.00906	0.00906	0.00651	0.00906
100	0.01142	0.01011	0.01142	0.01142	0.00921	0.01142
150	0.01307	0.01238	0.01307	0.01307	0.01128	0.01307
200	0.01438	0.0143	0.01438	0.01438	0.01303	0.01438
250	0.01549	0.01599	0.01599	0.01549	0.01456	0.01549
300	0.01646	0.01751	0.01751	0.01646	0.01595	0.01646
350	0.01733	0.01891	0.01891	0.01733	0.01723	0.01733
400	0.01812	0.02022	0.02022	0.01812	0.01842	0.01842
q(kN)	W _a = 15mm	f _y =250N/mm ²	t _c (m)	W _a = 15mm	f _y =415N/mm ²	t _c (m)
50	0.00792	0.01154	0.01154	0.00792	0.00715	0.00792
100	0.00997	0.01632	0.01632	0.00997	0.01011	0.01011
150	0.01142	0.01999	0.01999	0.01142	0.01238	0.01238
200	0.01256	0.02308	0.02308	0.01256	0.0143	0.0143
250	0.01353	0.0258	0.0258	0.01353	0.01599	0.01599
300	0.01438	0.02827	0.02827	0.01438	0.01751	0.01751
350	0.01514	0.03053	0.03053	0.01514	0.01891	0.01891
400	0.01583	0.03264	0.03264	0.01583	0.02022	0.02022
q(kN)	W _a = 15mm	f _y =500N/mm ²	t _c (m)	W _a = 20mm	f _y =250N/mm ²	t _c (m)
50	0.00792	0.00651	0.00792	0.00719	0.01154	0.01154
100	0.00997	0.00921	0.00997	0.00906	0.01632	0.01632
150	0.01142	0.01128	0.01142	0.01037	0.01999	0.01999
200	0.01256	0.01303	0.01303	0.01142	0.02308	0.02308
250	0.01353	0.01456	0.01456	0.0123	0.0258	0.0258
300	0.01438	0.01595	0.01595	0.01307	0.02827	0.02827
350	0.01514	0.01723	0.01723	0.01376	0.03053	0.03053

400	0.01583	0.01842	0.01842	0.01438	0.03264	0.03264
q(kN)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.00719	0.00715	0.00719	0.00719	0.00651	0.00719
100	0.00906	0.01011	0.01011	0.00906	0.00921	0.00921
150	0.01037	0.01238	0.01238	0.01037	0.01128	0.01128
200	0.01142	0.0143	0.0143	0.01142	0.01303	0.01303
250	0.0123	0.01599	0.01599	0.0123	0.01456	0.01456
300	0.01307	0.01751	0.01751	0.01307	0.01595	0.01595
350	0.01376	0.01891	0.01891	0.01376	0.01723	0.01723
400	0.01438	0.02022	0.02022	0.01438	0.01842	0.01842

Table 4.73: Critical thicknesses for CCCS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t_c)						
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.00946	0.00679	0.00946	0.00946	0.00527	0.00946
100	0.01192	0.0096	0.01192	0.01192	0.00745	0.01192
150	0.01365	0.01176	0.01365	0.01365	0.00913	0.01365
200	0.01502	0.01358	0.01502	0.01502	0.01054	0.01502
250	0.01618	0.01519	0.01618	0.01618	0.01179	0.01618
300	0.0172	0.01663	0.0172	0.0172	0.01291	0.0172
350	0.0181	0.01797	0.0181	0.0181	0.01395	0.0181
400	0.01893	0.01921	0.01893	0.01893	0.01491	0.01893
q(kN)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.00946	0.0048	0.00946	0.01365	0.00679	0.01365
100	0.01192	0.00679	0.01192	0.01437	0.0096	0.01437
150	0.01365	0.00832	0.01365	0.01502	0.01176	0.01502
200	0.01502	0.0096	0.01502	0.01365	0.01358	0.01365
250	0.01618	0.01074	0.01618	0.01437	0.01519	0.01519
300	0.0172	0.01176	0.0172	0.01502	0.01663	0.01663
350	0.0181	0.01271	0.0181	0.01365	0.01797	0.01797
400	0.01893	0.01358	0.01893	0.01437	0.01921	0.01921
q(kN)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.01365	0.00527	0.01365	0.01365	0.0048	0.01365
100	0.01437	0.00745	0.01437	0.01437	0.00679	0.01437

150	0.01502	0.00913	0.01502	0.01502	0.00832	0.01502
200	0.01365	0.01054	0.01365	0.01365	0.0096	0.01365
250	0.01437	0.01179	0.01437	0.01437	0.01074	0.01437
300	0.01502	0.01291	0.01502	0.01502	0.01176	0.01502
350	0.01365	0.01395	0.01395	0.01365	0.01271	0.01365
400	0.01437	0.01491	0.01491	0.01437	0.01358	0.01437
q(kN)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.00656	0.00679	0.00679	0.00656	0.00527	0.00656
100	0.00827	0.0096	0.0096	0.00827	0.00745	0.00827
150	0.00946	0.01176	0.01176	0.00946	0.00913	0.00946
200	0.01042	0.01358	0.01358	0.01042	0.01054	0.01054
250	0.01122	0.01519	0.01519	0.01122	0.01179	0.01179
300	0.01192	0.01663	0.01663	0.01192	0.01291	0.01291
350	0.01255	0.01797	0.01797	0.01255	0.01395	0.01395
400	0.01312	0.01921	0.01921	0.01312	0.01491	0.01491
q(kN)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.00656	0.0048	0.00656	0.00596	0.00679	0.00679
100	0.00827	0.00679	0.00827	0.00751	0.0096	0.0096
150	0.00946	0.00832	0.00946	0.0086	0.01176	0.01176
200	0.01042	0.0096	0.01042	0.00946	0.01358	0.01358
250	0.01122	0.01074	0.01122	0.01019	0.01519	0.01519
300	0.01192	0.01176	0.01192	0.01083	0.01663	0.01663
350	0.01255	0.01271	0.01271	0.0114	0.01797	0.01797
400	0.01312	0.01358	0.01358	0.01192	0.01921	0.01921
q(kN)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.00596	0.00527	0.00596	0.00596	0.0048	0.00596
100	0.00751	0.00745	0.00751	0.00751	0.00679	0.00751
150	0.0086	0.00913	0.00913	0.0086	0.00832	0.0086
200	0.00946	0.01054	0.01054	0.00946	0.0096	0.0096
250	0.01019	0.01179	0.01179	0.01019	0.01074	0.01074
300	0.01083	0.01291	0.01291	0.01083	0.01176	0.01176
350	0.0114	0.01395	0.01395	0.0114	0.01271	0.01271
400	0.01192	0.01491	0.01491	0.01192	0.01358	0.01358

Table 4.74: Critical thicknesses for SSSS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t_c)						
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.01612	0.01375	0.01612	0.01612	0.01067	0.01612
100	0.02031	0.01944	0.02031	0.02031	0.01509	0.02031
150	0.02324	0.02381	0.02381	0.02324	0.01848	0.02324
200	0.02558	0.0275	0.0275	0.02558	0.02134	0.02558
250	0.02756	0.03074	0.03074	0.02756	0.02386	0.02756
300	0.02928	0.03368	0.03368	0.02928	0.02614	0.02928
350	0.03083	0.03638	0.03638	0.03083	0.02823	0.03083
400	0.03223	0.03889	0.03889	0.03223	0.03018	0.03223
q(kN)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.01612	0.00972	0.01612	0.01279	0.01375	0.01375
100	0.02031	0.01375	0.02031	0.01612	0.01944	0.01944
150	0.02324	0.01684	0.02324	0.01845	0.02381	0.02381
200	0.02558	0.01944	0.02558	0.02031	0.0275	0.0275
250	0.02756	0.02174	0.02756	0.02187	0.03074	0.03074
300	0.02928	0.02381	0.02928	0.02324	0.03368	0.03368
350	0.03083	0.02572	0.03083	0.02447	0.03638	0.03638
400	0.03223	0.0275	0.03223	0.02558	0.03889	0.03889
q(kN)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.01279	0.01067	0.01279	0.01279	0.00972	0.01279
100	0.01612	0.01509	0.01612	0.01612	0.01375	0.01612
150	0.01845	0.01848	0.01848	0.01845	0.01684	0.01845
200	0.02031	0.02134	0.02134	0.02031	0.01944	0.02031
250	0.02187	0.02386	0.02386	0.02187	0.02174	0.02187
300	0.02324	0.02614	0.02614	0.02324	0.02381	0.02381
350	0.02447	0.02823	0.02823	0.02447	0.02572	0.02572
400	0.02558	0.03018	0.03018	0.02558	0.0275	0.0275
q(kN)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.01117	0.01375	0.01375	0.01117	0.01067	0.01117
100	0.01408	0.01944	0.01944	0.01408	0.01509	0.01509
150	0.01612	0.02381	0.02381	0.01612	0.01848	0.01848
200	0.01774	0.0275	0.0275	0.01774	0.02134	0.02134
250	0.01911	0.03074	0.03074	0.01911	0.02386	0.02386

300	0.02031	0.03368	0.03368	0.02031	0.02614	0.02614
350	0.02138	0.03638	0.03638	0.02138	0.02823	0.02823
400	0.02235	0.03889	0.03889	0.02235	0.03018	0.03018
q(kN)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.01117	0.00972	0.01117	0.01015	0.01375	0.01375
100	0.01408	0.01375	0.01408	0.01279	0.01944	0.01944
150	0.01612	0.01684	0.01684	0.01464	0.02381	0.02381
200	0.01774	0.01944	0.01944	0.01612	0.0275	0.0275
250	0.01911	0.02174	0.02174	0.01736	0.03074	0.03074
300	0.02031	0.02381	0.02381	0.01845	0.03368	0.03368
350	0.02138	0.02572	0.02572	0.01942	0.03638	0.03638
400	0.02235	0.0275	0.0275	0.02031	0.03889	0.03889
q(kN)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.0102	0.0107	0.0107	0.0102	0.0097	0.0102
100	0.0128	0.0151	0.0151	0.0128	0.0138	0.0138
150	0.0146	0.0185	0.0185	0.0146	0.0168	0.0168
200	0.0161	0.0213	0.0213	0.0161	0.0194	0.0194
250	0.0174	0.0239	0.0239	0.0174	0.0217	0.0217
300	0.0185	0.0261	0.0261	0.0185	0.0238	0.0238
350	0.0194	0.0282	0.0282	0.0194	0.0257	0.0257
400	0.0203	0.0302	0.0302	0.0203	0.0275	0.0275

Table 4.75: Critical thicknesses for CCCC classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t_c)						
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.01075	0.00817	0.01075	0.01075	0.00634	0.01075
100	0.01355	0.01155	0.01355	0.01355	0.00897	0.01355
150	0.01551	0.01415	0.01551	0.01551	0.01098	0.01551
200	0.01707	0.01634	0.01707	0.01707	0.01268	0.01707
250	0.01839	0.01827	0.01839	0.01839	0.01418	0.01839
300	0.01954	0.02001	0.02001	0.01954	0.01553	0.01954
350	0.02057	0.02161	0.02161	0.02057	0.01678	0.02057
400	0.0215	0.02311	0.02311	0.0215	0.01793	0.0215
q(kN)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)

50	0.01075	0.00578	0.01075	0.00853	0.00817	0.00853
100	0.01355	0.00817	0.01355	0.01075	0.01155	0.01155
150	0.01551	0.01001	0.01551	0.01231	0.01415	0.01415
200	0.01707	0.01155	0.01707	0.01355	0.01634	0.01634
250	0.01839	0.01292	0.01839	0.01459	0.01827	0.01827
300	0.01954	0.01415	0.01954	0.01551	0.02001	0.02001
350	0.02057	0.01528	0.02057	0.01633	0.02161	0.02161
400	0.0215	0.01634	0.0215	0.01707	0.02311	0.02311
q(kN)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.00853	0.00634	0.00853	0.00853	0.00578	0.00853
100	0.01075	0.00897	0.01075	0.01075	0.00817	0.01075
150	0.01231	0.01098	0.01231	0.01231	0.01001	0.01231
200	0.01355	0.01268	0.01355	0.01355	0.01155	0.01355
250	0.01459	0.01418	0.01459	0.01459	0.01292	0.01459
300	0.01551	0.01553	0.01553	0.01551	0.01415	0.01551
350	0.01633	0.01678	0.01678	0.01633	0.01528	0.01633
400	0.01707	0.01793	0.01793	0.01707	0.01634	0.01707
q(kN)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.00746	0.00817	0.00817	0.00746	0.00634	0.00746
100	0.00939	0.01155	0.01155	0.00939	0.00897	0.00939
150	0.01075	0.01415	0.01415	0.01075	0.01098	0.01098
200	0.01183	0.01634	0.01634	0.01183	0.01268	0.01268
250	0.01275	0.01827	0.01827	0.01275	0.01418	0.01418
300	0.01355	0.02001	0.02001	0.01355	0.01553	0.01553
350	0.01426	0.02161	0.02161	0.01426	0.01678	0.01678
400	0.01491	0.02311	0.02311	0.01491	0.01793	0.01793
q(kN)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.007455	0.005776	0.007455	0.006773	0.008169	0.008169
100	0.009393	0.008169	0.009393	0.008534	0.011553	0.011553
150	0.010752	0.010005	0.010752	0.009769	0.014149	0.014149
200	0.011834	0.011553	0.011834	0.010752	0.016338	0.016338
250	0.012748	0.012917	0.012917	0.011582	0.018267	0.018267
300	0.013547	0.014149	0.014149	0.012308	0.02001	0.02001
350	0.014261	0.015283	0.015283	0.012957	0.021614	0.021614
400	0.01491	0.016338	0.016338	0.013547	0.023106	0.023106
q(kN)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.00677	0.00634	0.00677	0.00677	0.00578	0.00677
100	0.00853	0.00897	0.00897	0.00853	0.00817	0.00853
150	0.00977	0.01098	0.01098	0.00977	0.01001	0.01001
200	0.01075	0.01268	0.01268	0.01075	0.01155	0.01155
250	0.01158	0.01418	0.01418	0.01158	0.01292	0.01292
300	0.01231	0.01553	0.01553	0.01231	0.01415	0.01415

350	0.01296	0.01678	0.01678	0.01296	0.01528	0.01528
400	0.01355	0.01793	0.01793	0.01355	0.01634	0.01634

Table 4.76: Critical thicknesses for CISC classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t_c)						
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$					
	$W_a = 5\text{mm}$	$f_y = 250\text{N/mm}^2$	t_c (m)	$W_a = 5\text{mm}$	$f_y = 415\text{N/mm}^2$	t_c (m)
50	0.01427	0.01058	0.01427	0.01427	0.00821	0.01427
100	0.01798	0.01496	0.01798	0.01798	0.01161	0.01798
150	0.02058	0.01832	0.02058	0.02058	0.01422	0.02058
200	0.02265	0.02116	0.02265	0.02265	0.01642	0.02265
250	0.0244	0.02365	0.0244	0.0244	0.01836	0.0244
300	0.02592	0.02591	0.02592	0.02592	0.02011	0.02592
350	0.02729	0.02799	0.02799	0.02729	0.02172	0.02729
400	0.02853	0.02992	0.02992	0.02853	0.02322	0.02853
q(kN)	$W_a = 5\text{mm}$	$f_y = 500\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y = 250\text{N/mm}^2$	t_c (m)
50	0.01427	0.00748	0.01427	0.01132	0.01058	0.01132
100	0.01798	0.01058	0.01798	0.01427	0.01496	0.01496
150	0.02058	0.01296	0.02058	0.01633	0.01832	0.01832
200	0.02265	0.01496	0.02265	0.01798	0.02116	0.02116
250	0.0244	0.01673	0.0244	0.01936	0.02365	0.02365
300	0.02592	0.01832	0.02592	0.02058	0.02591	0.02591
350	0.02729	0.01979	0.02729	0.02166	0.02799	0.02799
400	0.02853	0.02116	0.02853	0.02265	0.02992	0.02992
q(kN)	$W_a = 10\text{mm}$	$f_y = 415\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y = 500\text{N/mm}^2$	t_c (m)
50	0.01132	0.00821	0.01132	0.01132	0.00748	0.01132
100	0.01427	0.01161	0.01427	0.01427	0.01058	0.01427
150	0.01633	0.01422	0.01633	0.01633	0.01296	0.01633
200	0.01798	0.01642	0.01798	0.01798	0.01496	0.01798
250	0.01936	0.01836	0.01936	0.01936	0.01673	0.01936
300	0.02058	0.02011	0.02058	0.02058	0.01832	0.02058
350	0.02166	0.02172	0.02172	0.02166	0.01979	0.02166
400	0.02265	0.02322	0.02322	0.02265	0.02116	0.02265
q(kN)	$W_a = 15\text{mm}$	$f_y = 250\text{N/mm}^2$	t_c (m)	$W_a = 15\text{mm}$	$f_y = 415\text{N/mm}^2$	t_c (m)
50	0.00989	0.01058	0.01058	0.00989	0.00821	0.00989

100	0.01246	0.01496	0.01496	0.01246	0.01161	0.01246
150	0.01427	0.01832	0.01832	0.01427	0.01422	0.01427
200	0.0157	0.02116	0.02116	0.0157	0.01642	0.01642
250	0.01692	0.02365	0.02365	0.01692	0.01836	0.01836
300	0.01798	0.02591	0.02591	0.01798	0.02011	0.02011
350	0.01892	0.02799	0.02799	0.01892	0.02172	0.02172
400	0.01978	0.02992	0.02992	0.01978	0.02322	0.02322
q(kN)	W _a = 15mm	f _y =500N/mm ²	t _c (m)	W _a = 20mm	f _y =250N/mm ²	t _c (m)
50	0.00989	0.00748	0.00989	0.00899	0.01058	0.01058
100	0.01246	0.01058	0.01246	0.01132	0.01496	0.01496
150	0.01427	0.01296	0.01427	0.01296	0.01832	0.01832
200	0.0157	0.01496	0.0157	0.01427	0.02116	0.02116
250	0.01692	0.01673	0.01692	0.01537	0.02365	0.02365
300	0.01798	0.01832	0.01832	0.01633	0.02591	0.02591
350	0.01892	0.01979	0.01979	0.01719	0.02799	0.02799
400	0.01978	0.02116	0.02116	0.01798	0.02992	0.02992
q(kN)	W _a = 20mm	f _y =415N/mm ²	t _c (m)	W _a = 20mm	f _y =500N/mm ²	t _c (m)
50	0.00899	0.00821	0.00899	0.00899	0.00748	0.00899
100	0.01132	0.01161	0.01161	0.01132	0.01058	0.01132
150	0.01296	0.01422	0.01422	0.01296	0.01296	0.01296
200	0.01427	0.01642	0.01642	0.01427	0.01496	0.01496
250	0.01537	0.01836	0.01836	0.01537	0.01673	0.01673
300	0.01633	0.02011	0.02011	0.01633	0.01832	0.01832
350	0.01719	0.02172	0.02172	0.01719	0.01979	0.01979
400	0.01798	0.02322	0.02322	0.01798	0.02116	0.02116

Table 4.77: Critical thicknesses for CCSS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t _c)						
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$					
	W _a = 5mm	f _y =250N/mm ²	t _c (m)	W _a = 5mm	f _y =415N/mm ²	t _c (m)
50	0.01267	0.00929	0.01267	0.01267	0.00721	0.01267
100	0.01596	0.01314	0.01596	0.01596	0.0102	0.01596
150	0.01827	0.01609	0.01827	0.01827	0.01249	0.01827
200	0.02011	0.01858	0.02011	0.02011	0.01442	0.02011
250	0.02166	0.02077	0.02166	0.02166	0.01612	0.02166
300	0.02302	0.02276	0.02302	0.02302	0.01766	0.02302

350	0.02423	0.02458	0.02458	0.02423	0.01908	0.02423
400	0.02533	0.02628	0.02628	0.02533	0.0204	0.02533
q(kN)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.01267	0.00657	0.01267	0.01005	0.00929	0.01005
100	0.01596	0.00929	0.01596	0.01267	0.01314	0.01314
150	0.01827	0.01138	0.01827	0.0145	0.01609	0.01609
200	0.02011	0.01314	0.02011	0.01596	0.01858	0.01858
250	0.02166	0.01469	0.02166	0.01719	0.02077	0.02077
300	0.02302	0.01609	0.02302	0.01827	0.02276	0.02276
350	0.02423	0.01738	0.02423	0.01923	0.02458	0.02458
400	0.02533	0.01858	0.02533	0.02011	0.02628	0.02628
q(kN)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.01005	0.00721	0.01005	0.01005	0.00657	0.01005
100	0.01267	0.0102	0.01267	0.01267	0.00929	0.01267
150	0.0145	0.01249	0.0145	0.0145	0.01138	0.0145
200	0.01596	0.01442	0.01596	0.01596	0.01314	0.01596
250	0.01719	0.01612	0.01719	0.01719	0.01469	0.01719
300	0.01827	0.01766	0.01827	0.01827	0.01609	0.01827
350	0.01923	0.01908	0.01923	0.01923	0.01738	0.01923
400	0.02011	0.0204	0.0204	0.02011	0.01858	0.02011
q(kN)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.00878	0.00929	0.00929	0.00878	0.00721	0.00878
100	0.01107	0.01314	0.01314	0.01107	0.0102	0.01107
150	0.01267	0.01609	0.01609	0.01267	0.01249	0.01267
200	0.01394	0.01858	0.01858	0.01394	0.01442	0.01442
250	0.01502	0.02077	0.02077	0.01502	0.01612	0.01612
300	0.01596	0.02276	0.02276	0.01596	0.01766	0.01766
350	0.0168	0.02458	0.02458	0.0168	0.01908	0.01908
400	0.01756	0.02628	0.02628	0.01756	0.0204	0.0204
q(kN)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.00878	0.00657	0.00878	0.00798	0.00929	0.00929
100	0.01107	0.00929	0.01107	0.01005	0.01314	0.01314
150	0.01267	0.01138	0.01267	0.01151	0.01609	0.01609
200	0.01394	0.01314	0.01394	0.01267	0.01858	0.01858
250	0.01502	0.01469	0.01502	0.01364	0.02077	0.02077
300	0.01596	0.01609	0.01609	0.0145	0.02276	0.02276
350	0.0168	0.01738	0.01738	0.01526	0.02458	0.02458
400	0.01756	0.01858	0.01858	0.01596	0.02628	0.02628
q(kN)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.00798	0.00721	0.00798	0.00798	0.00657	0.00798
100	0.01005	0.0102	0.0102	0.01005	0.00929	0.01005

150	0.01151	0.01249	0.01249	0.01151	0.01138	0.01151
200	0.01267	0.01442	0.01442	0.01267	0.01314	0.01314
250	0.01364	0.01612	0.01612	0.01364	0.01469	0.01469
300	0.0145	0.01766	0.01766	0.0145	0.01609	0.01609
350	0.01526	0.01908	0.01908	0.01526	0.01738	0.01738
400	0.01596	0.0204	0.0204	0.01596	0.01858	0.01858

Table 4.78: Critical thicknesses for CSSS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t_c)						
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$					
	$W_a = 5\text{mm}$	$f_y = 250\text{N/mm}^2$	t_c (m)	$W_a = 5\text{mm}$	$f_y = 415\text{N/mm}^2$	t_c (m)
50	0.01505	0.01292	0.01505	0.01505	0.00896	0.01505
100	0.01897	0.01827	0.01897	0.01897	0.01267	0.01897
150	0.02171	0.02237	0.02237	0.02171	0.01551	0.02171
200	0.0239	0.02584	0.02584	0.0239	0.01791	0.0239
250	0.02574	0.02889	0.02889	0.02574	0.02003	0.02574
300	0.02736	0.03164	0.03164	0.02736	0.02194	0.02736
350	0.0288	0.03418	0.03418	0.0288	0.0237	0.0288
400	0.03011	0.03654	0.03654	0.03011	0.02533	0.03011
q(kN)	$W_a = 5\text{mm}$	$f_y = 500\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y = 250\text{N/mm}^2$	t_c (m)
50	0.01505	0.00816	0.01505	0.01195	0.01292	0.01292
100	0.01897	0.01154	0.01897	0.01505	0.01827	0.01827
150	0.02171	0.01413	0.02171	0.01723	0.02237	0.02237
200	0.0239	0.01632	0.0239	0.01897	0.02584	0.02584
250	0.02574	0.01825	0.02574	0.02043	0.02889	0.02889
300	0.02736	0.01999	0.02736	0.02171	0.03164	0.03164
350	0.0288	0.02159	0.0288	0.02286	0.03418	0.03418
400	0.03011	0.02308	0.03011	0.0239	0.03654	0.03654
q(kN)	$W_a = 10\text{mm}$	$f_y = 415\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y = 500\text{N/mm}^2$	t_c (m)
50	0.01195	0.00896	0.01195	0.01195	0.00816	0.01195
100	0.01505	0.01267	0.01505	0.01505	0.01154	0.01505
150	0.01723	0.01551	0.01723	0.01723	0.01413	0.01723
200	0.01897	0.01791	0.01897	0.01897	0.01632	0.01897
250	0.02043	0.02003	0.02043	0.02043	0.01825	0.02043
300	0.02171	0.02194	0.02194	0.02171	0.01999	0.02171
350	0.02286	0.0237	0.0237	0.02286	0.02159	0.02286

400	0.0239	0.02533	0.02533	0.0239	0.02308	0.0239
q(kN)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.01044	0.01292	0.01292	0.01044	0.00896	0.01044
100	0.01315	0.01827	0.01827	0.01315	0.01267	0.01315
150	0.01505	0.02237	0.02237	0.01505	0.01551	0.01551
200	0.01657	0.02584	0.02584	0.01657	0.01791	0.01791
250	0.01785	0.02889	0.02889	0.01785	0.02003	0.02003
300	0.01897	0.03164	0.03164	0.01897	0.02194	0.02194
350	0.01997	0.03418	0.03418	0.01997	0.0237	0.0237
400	0.02088	0.03654	0.03654	0.02088	0.02533	0.02533
q(kN)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.01044	0.00816	0.01044	0.00948	0.01292	0.01292
100	0.01315	0.01154	0.01315	0.01195	0.01827	0.01827
150	0.01505	0.01413	0.01505	0.01368	0.02237	0.02237
200	0.01657	0.01632	0.01657	0.01505	0.02584	0.02584
250	0.01785	0.01825	0.01825	0.01622	0.02889	0.02889
300	0.01897	0.01999	0.01999	0.01723	0.03164	0.03164
350	0.01997	0.02159	0.02159	0.01814	0.03418	0.03418
400	0.02088	0.02308	0.02308	0.01897	0.03654	0.03654
q(kN)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.00948	0.00896	0.00948	0.00948	0.00816	0.00948
100	0.01195	0.01267	0.01267	0.01195	0.01154	0.01195
150	0.01368	0.01551	0.01551	0.01368	0.01413	0.01413
200	0.01505	0.01791	0.01791	0.01505	0.01632	0.01632
250	0.01622	0.02003	0.02003	0.01622	0.01825	0.01825
300	0.01723	0.02194	0.02194	0.01723	0.01999	0.01999
350	0.01814	0.0237	0.0237	0.01814	0.02159	0.02159
400	0.01897	0.02533	0.02533	0.01897	0.02308	0.02308

Table 4.79: Critical thicknesses for CCCS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t_c)						
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 1.5$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.01233	0.00897	0.01233	0.01233	0.00696	0.01233
100	0.01553	0.01269	0.01553	0.01553	0.00985	0.01553

150	0.01778	0.01554	0.01778	0.01778	0.01206	0.01778
200	0.01957	0.01794	0.01957	0.01957	0.01393	0.01957
250	0.02108	0.02006	0.02108	0.02108	0.01557	0.02108
300	0.0224	0.02198	0.0224	0.0224	0.01706	0.0224
350	0.02358	0.02374	0.02374	0.02358	0.01842	0.02358
400	0.02465	0.02538	0.02538	0.02465	0.0197	0.02465
q(kN)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.01233	0.00634	0.01233	0.01778	0.00897	0.01778
100	0.01553	0.00897	0.01553	0.01872	0.01269	0.01872
150	0.01778	0.01099	0.01778	0.01957	0.01554	0.01957
200	0.01957	0.01269	0.01957	0.01778	0.01794	0.01794
250	0.02108	0.01419	0.02108	0.01872	0.02006	0.02006
300	0.0224	0.01554	0.0224	0.01957	0.02198	0.02198
350	0.02358	0.01678	0.02358	0.01778	0.02374	0.02374
400	0.02465	0.01794	0.02465	0.01872	0.02538	0.02538
q(kN)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.01778	0.00696	0.01778	0.01778	0.00634	0.01778
100	0.01872	0.00985	0.01872	0.01872	0.00897	0.01872
150	0.01957	0.01206	0.01957	0.01957	0.01099	0.01957
200	0.01778	0.01393	0.01778	0.01778	0.01269	0.01778
250	0.01872	0.01557	0.01872	0.01872	0.01419	0.01872
300	0.01957	0.01706	0.01957	0.01957	0.01554	0.01957
350	0.01778	0.01842	0.01842	0.01778	0.01678	0.01778
400	0.01872	0.0197	0.0197	0.01872	0.01794	0.01872
q(kN)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.00855	0.00897	0.00897	0.00855	0.00696	0.00855
100	0.01077	0.01269	0.01269	0.01077	0.00985	0.01077
150	0.01233	0.01554	0.01554	0.01233	0.01206	0.01233
200	0.01357	0.01794	0.01794	0.01357	0.01393	0.01393
250	0.01461	0.02006	0.02006	0.01461	0.01557	0.01557
300	0.01553	0.02198	0.02198	0.01553	0.01706	0.01706
350	0.01635	0.02374	0.02374	0.01635	0.01842	0.01842
400	0.01709	0.02538	0.02538	0.01709	0.0197	0.0197
q(kN)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.00855	0.00634	0.00855	0.00777	0.00897	0.00897
100	0.01077	0.00897	0.01077	0.00978	0.01269	0.01269
150	0.01233	0.01099	0.01233	0.0112	0.01554	0.01554
200	0.01357	0.01269	0.01357	0.01233	0.01794	0.01794
250	0.01461	0.01419	0.01461	0.01328	0.02006	0.02006
300	0.01553	0.01554	0.01554	0.01411	0.02198	0.02198
350	0.01635	0.01678	0.01678	0.01485	0.02374	0.02374
400	0.01709	0.01794	0.01794	0.01553	0.02538	0.02538

q(kN)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.00777	0.00696	0.00777	0.00777	0.00634	0.00777
100	0.00978	0.00985	0.00985	0.00978	0.00897	0.00978
150	0.0112	0.01206	0.01206	0.0112	0.01099	0.0112
200	0.01233	0.01393	0.01393	0.01233	0.01269	0.01269
250	0.01328	0.01557	0.01557	0.01328	0.01419	0.01419
300	0.01411	0.01706	0.01706	0.01411	0.01554	0.01554
350	0.01485	0.01842	0.01842	0.01485	0.01678	0.01678
400	0.01553	0.0197	0.0197	0.01553	0.01794	0.01794

Table 4.80: Critical thicknesses for SSSS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t_c)						
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 2$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.01775	0.01434	0.01775	0.01775	0.01113	0.01775
100	0.02236	0.02028	0.02236	0.02236	0.01574	0.02236
150	0.02559	0.02484	0.02559	0.02559	0.01928	0.02559
200	0.02817	0.02869	0.02869	0.02817	0.02226	0.02817
250	0.03035	0.03207	0.03207	0.03035	0.02489	0.03035
300	0.03225	0.03513	0.03513	0.03225	0.02727	0.03225
350	0.03395	0.03795	0.03795	0.03395	0.02945	0.03395
400	0.03549	0.04057	0.04057	0.03549	0.03149	0.03549
q(kN)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.01775	0.01014	0.01775	0.01409	0.01434	0.01434
100	0.02236	0.01434	0.02236	0.01775	0.02028	0.02028
150	0.02559	0.01757	0.02559	0.02031	0.02484	0.02484
200	0.02817	0.02028	0.02817	0.02236	0.02869	0.02869
250	0.03035	0.02268	0.03035	0.02409	0.03207	0.03207
300	0.03225	0.02484	0.03225	0.02559	0.03513	0.03513
350	0.03395	0.02683	0.03395	0.02694	0.03795	0.03795
400	0.03549	0.02869	0.03549	0.02817	0.04057	0.04057
q(kN)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.01409	0.01113	0.01409	0.01409	0.01014	0.01409
100	0.01775	0.01574	0.01775	0.01775	0.01434	0.01775
150	0.02031	0.01928	0.02031	0.02031	0.01757	0.02031

200	0.02236	0.02226	0.02236	0.02236	0.02028	0.02236
250	0.02409	0.02489	0.02489	0.02409	0.02268	0.02409
300	0.02559	0.02727	0.02727	0.02559	0.02484	0.02559
350	0.02694	0.02945	0.02945	0.02694	0.02683	0.02694
400	0.02817	0.03149	0.03149	0.02817	0.02869	0.02869
q(kN)	W _a = 15mm	f _y =250N/mm ²	t _c (m)	W _a = 15mm	f _y =415N/mm ²	t _c (m)
50	0.0123	0.01434	0.01434	0.0123	0.01113	0.0123
100	0.0155	0.02028	0.02028	0.0155	0.01574	0.01574
150	0.01775	0.02484	0.02484	0.01775	0.01928	0.01928
200	0.01953	0.02869	0.02869	0.01953	0.02226	0.02226
250	0.02104	0.03207	0.03207	0.02104	0.02489	0.02489
300	0.02236	0.03513	0.03513	0.02236	0.02727	0.02727
350	0.02354	0.03795	0.03795	0.02354	0.02945	0.02945
400	0.02461	0.04057	0.04057	0.02461	0.03149	0.03149
q(kN)	W _a = 15mm	f _y =500N/mm ²	t _c (m)	W _a = 20mm	f _y =250N/mm ²	t _c (m)
50	0.0123	0.01014	0.0123	0.01118	0.01434	0.01434
100	0.0155	0.01434	0.0155	0.01409	0.02028	0.02028
150	0.01775	0.01757	0.01775	0.01612	0.02484	0.02484
200	0.01953	0.02028	0.02028	0.01775	0.02869	0.02869
250	0.02104	0.02268	0.02268	0.01912	0.03207	0.03207
300	0.02236	0.02484	0.02484	0.02031	0.03513	0.03513
350	0.02354	0.02683	0.02683	0.02139	0.03795	0.03795
400	0.02461	0.02869	0.02869	0.02236	0.04057	0.04057
q(kN)	W _a = 20mm	f _y =415N/mm ²	t _c (m)	W _a = 20mm	f _y =500N/mm ²	t _c (m)
50	0.01118	0.01113	0.01118	0.01118	0.01014	0.01118
100	0.01409	0.01574	0.01574	0.01409	0.01434	0.01434
150	0.01612	0.01928	0.01928	0.01612	0.01757	0.01757
200	0.01775	0.02226	0.02226	0.01775	0.02028	0.02028
250	0.01912	0.02489	0.02489	0.01912	0.02268	0.02268
300	0.02031	0.02727	0.02727	0.02031	0.02484	0.02484
350	0.02139	0.02945	0.02945	0.02139	0.02683	0.02683
400	0.02236	0.03149	0.03149	0.02236	0.02869	0.02869

Table 4.81: Critical thicknesses for CCCC classical rectangular plate on aspect ratio of 2, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t _c)	
q(kN)	Aspect ration $\alpha = \frac{b}{a} = 2$

	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.01144	0.00868	0.01144	0.01144	0.00674	0.01144
100	0.01441	0.01228	0.01441	0.01441	0.00953	0.01441
150	0.0165	0.01504	0.0165	0.0165	0.01167	0.0165
200	0.01816	0.01737	0.01816	0.01816	0.01348	0.01816
250	0.01956	0.01941	0.01956	0.01956	0.01507	0.01956
300	0.02079	0.02127	0.02127	0.02079	0.01651	0.02079
350	0.02188	0.02297	0.02297	0.02188	0.01783	0.02188
400	0.02288	0.02456	0.02456	0.02288	0.01906	0.02288
q(kN)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a=10\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.01144	0.00614	0.01144	0.00908	0.00868	0.00908
100	0.01441	0.00868	0.01441	0.01144	0.01228	0.01228
150	0.0165	0.01063	0.0165	0.0131	0.01504	0.01504
200	0.01816	0.01228	0.01816	0.01441	0.01737	0.01737
250	0.01956	0.01373	0.01956	0.01553	0.01941	0.01941
300	0.02079	0.01504	0.02079	0.0165	0.02127	0.02127
350	0.02188	0.01624	0.02188	0.01737	0.02297	0.02297
400	0.02288	0.01737	0.02288	0.01816	0.02456	0.02456
q(kN)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a=10\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.00908	0.00674	0.00908	0.00908	0.00614	0.00908
100	0.01144	0.00953	0.01144	0.01144	0.00868	0.01144
150	0.0131	0.01167	0.0131	0.0131	0.01063	0.0131
200	0.01441	0.01348	0.01441	0.01441	0.01228	0.01441
250	0.01553	0.01507	0.01553	0.01553	0.01373	0.01553
300	0.0165	0.01651	0.01651	0.0165	0.01504	0.0165
350	0.01737	0.01783	0.01783	0.01737	0.01624	0.01737
400	0.01816	0.01906	0.01906	0.01816	0.01737	0.01816
q(kN)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a=15\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.00793	0.00868	0.00868	0.00793	0.00674	0.00793
100	0.00999	0.01228	0.01228	0.00999	0.00953	0.00999
150	0.01144	0.01504	0.01504	0.01144	0.01167	0.01167
200	0.01259	0.01737	0.01737	0.01259	0.01348	0.01348
250	0.01356	0.01941	0.01941	0.01356	0.01507	0.01507
300	0.01441	0.02127	0.02127	0.01441	0.01651	0.01651
350	0.01517	0.02297	0.02297	0.01517	0.01783	0.01783
400	0.01586	0.02456	0.02456	0.01586	0.01906	0.01906
q(kN)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a=20\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.00793	0.00614	0.00793	0.00721	0.00868	0.00868
100	0.00999	0.00868	0.00999	0.00908	0.01228	0.01228

150	0.01144	0.01063	0.01144	0.01039	0.01504	0.01504
200	0.01259	0.01228	0.01259	0.01144	0.01737	0.01737
250	0.01356	0.01373	0.01373	0.01232	0.01941	0.01941
300	0.01441	0.01504	0.01504	0.0131	0.02127	0.02127
350	0.01517	0.01624	0.01624	0.01379	0.02297	0.02297
400	0.01586	0.01737	0.01737	0.01441	0.02456	0.02456
q(kN)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a=20\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.00721	0.00674	0.00721	0.00721	0.00614	0.00721
100	0.00908	0.00953	0.00953	0.00908	0.00868	0.00908
150	0.01039	0.01167	0.01167	0.01039	0.01063	0.01063
200	0.01144	0.01348	0.01348	0.01144	0.01228	0.01228
250	0.01232	0.01507	0.01507	0.01232	0.01373	0.01373
300	0.0131	0.01651	0.01651	0.0131	0.01504	0.01504
350	0.01379	0.01783	0.01783	0.01379	0.01624	0.01624
400	0.01441	0.01906	0.01906	0.01441	0.01737	0.01737

Table 4.82: Critical thicknesses for CSCS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t_c)						
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 2$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.01671	0.01251	0.01671	0.01671	0.00971	0.01671
100	0.02105	0.0177	0.02105	0.02105	0.01373	0.02105
150	0.0241	0.02167	0.0241	0.0241	0.01682	0.0241
200	0.02652	0.02503	0.02652	0.02652	0.01942	0.02652
250	0.02857	0.02798	0.02857	0.02857	0.02172	0.02857
300	0.03036	0.03065	0.03065	0.03036	0.02379	0.03036
350	0.03196	0.03311	0.03311	0.03196	0.02569	0.03196
400	0.03342	0.03539	0.03539	0.03342	0.02747	0.03342
q(kN)	$W_a = 5\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.01671	0.00885	0.01671	0.01326	0.01251	0.01326
100	0.02105	0.01251	0.02105	0.01671	0.0177	0.0177
150	0.0241	0.01533	0.0241	0.01913	0.02167	0.02167
200	0.02652	0.0177	0.02652	0.02105	0.02503	0.02503
250	0.02857	0.01978	0.02857	0.02268	0.02798	0.02798
300	0.03036	0.02167	0.03036	0.0241	0.03065	0.03065
350	0.03196	0.02341	0.03196	0.02537	0.03311	0.03311

400	0.03342	0.02503	0.03342	0.02652	0.03539	0.03539
q(kN)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.01326	0.00971	0.01326	0.01326	0.00885	0.01326
100	0.01671	0.01373	0.01671	0.01671	0.01251	0.01671
150	0.01913	0.01682	0.01913	0.01913	0.01533	0.01913
200	0.02105	0.01942	0.02105	0.02105	0.0177	0.02105
250	0.02268	0.02172	0.02268	0.02268	0.01978	0.02268
300	0.0241	0.02379	0.0241	0.0241	0.02167	0.0241
350	0.02537	0.02569	0.02569	0.02537	0.02341	0.02537
400	0.02652	0.02747	0.02747	0.02652	0.02503	0.02652
q(kN)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.01158	0.01251	0.01251	0.01158	0.00971	0.01158
100	0.0146	0.0177	0.0177	0.0146	0.01373	0.0146
150	0.01671	0.02167	0.02167	0.01671	0.01682	0.01682
200	0.01839	0.02503	0.02503	0.01839	0.01942	0.01942
250	0.01981	0.02798	0.02798	0.01981	0.02172	0.02172
300	0.02105	0.03065	0.03065	0.02105	0.02379	0.02379
350	0.02216	0.03311	0.03311	0.02216	0.02569	0.02569
400	0.02317	0.03539	0.03539	0.02317	0.02747	0.02747
q(kN)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.01158	0.00885	0.01158	0.01053	0.01251	0.01251
100	0.0146	0.01251	0.0146	0.01326	0.0177	0.0177
150	0.01671	0.01533	0.01671	0.01518	0.02167	0.02167
200	0.01839	0.0177	0.01839	0.01671	0.02503	0.02503
250	0.01981	0.01978	0.01981	0.018	0.02798	0.02798
300	0.02105	0.02167	0.02167	0.01913	0.03065	0.03065
350	0.02216	0.02341	0.02341	0.02013	0.03311	0.03311
400	0.02317	0.02503	0.02503	0.02105	0.03539	0.03539
q(kN)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.01053	0.00971	0.01053	0.01053	0.00885	0.01053
100	0.01326	0.01373	0.01373	0.01326	0.01251	0.01326
150	0.01518	0.01682	0.01682	0.01518	0.01533	0.01533
200	0.01671	0.01942	0.01942	0.01671	0.0177	0.0177
250	0.018	0.02172	0.02172	0.018	0.01978	0.01978
300	0.01913	0.02379	0.02379	0.01913	0.02167	0.02167
350	0.02013	0.02569	0.02569	0.02013	0.02341	0.02341
400	0.02105	0.02747	0.02747	0.02105	0.02503	0.02503

Table 4.83: Critical thicknesses for CCSS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t_c)						
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 2$					
	$W_a = 5\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 5\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.01368	0.00999	0.01368	0.01368	0.00775	0.01368
100	0.01723	0.01412	0.01723	0.01723	0.01096	0.01723
150	0.01973	0.0173	0.01973	0.01973	0.01342	0.01973
200	0.02171	0.01997	0.02171	0.02171	0.0155	0.02171
250	0.02339	0.02233	0.02339	0.02339	0.01733	0.02339
300	0.02485	0.02446	0.02485	0.02485	0.01898	0.02485
350	0.02616	0.02642	0.02642	0.02616	0.02051	0.02616
400	0.02735	0.02824	0.02824	0.02735	0.02192	0.02735
q(kN)	$W_a= 5\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.01368	0.00706	0.01368	0.01086	0.00999	0.01086
100	0.01723	0.00999	0.01723	0.01368	0.01412	0.01412
150	0.01973	0.01223	0.01973	0.01566	0.0173	0.0173
200	0.02171	0.01412	0.02171	0.01723	0.01997	0.01997
250	0.02339	0.01579	0.02339	0.01856	0.02233	0.02233
300	0.02485	0.0173	0.02485	0.01973	0.02446	0.02446
350	0.02616	0.01868	0.02616	0.02077	0.02642	0.02642
400	0.02735	0.01997	0.02735	0.02171	0.02824	0.02824
q(kN)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.01086	0.00775	0.01086	0.01086	0.00706	0.01086
100	0.01368	0.01096	0.01368	0.01368	0.00999	0.01368
150	0.01566	0.01342	0.01566	0.01566	0.01223	0.01566
200	0.01723	0.0155	0.01723	0.01723	0.01412	0.01723
250	0.01856	0.01733	0.01856	0.01856	0.01579	0.01856
300	0.01973	0.01898	0.01973	0.01973	0.0173	0.01973
350	0.02077	0.02051	0.02077	0.02077	0.01868	0.02077
400	0.02171	0.02192	0.02192	0.02171	0.01997	0.02171
q(kN)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.00948	0.00999	0.00999	0.00948	0.00775	0.00948
100	0.01195	0.01412	0.01412	0.01195	0.01096	0.01195
150	0.01368	0.0173	0.0173	0.01368	0.01342	0.01368
200	0.01505	0.01997	0.01997	0.01505	0.0155	0.0155
250	0.01622	0.02233	0.02233	0.01622	0.01733	0.01733

300	0.01723	0.02446	0.02446	0.01723	0.01898	0.01898
350	0.01814	0.02642	0.02642	0.01814	0.02051	0.02051
400	0.01897	0.02824	0.02824	0.01897	0.02192	0.02192
q(kN)	W _a = 15mm	f _y =500N/mm ²	t _c (m)	W _a = 20mm	f _y =250N/mm ²	t _c (m)
50	0.00948	0.00706	0.00948	0.00862	0.00999	0.00999
100	0.01195	0.00999	0.01195	0.01086	0.01412	0.01412
150	0.01368	0.01223	0.01368	0.01243	0.0173	0.0173
200	0.01505	0.01412	0.01505	0.01368	0.01997	0.01997
250	0.01622	0.01579	0.01622	0.01473	0.02233	0.02233
300	0.01723	0.0173	0.0173	0.01566	0.02446	0.02446
350	0.01814	0.01868	0.01868	0.01648	0.02642	0.02642
400	0.01897	0.01997	0.01997	0.01723	0.02824	0.02824
q(kN)	W _a = 20mm	f _y =415N/mm ²	t _c (m)	W _a = 20mm	f _y =500N/mm ²	t _c (m)
50	0.00862	0.00775	0.00862	0.00862	0.00706	0.00862
100	0.01086	0.01096	0.01096	0.01086	0.00999	0.01086
150	0.01243	0.01342	0.01342	0.01243	0.01223	0.01243
200	0.01368	0.0155	0.0155	0.01368	0.01412	0.01412
250	0.01473	0.01733	0.01733	0.01473	0.01579	0.01579
300	0.01566	0.01898	0.01898	0.01566	0.0173	0.0173
350	0.01648	0.02051	0.02051	0.01648	0.01868	0.01868
400	0.01723	0.02192	0.02192	0.01723	0.01997	0.01997

Table 4.84: Critical thicknesses for CSSS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t _c)						
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 2$					
	W _a = 5mm	f _y =250N/mm ²	t _c (m)	W _a = 5mm	f _y =415N/mm ²	t _c (m)
50	0.01704	0.00921	0.01704	0.01704	0.01003	0.01704
100	0.02147	0.01303	0.02147	0.02147	0.01418	0.02147
150	0.02458	0.01595	0.02458	0.02458	0.01737	0.02458
200	0.02705	0.01842	0.02705	0.02705	0.02005	0.02705
250	0.02914	0.0206	0.02914	0.02914	0.02242	0.02914
300	0.03096	0.02256	0.03096	0.03096	0.02456	0.03096
350	0.0326	0.02437	0.0326	0.0326	0.02653	0.0326
400	0.03408	0.02605	0.03408	0.03408	0.02836	0.03408
q(kN)	W _a = 5mm	f _y =500N/mm ²	t _c (m)	W _a = 10mm	f _y =250N/mm ²	t _c (m)

50	0.01704	0.00913	0.01704	0.01352	0.00921	0.01352
100	0.02147	0.01292	0.02147	0.01704	0.01303	0.01704
150	0.02458	0.01582	0.02458	0.01951	0.01595	0.01951
200	0.02705	0.01827	0.02705	0.02147	0.01842	0.02147
250	0.02914	0.02043	0.02914	0.02313	0.0206	0.02313
300	0.03096	0.02237	0.03096	0.02458	0.02256	0.02458
350	0.0326	0.02417	0.0326	0.02587	0.02437	0.02587
400	0.03408	0.02584	0.03408	0.02705	0.02605	0.02705
q(kN)	$W_a = 10\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.01352	0.01003	0.01352	0.01352	0.00913	0.01352
100	0.01704	0.01418	0.01704	0.01704	0.01292	0.01704
150	0.01951	0.01737	0.01951	0.01951	0.01582	0.01951
200	0.02147	0.02005	0.02147	0.02147	0.01827	0.02147
250	0.02313	0.02242	0.02313	0.02313	0.02043	0.02313
300	0.02458	0.02456	0.02458	0.02458	0.02237	0.02458
350	0.02587	0.02653	0.02587	0.02587	0.02417	0.02587
400	0.02705	0.02836	0.02705	0.02705	0.02584	0.02705
q(kN)	$W_a = 15\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)	$W_a = 15\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)
50	0.01181	0.00921	0.01181	0.01181	0.01003	0.01181
100	0.01489	0.01303	0.01489	0.01489	0.01418	0.01489
150	0.01704	0.01595	0.01704	0.01704	0.01737	0.01737
200	0.01875	0.01842	0.01875	0.01875	0.02005	0.02005
250	0.0202	0.0206	0.0202	0.0202	0.02242	0.02242
300	0.02147	0.02256	0.02256	0.02147	0.02456	0.02456
350	0.0226	0.02437	0.02437	0.0226	0.02653	0.02653
400	0.02363	0.02605	0.02605	0.02363	0.02836	0.02836
q(kN)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.01181	0.00913	0.01181	0.01073	0.00921	0.01073
100	0.01489	0.01292	0.01489	0.01352	0.01303	0.01352
150	0.01704	0.01582	0.01704	0.01548	0.01595	0.01595
200	0.01875	0.01827	0.01875	0.01704	0.01842	0.01842
250	0.0202	0.02043	0.0202	0.01836	0.0206	0.0206
300	0.02147	0.02237	0.02147	0.01951	0.02256	0.02256
350	0.0226	0.02417	0.0226	0.02053	0.02437	0.02437
400	0.02363	0.02584	0.02363	0.02147	0.02605	0.02605
q(kN)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.01073	0.01003	0.01073	0.01073	0.00913	0.01073
100	0.01352	0.01418	0.01418	0.01352	0.01292	0.01352
150	0.01548	0.01737	0.01737	0.01548	0.01582	0.01582
200	0.01704	0.02005	0.02005	0.01704	0.01827	0.01827
250	0.01836	0.02242	0.02242	0.01836	0.02043	0.02043
300	0.01951	0.02456	0.02456	0.01951	0.02237	0.02237

350	0.02053	0.02653	0.02653	0.02053	0.02417	0.02417
400	0.02147	0.02836	0.02836	0.02147	0.02584	0.02584

Table 4.85: Critical thicknesses for CCCS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t_c)						
q(kN)	Aspect ratio $\alpha = \frac{b}{a} = 2$					
	$W_a = 5\text{mm}$	$f_y = 250\text{N/mm}^2$	t_c (m)	$W_a = 5\text{mm}$	$f_y = 415\text{N/mm}^2$	t_c (m)
50	0.01365	0.00999	0.01365	0.01365	0.00776	0.01365
100	0.0172	0.01413	0.0172	0.0172	0.01097	0.0172
150	0.01969	0.01731	0.01969	0.01969	0.01343	0.01969
200	0.02167	0.01998	0.02167	0.02167	0.01551	0.02167
250	0.02334	0.02234	0.02334	0.02334	0.01734	0.02334
300	0.0248	0.02447	0.0248	0.0248	0.019	0.0248
350	0.02611	0.02643	0.02643	0.02611	0.02052	0.02611
400	0.0273	0.02826	0.02826	0.0273	0.02193	0.0273
q(kN)	$W_a = 5\text{mm}$	$f_y = 500\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y = 250\text{N/mm}^2$	t_c (m)
50	0.01365	0.00707	0.01365	0.01969	0.00999	0.01969
100	0.0172	0.00999	0.0172	0.02072	0.01413	0.02072
150	0.01969	0.01224	0.01969	0.02167	0.01731	0.02167
200	0.02167	0.01413	0.02167	0.01969	0.01998	0.01998
250	0.02334	0.0158	0.02334	0.02072	0.02234	0.02234
300	0.0248	0.01731	0.0248	0.02167	0.02447	0.02447
350	0.02611	0.01869	0.02611	0.01969	0.02643	0.02643
400	0.0273	0.01998	0.0273	0.02072	0.02826	0.02826
q(kN)	$W_a = 10\text{mm}$	$f_y = 415\text{N/mm}^2$	t_c (m)	$W_a = 10\text{mm}$	$f_y = 500\text{N/mm}^2$	t_c (m)
50	0.01969	0.00776	0.01969	0.01969	0.00707	0.01969
100	0.02072	0.01097	0.02072	0.02072	0.00999	0.02072
150	0.02167	0.01343	0.02167	0.02167	0.01224	0.02167
200	0.01969	0.01551	0.01969	0.01969	0.01413	0.01969
250	0.02072	0.01734	0.02072	0.02072	0.0158	0.02072
300	0.02167	0.019	0.02167	0.02167	0.01731	0.02167
350	0.01969	0.02052	0.02052	0.01969	0.01869	0.01969
400	0.02072	0.02193	0.02193	0.02072	0.01998	0.02072
q(kN)	$W_a = 15\text{mm}$	$f_y = 250\text{N/mm}^2$	t_c (m)	$W_a = 15\text{mm}$	$f_y = 415\text{N/mm}^2$	t_c (m)
50	0.00946	0.00999	0.00999	0.00946	0.00776	0.00946

100	0.01192	0.01413	0.01413	0.01192	0.01097	0.01192
150	0.01365	0.01731	0.01731	0.01365	0.01343	0.01365
200	0.01502	0.01998	0.01998	0.01502	0.01551	0.01551
250	0.01618	0.02234	0.02234	0.01618	0.01734	0.01734
300	0.0172	0.02447	0.02447	0.0172	0.019	0.019
350	0.0181	0.02643	0.02643	0.0181	0.02052	0.02052
400	0.01893	0.02826	0.02826	0.01893	0.02193	0.02193
q(kN)	$W_a = 15\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=250\text{N/mm}^2$	t_c (m)
50	0.00946	0.00707	0.00946	0.0086	0.00999	0.00999
100	0.01192	0.00999	0.01192	0.01083	0.01413	0.01413
150	0.01365	0.01224	0.01365	0.0124	0.01731	0.01731
200	0.01502	0.01413	0.01502	0.01365	0.01998	0.01998
250	0.01618	0.0158	0.01618	0.0147	0.02234	0.02234
300	0.0172	0.01731	0.01731	0.01563	0.02447	0.02447
350	0.0181	0.01869	0.01869	0.01645	0.02643	0.02643
400	0.01893	0.01998	0.01998	0.0172	0.02826	0.02826
q(kN)	$W_a = 20\text{mm}$	$f_y=415\text{N/mm}^2$	t_c (m)	$W_a = 20\text{mm}$	$f_y=500\text{N/mm}^2$	t_c (m)
50	0.0086	0.00776	0.0086	0.0086	0.00707	0.0086
100	0.01083	0.01097	0.01097	0.01083	0.00999	0.01083
150	0.0124	0.01343	0.01343	0.0124	0.01224	0.0124
200	0.01365	0.01551	0.01551	0.01365	0.01413	0.01413
250	0.0147	0.01734	0.01734	0.0147	0.0158	0.0158
300	0.01563	0.019	0.019	0.01563	0.01731	0.01731
350	0.01645	0.02052	0.02052	0.01645	0.01869	0.01869
400	0.0172	0.02193	0.02193	0.0172	0.01998	0.01998

4.2 Discussion of Results

The results presented in section 4.1 are discussed herein

4.2.1 Discussion on Third-Order Total Energy Functional of a Classical Rectangular Plate

Previous researchers have done investigations on pure bending analysis of classical rectangular plate using total potential energy functional and the mainly used method are second order (Ritz energy function) and fourth order (Galerkin and work error energy functional). In this study the third-order total energy functional of a classical rectangular plate was derived successfully and the outcome to this was Equation (4.1).

In arriving at this result, the equations and principles of theory of elasticity were introduced, where the sequence of the formulation includes: total potential energy, kinematic, strains-deflection relationships, stress-strain (constitutive) relationship and also Kirchhoff's plate theory assumptions were satisfied.

4.2.2 Discussion on Deflection Function

We were able to obtain the general deflection equation of classical rectangular plate and this result was presented on Equation (4.2). With the help of the obtained general deflection equation, the peculiar deflection equations for the six boundary conditions of classical rectangular plates were formulated and the outcome presented on Table 4.1.

In arriving at these results, the total potential energy functional was minimized with respect to deflection to obtain the governing equation of equilibrium of forces of a rectangular plate, where the governing equation of a plate is an equilibrium equation of all forces acting on the plate. Since the load and the plate are of same displacement, then the total work performed by them is equilibrium. That means, all the forces sum are equal to zero.

Solving the governing resultant force equation gives the general deflection equation, where split-deflection approach was used to obtain exact general orthogonal polynomial deflection equation of a rectangular plate.

The solution of the governing equation is the general deflection equation and satisfying the boundary conditions of the plate results into peculiar deflection equation.

4.2.3 Discussion on Stiffness Coefficients

The established load and material stiffness function for a thin rectangular plate on Equation (4.4) to Equation (4.7) was used to get the stiffness coefficients for various boundary conditions; this was tabulated on Table 4.2.

Putting the stiffness coefficient results from Table 4.2 into Equation (4.3) for various boundary conditions and varying aspect ratios ranging from $(0.1 \leq \alpha \leq 2.0)$ gives the value of non-dimensional deflection coefficients, K , for the six plates are presented on Table 4.4.

4.2.4 Discussion on the Determination of Critical Design Parameters of Classical Rectangular Plates under Uniformly Distributed Lateral Load

From Table 4.6 the center deflection was presented for various boundary conditions with varying aspect ratios, and these were compared to those obtained from Ibearugbulem (2014) and are presented on Table 4.86 – Table 4.91. According to Table 4.86 – Table 4.91, the percentage difference between the present study and Ibearugbulem (2014) shows that their outcomes are around equivalent and the percentage differences are within the acceptable limit in engineering. This suggests that the outcomes acquired from present study concur with values from past investigations and the difference in the above results could be credited to the approximation made in both techniques.

Table 4.86: Center deflection values for different aspect ratios for (SSSS) classical rectangular plate under uniformly distributed lateral load

$K_c = \text{Maximum deflection Coefficient}$			
$\alpha = \frac{b}{a}$	Present Study	Ibearugbulem et al, (2014)	Percentage difference (%)
1.0	0.004139	0.00414	-0.02415

1.1	0.004963	0.00496	0.060484
1.2	0.005766	0.00576	0.104167
1.3	0.006534	0.00653	0.061256
1.4	0.007258	0.00735	-1.2517
1.5	0.007934	0.00793	0.050441
1.6	0.00856	0.00856	0
1.7	0.009137	0.00913	0.07667
1.8	0.009666	0.00966	0.062112
1.9	0.01015	0.01015	0
2.0	0.010593	0.01059	0.028329

Table 4.87: Center deflection values for different aspect ratios for (CCCC) classical rectangular plate under uniformly distributed lateral load

$K_c = \text{Maximum deflection Coefficient}$			
$\alpha = \frac{b}{a}$	Present Study	Ibearugbulem et al, (2014)	Percentage difference (%)
1.0	0.00133	0.00133	0
1.1	0.001587	0.00159	-0.18868
1.2	0.00182	0.00182	0
1.3	0.002026	0.00203	-0.19704
1.4	0.002204	0.00221	-0.27149
1.5	0.002356	0.00236	-0.16949
1.6	0.002486	0.00249	-0.16064
1.7	0.002596	0.00260	-0.15385
1.8	0.00269	0.00269	0
1.9	0.002769	0.00277	-0.0361
2.0	0.002838	0.00284	-0.07042

Table 4.88: Center deflection values for different aspect ratios for (CSCS) classical rectangular plate under uniformly distributed lateral load

$K_c = \text{Maximum deflection Coefficient}$			
$\alpha = \frac{b}{a}$	Present Study	Ibearugbulem et al, (2014)	Percentage difference (%)
1.0	0.001985	0.00199	-0.25126
1.1	0.002611	0.00262	-0.34351
1.2	0.003295	0.00330	-0.15152
1.3	0.004019	0.00403	-0.27295
1.4	0.004761	0.00477	-0.18868
1.5	0.005504	0.00551	-0.10889
1.6	0.006234	0.00624	-0.09615
1.7	0.006939	0.00695	-0.15827
1.8	0.007612	0.00762	-0.10499
1.9	0.008246	0.00826	-0.16949
2.0	0.008841	0.00885	-0.10169

Table 4.89: Center deflection values for different aspect ratios for (CCSS) classical rectangular plate under uniformly distributed lateral load

$K_c = \text{Maximum deflection Coefficient}$			
$\alpha = \frac{b}{a}$	Present Study	Ibearugbulem et al, (2014)	Percentage difference (%)
1.0	0.002097	0.00210	-0.14286
1.1	0.002508	0.00251	-0.07968
1.2	0.002894	0.00290	-0.2069
1.3	0.003248	0.00325	-0.06154
1.4	0.003567	0.00357	-0.08403
1.5	0.003852	0.00386	-0.20725

1.6	0.004104	0.00411	-0.14599
1.7	0.004327	0.00433	-0.06928
1.8	0.004523	0.00453	-0.15453
1.9	0.004697	0.00470	-0.06383
2.0	0.00485	0.00485	0

Table 4.90: Center deflection values for different aspect ratios for (CSSS) classical rectangular plate under uniformly distributed lateral load

$K_c = \text{Maximum deflection Coefficient}$			
$\alpha = \frac{b}{a}$	Present Study	Ibearugbulem et al, (2014)	Percentage difference (%)
1.0	0.002819	0.00282	-0.03546
1.1	0.003544	0.00354	0.112994
1.2	0.00429	0.00429	0
1.3	0.005036	0.00503	0.119284
1.4	0.005765	0.00576	0.086806
1.5	0.006467	0.00646	0.108359
1.6	0.007132	0.00713	0.02805
1.7	0.007757	0.00775	0.090323
1.8	0.00834	0.00833	0.120048
1.9	0.008879	0.00887	0.101466
2.0	0.009378	0.00937	0.085379

Table 4.91: Center deflection values for different aspect ratios for (CCCS) classical rectangular plate under uniformly distributed lateral load

$K_c = \text{Maximum deflection Coefficient}$			
$\alpha = \frac{b}{a}$	Present Study	Ibearugbulem et al, (2014)	Percentage difference (%)
1.0	0.001606	0.00159	1.006289
1.1	0.002018	0.00200	0.9
1.2	0.00243	0.00241	0.829876
1.3	0.002829	0.00280	1.035714
1.4	0.003204	0.00317	1.072555
1.5	0.00355	0.00352	0.852273
1.6	0.003864	0.00383	0.887728
1.7	0.004146	0.00411	0.875912
1.8	0.004398	0.00436	0.87156
1.9	0.004622	0.00459	0.697168
2.0	0.00482	0.00479	0.626305

Formulated equations of the critical imposed load and critical thickness of a classical rectangular plate for the serviceability limit state of deflection and ultimate limit state of stress results were presented in Equation (4.9), Equation (4.11), Equation (4.13), and Equation (4.16).

In arriving at these results, 0.5 being the center of the non-dimensional axis was used in solving the shape function in the peculiar deflection equations in Table 4.1, the solution of this statement gives the shape function at the center of the plate h_c and the results are presented in Table 4.5. Satisfying the serviceability Limit state condition of deflection ($W_{max} < W_a$) with the derived maximum deflection solution in Equation (4.8), the critical imposed load and critical thickness were obtained and presented in Equation (4.9) and Equation (4.11).

Using the total strain energy per volume and allowable strain energy of a classical rectangular plate to satisfy the ultimate limit state of stress ($\bar{U} \leq U_0$), the critical imposed load and critical thickness for ultimate limit state of stress were obtained and presented in Equation (4.13) and Equation (4.16).

For ease of usage of the critical parameter equations, values of ϕ_1 , ϕ_2 , ϕ_3 and ϕ_4 were formulated and presented in Table 4.7, Table 4.8, Table 4.9 and Table 4.10 respectively.

4.2.4 Discussion on the Numerical Studies

Numerical studies were carried out to determine the most efficient material capable of resisting a given system forces under specified conditions and vice versa.

In arriving at these results, allowable deflections $5\text{mm} \leq w_a \leq 20\text{mm}$ with an increment of 5 and material strength (f_y) 250N/mm^2 ; 415N/mm^2 ; 500N/mm^2 were specified, other physical and geometric properties were used which include: Poisson's ratio, $\mu = 0.3$; length, $a = 1000\text{mm}$; young's modulus of elasticity, $E = 207 \times 10^9 \text{N/m}^2$; unit weight of material $\varphi = 77 \text{kN/m}^3$, the range of the aspect ratios considered are, $1 \leq \frac{b}{a} \leq 2$ with an increment of 0.5.

Table 4.11, Table 4.12, aspect ratios, poisson's ratio, $\mu = 0.3$, were substituted into Equation (4.15) in getting the values of n and this was tabulated on Table 4.13.

For critical lateral imposed load numerical studies, plate thicknesses of $5\text{mm} \leq t \leq 40\text{mm}$ with an increment of 5 were considered. The specified deflections, material strength, physical and geometric properties above and Table 4.7, Table 4.9, Table 4.13 were substituted into the critical lateral imposed load equation for serviceability limit state of deflection (q_{icD}) in Equation (4.9) and also the critical lateral imposed load equation for ultimate limit state of stress (q_{icE}) in

Equation (4.13). Results of this substitutions with respect to the considered aspect ratios in the numerical examples parameters were tabulated on Table 4.14 to Table 4.31. The critical lateral imposed load was considered from Table 4.14 to Table 4.31, choosing the lesser load between the deflection and the stress loads of a specified plate thickness, aspect ratio and boundary condition. This load is said to be the critical lateral imposed load the plate thickness can withstand without failure and also satisfying the design limit state conditions and these are tabulated on Table 4.50 to Table 4.67.

For critical thickness numerical studies, lateral loads, $50\text{kN} \leq q \leq 450\text{kN}$ with an increment of 50 were considered. The specified deflections, material strength, physical and geometric properties above and Table 4.8, Table 4.10, Table 4.13 were substituted into the critical thickness equation for serviceability limit state of deflection (t_{cD}) in Equation (4.11) and also the critical thickness equation for ultimate limit state of stress (t_{cE}) in Equation (4.16). Results of this substitutions with respect to the considered aspect ratios in the numerical examples parameters were tabulated on Table 4.32 to Table 4.49. The critical thicknesses were considered from Table 4.32 to Table 4.49, choosing the larger thickness between the deflection and the stress thicknesses of a specified loads intensity, aspect ratio and boundary condition. This thickness is said to be the critical classical plate thickness that can withstand the specified lateral load intensity without failure and also satisfying the design limit state conditions and these are tabulated on Table 4.68 to Table 4.85.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

Based on the research results obtained from this present study, the third-order energy functional was derived to ease the pure bending analysis of classical rectangular plates, and also provides an alternative equation for the analysis of thin plate. This energy functional with third derivative, prove to produce the same response with the Galerkin's functional (fourth-order functional) and Ritz functional (second-order functional).

The peculiar deflected shape functions, stress coefficients, stiffness components, center deflection at different aspect ratios of various boundary conditions (SSSS, CCCC, CSCS, CCSS, CSSS, CCCS) of classical rectangular plates of this study are reliable and can be used in confidence for plate analysis.

The boundary conditions, aspect ratios, allowable deflections and material strength of classical rectangular plates which play a significant effect on the critical lateral imposed load and critical thickness were considered in the formulation of the critical design parameters (q_{ic}, t_c) of classical rectangular plate.

The critical design parameters herein are very reliable and can be used in confidence for design of classical rectangular plates.

From the numerical example results, it can be concluded that the designer can easily achieve the suitable thickness of classical rectangular plate from Table 4.68 to Table 4.85, for a specified lateral load, allowable deflection and material strength with respect to $1 \leq \frac{b}{a} \leq 2$ with an

increment of 0.5 aspect ratios, the designer can also achieve the lateral load a specified plate thickness can withstand from Table 4.50 to Table 4.67, for a specified, allowable deflection and material strength with respect of $1 \leq \frac{b}{a} \leq 2$ with an increment of 0.5 aspect ratios.

In addition, this present study (The determination of critical design parameters of classical rectangular plates under uniformly distributed lateral load) has proven to be more exact in the determination for critical thickness of classical rectangular plate a specified lateral load can withstand and also the lateral imposed load a specified classical rectangular plate thickness can withstand, under specified condition of operations.

5.2 Recommendations

- i. The derived equations for the critical parameters of classical rectangular plate in this work are recommended for practical designs.

5.3 Contributions to Knowledge

This research work “Determination of critical design parameters of classical rectangular plates under uniformly distributed lateral load” contributed the following to knowledge:

- i. This study provided critical design equations (Equation 3.346–Equation 3.349) for design functions in determining the classical plate thickness that is most efficient for withstanding a given system lateral load, as well as the lateral imposed load that a specified classical rectangular plate thickness can withstand under specified operating conditions for classical rectangular plates.

- ii. This study has presented coefficients (Table 4.5-Table 4.12) for simplified computation, for the design of economical thickness and lateral imposed load of a classical rectangular plates for SSSS, CCCC, CSCS, CCSS, CSSS and CCCS boundary conditions.
- iii. This study has increased the boundaries of knowledge on classical rectangular plate analysis especially in the areas of plates subjected to bending.

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